ROBUST OPTIMIZATION FOR PETROLEUM REFINERY PLANNING UNDER UNCERTAINTY

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RESUMO
Neste artigo, a metodologia de otimização robusta de Bertsimas and Sim (2004) foi aplicada para lidar com incertezas em preços de produtos, custos de operação, demanda e rendimento de produtos no problema de planejamento operacional de refinarias. Além disso, limites probabilísticos de violação das restrições foram calculados para ajudar o tomador de decisão em uma escolha mais apropriada do parâmetro para controlar a robustez da solução. O modelo foi implementado em AIMMS e resolvido com CPLEX 11.1. Um estudo numérico baseado em um modelo de planejamento operacional de refinarias foi utilizado para demonstrar a efetividade da abordagem robusta proposta. O valor da função objetivo é marginalmente afetado pela introdução do parâmetro para controlar o nível de proteção contra incertezas.


ABSTRACT
In this paper the robust optimization methodology of Bertsimas and Sim (2004) was applied to deal with uncertainties in prices of saleable products, operating costs, product demands, and product yields in the refinery operational planning problem. In addition, probability bounds of constraint violation were calculated to help the decision-maker in a more appropriate choice of the parameter to control robustness solution. The model was implemented in AIMMS and solved with CPLEX 11.1. A numerical study based on a refinery planning model was utilized to demonstrate the effectiveness of the robust approach proposed. The objective function value is marginally affected by introducing the parameter to control the level of protection against uncertainties.

1. Introduction

The development of global competition and search for cost reductions is forcing the refining industry to do modifications in their operations to improve their economic performance (Moro, 2003). As a result, the decisions involved in the production planning activity must be aided by decision-making systems, especially those that employ mathematical programming or optimization – for example, RPMS - Refinery and Petrochemical Modeling System (Bonner and Moore, 1979), OMEGA - Optimization Method for the Estimation of Gasoline Attributes (Dewitt et al., 1989), and PIMS - Process Industry Modeling System (Bechtel, 1993). Moro (2003) emphasizes that usage of mathematical programming can increase profits by $10 per ton of product refined.

Petroleum refineries extract and upgrade the valuable components of crude oil to produce a variety of marketable petroleum products. The refinery topology is defined by sets that specify the connections among units as well as among streams and units. Refineries are composed by process units and tanks to blend products and produce several streams of intermediate products which can be blended to specify a commercial product. A planning model for oil refineries must allow the proper selection of oil blending and destinations for each of intermediary streams to obtain the final products in quantities and qualities desired. At the same time, the production cost must be minimized or profitability must be maximized (Moro, 2000). This work focuses on the activity of refineries operational planning.

In addition, the production planning activity in the oil sector clearly involves uncertainties, since it is hard to anticipate some of the parameters that need to be taken into account, such as prices, oil supply and product demand. There are several approaches to incorporate and deal with uncertainties inherent to the refinery planning problem, for example, the fuzzy, stochastic, and robust programming techniques (Khor et al., 2008; Li et al. 2004; Neiro and Pinto, 2005; Ravi and Reddy, 1998; Ribas et al. 2008).

The robust optimization focuses on developing models that ensure the solution feasibility to the possible outcomes of uncertain parameters. Under this approach, the decision-maker is willing to accept a suboptimal solution for the nominal values in order to ensure that the solution remains feasible and near optimal when the data changes.

Many works have devoted significant efforts to develop methodologies based on robust technique to deal with optimization under uncertainty, as can be seen in Beyer and Sendhoff (2007). The first step in this direction was taken by Soyster (1973), who proposed a conservative approach that assumes that all random parameters are equal to their worst case possible value. The robust Soyster approach has recently been extensively studied and extended. Ben-Tal and Nemirovski (1998, 1999, 2000), El-Ghaoui et al. (1998) and El-Ghaoui and Lebret (1997) presented a robust method that is less conservative, but introduce a nonlinear term in the objective function.

In this paper was applied the robust optimization methodology proposed by Bertsimas and Sim (2003, 2004). These authors proposed an approach that attempts to make the trade-off between solution optimality and solution robustness more attractive. An important aspect of this method is that the new robust formulation does not add complexity to the original problem.

No refinery planning models that consider the robust optimization approach of Bertsimas and Sim (2003, 2004) were found in the literature.

The operational refinery production planning problem addressed in this paper can be stated as follows. It is assumed that the physical resources of the plant are fixed and that the associated prices, costs, and demands are externally imposed (Reklaitis, 1982). The objective is to maximize the profitability and to determine the optimal planning by computing the amount of materials that are processed at each time in each unit (Khor et al., 2008). This paper proposes a robust programming model within the framework of Bertsimas and Sim (2003, 2004) that addresses three factors of uncertainties: purchasing and operating costs and prices of saleable products (in the objective function), product demands (in the right-hand-side coefficient constraints - RHS), and product yields (in the left-hand-side coefficient constraints - LHS). Without loss of
generality, a numerical study based on the refinery planning model of Allen (1971) is utilized to demonstrate the implementation of the proposed approach.

The remainder of this paper is organized as follows. In the section 2, the Robust Optimization Framework is explained and this methodology is applied to the refinery operational planning problem in the section 3. Section 4 presents the results and discussions of the numerical example conducted in this research. Finally, section 5 reports the main conclusions of the work.

2. Robust Optimization Framework

The main idea of Bertsimas and Sim’s approach is to control the conservatism of the robust solution by introducing a parameter that can be defined by the decision-maker. Since in practice it is unlikely that all the uncertain coefficient are equal to their worst case value (such as Soyster’s method), the authors propose a less conservative approach where the decision-maker can choose the number of uncertain factors on which he wishes to be protected.

Consider the following linear optimization problem on a set of $n$ variables:

$$\begin{align*}
\text{Minimize} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{Subject to} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \forall i \\
& l_j \leq x_j \leq u_j, \forall j
\end{align*}$$ (1)

Assuming that the coefficients of the technological matrix $A$ are subject to uncertainty and modeled as a symmetric and bounded random variable $\tilde{a}_{ij}, j \in J_i^a$ that take values in the interval $[\tilde{a}_{ij}, a_{ij}, a_{ij} + \hat{a}_{ij}]$. For every constraint $i$, it is introduced a parameter $\Gamma_i^a$ (not necessarily integer) that takes values in the interval $[0, |J_i^a|]$, where $J_i^a = \{ j | \hat{a}_{ij} > 0 \}$. $\Gamma_i^a$ can be seen as parameter to adjust the model robustness against the level of the solution conservatism. If $\Gamma_i^a = 0$, the uncertainties in the parameters of the constraint $i$ are ignored (deterministic problem). On the other hand, $\Gamma_i^a = |J_i^a|$ represents the most conservative case in which all the uncertainty parameters of the constraint $i$ are considered (Soyster’s Model). So, this parameter limits the number of coefficients that simultaneously take its worst value, unlike a formulation where all values assumes their worst value.

If there is uncertainty in the independent coefficients of the constraints ($b_i$), a new variable $x_{n+1}$ can be introduced to the model, and then the model can be rewritten as

$$\begin{align*}
\text{Minimize} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{Subject to} \quad & \sum_{j=1}^{n} a_{ij} x_j - b_i x_{n+1} \leq 0 \\
& l_j \leq x_j \leq u_j, \quad 1 \leq x_{n+1} \leq 1
\end{align*}$$ (2)

The costs of the objective function can also be subject to uncertainty and modeled as a symmetric and bounded random variable $\tilde{c}_j, j \in J^c$, which take values in the range $[c_j, c_j + d_j]$.
where $J^c = \{ j \mid d_j > 0 \}$ is the set of indices $j$ with a positive deviation $d_j$. In this case, $\Gamma^c \in [0, |J^c|]$ is the parameter to control robustness in the objective function.

To deal with randomness in the objective function coefficients, RHS and LHS coefficients of constraints, Model (1) has an equivalent robust linear counterpart as follows - see Bertsimas and Sim (2003) for proofs:

Minimize $\sum_{j=1}^n c_j x_j + \lambda^c \Gamma^c + \sum_{j \in J^c} \mu^c_j$

Subject to $\sum_{j} a_{ij} x_j - b_i x_{n+1} + \lambda^a_i \Gamma^a_i + \sum_{j \in J^a} \mu^a_{ij} \leq 0$ \quad \forall \, i$

$\lambda^c + \mu^c_j \geq d_j w_j$ \quad $\forall \, j \in J^c$

$\lambda^a_i + \mu^a_{ij} \geq \tilde{a}_{ij} w_j$ \quad $\forall \, i, \forall \, j \in J^a$

$l_j \leq x_j \leq u_j$, \quad $\forall \, j \in J$

$- w_j \leq x_j \leq w_j$, \quad $\forall \, j \in J$

$1 \leq x_{n+1} \leq 1$ \quad $\forall \, i$

$w_j \geq 0$ \quad $\forall \, j \in J$

$\lambda^a_i \geq 0$ \quad $\forall \, i$

$\mu^a_{ij} \geq 0$ \quad $\forall \, i, \forall \, j \in J^a$

$\mu^c_j \geq 0$ \quad $\forall \, i, \forall \, j \in J^c$

$\lambda^c \geq 0$

The variables $\lambda^c$ and $\mu^c_j$, $\lambda^a_i$ and $\mu^a_{ij}$, quantify the system’s sensitivity to changes in the uncertain parameters $c_j$, $a_{ij}$ and $b_i$. The sum of the variables $\lambda$ and $\mu$ represents the minimum deviation.

At optimality, $w_j = |x_j^*|$ for all $j$, where $x_j^*$ is an optimal solution of the problem (3). In fact, only deviations with negative impact in the objective function are controlled, because the others led automatically to feasible solutions.

2.1. Probability Bounds of Constraint Violation

The parameter $\Gamma^a_i$ controls the trade-off between the probability of violation and the effect to the objective function of the nominal problem. If $\Gamma^a_i \in [0, |J^a|]$, then the robust solution will be feasible deterministically. Even if more than $\lfloor \Gamma^a_i \rfloor$ change, then the robust solution will be feasible with very high probability. Assuming a symmetrical distribution of random variables, Bertsimas and Sim (2003, 2004) calculated the probability that the $i^{th}$ constraint is violated, if more than $\lfloor \Gamma^a_i \rfloor$ coefficients vary. This probability can be approximated by the following expression:

$$\Pr \left( \sum_{j} \tilde{a}_{ij} x_j^* > b_i \right) \leq 1 - \Phi \left( \frac{\Gamma^a_i - 1}{\sqrt{n_i}} \right)$$

(4)

Where $n = |J^a|$, and $\Phi(\theta)$ is the cumulative distribution function of a standard normal.
The limiting (4) is particularly interesting because it helps in a more appropriate choice for \( \Gamma^a \), as shown below in the computational results.

3. Robust optimization model to refinery operational planning

Let the following deterministic operational refinery planning problem (adapted from Allen, 1971):

**Nomenclature**

*Sets and indices*
- \( J \) set of streams \( j, i \)
- \( J_{\text{feed}} \) subset of streams \( j \) that are feed flow of a process unit
- \( J_{\text{prod}} \) subset of streams \( j \) that are finished products
- \( T \) set of time periods \( t \in [0, T] \)

*Deterministic Decision Variable*
- \( x_{jt} \) production flow rate of stream \( j \) in period \( t \)

*Deterministic Parameters*
- \( p_{jt} \) sales price of finished products in period \( t \)
- \( c_{jt} \) purchasing and operating costs in period \( t \)
- \( \text{cap}_{jt} \) production capacity of process unit that is fed by the feed flow \( j \) in period \( t \)
- \( \eta_{ijt} \) yield for production of the flow \( j \) from the flow \( i \) in period \( t \)
- \( \sigma_{ijt} \) blend coefficient for production of the flow \( j \) from the flow \( i \) in period \( t \)
- \( \alpha_{ijt} \in \{0,1\} \), 1 if, in period \( t \), the flow rate \( x_j \) is produced from a split of the flow rate \( x_i \), 0 otherwise
- \( \text{prod}_{jt} \) maximum production requirement for the flow rate \( x_j \) in period \( t \)

*Robust Variables*
- \( \mu_{jt}^{\text{cost}}, \mu_{jt}^{\text{price}}, \mu_{jt}^{\text{yield}}, \mu_{jt}^{\text{prod}} \) quantify the sensitivity to the changes in cost, price, yield, and demand, respectively
- \( \lambda_{jt}^{\text{cost}}, \lambda_{jt}^{\text{price}}, \lambda_{jt}^{\text{yield}}, \lambda_{jt}^{\text{prod}} \) quantify the sensitivity to the changes in cost, price, yield, and demand, respectively

*Robust Parameters*
- \( \Gamma_{\text{cost}}, \Gamma_{\text{price}}, \Gamma_{\text{yield}}^{\text{prod}}, \Gamma_{\text{prod}}^{\text{prod}} \) parameter to adjust cost, price, yield, and demand robustness, respectively
- \( d_{\text{cost}}, d_{\text{price}}, d_{\text{yield}}^{\text{prod}}, d_{\text{prod}}^{\text{prod}} \) cost, price, yield, and demand deviations, respectively

(5)
Maximize \( \sum_{n \in T} \sum_{jt \in \text{Jprod}} (p_{jt} x_{jt} - c_{jt} x_{jt}) \)

Subject to

\[(5.1) \text{ Limits on plant capacity:} \quad x_{jt} \leq cap_{jt} \quad \forall j \in J^{\text{feed}} \subset J, t \in T,\]

\[(5.2) \text{ Mass balance for yields:} \quad x_{jt} \leq \sum_{i} \eta_{ijt} x_{it} \quad \forall j \in J, t \in T,\]

\[(5.3) \text{ Mass balance for blends:} \quad x_{it} = \sum_{j} \sigma_{ijt} x_{jt} \quad \forall i \in I, t \in T,\]

\[(5.4) \text{ Unrestricted balance:} \quad x_{it} = \sum_{j} a_{ijt} x_{jt} \quad \forall i \in I, t \in T,\]

\[(5.5) \text{ Maximum production requirement:} \quad x_{jt} \leq \text{prod}_{jt} \quad \forall j \in J^{\text{prod}} \subset J, t \in T,\]

\[(5.6) \text{ Non-negativity:} \quad x_{jt} \geq 0 \quad \forall j \in J, t \in T.\]

Mass balance constraints are in the form of equalities. There are three types of such constraints: yields, blends, and unrestricted balances. The production requirement constraints are directly impacted by the market demand for the final refinery products. Therefore, in the robust model, the random decision variables will be introduced into these constraints. For this model, there are no restrictions on crude oil availability or minimum production required.

The robust formulation for Model (5) to account for the uncertainty in prices and costs (economic risk) and in product yields and market demand (operational risk) is presented below.

**Costs/Prices randomness (in the objective function)**

The parameters \( \Gamma_{\text{price}} \) and \( \Gamma_{\text{cost}} \) control the effect of uncertainty in the objective function and take values in the intervals \([0, |J_{\text{price}}|]\) and \([0, |J_{\text{cost}}|]\), where \( J \) is the set of uncertain coefficients with a deviation \( d \).

**Yield randomness (in LHS constraint coefficients)**

Uncertainty in yields introduces randomness in mass balance for yields constraints (5.2). The parameter \( \Gamma_{\text{yield}} \) assumes values in the interval \([0, |J_{\text{yield}}|]\), where \(|J_{\text{yield}}|\) equals to the number of different types of feed flow of the process unit. The yield deviation is represented by \( \eta_{ijt} \).

**Demand randomness (in RHS constraint coefficients)**

Finally, incorporating uncertainty in market demand introduces randomness in constraints of maximum production requirement (5.5). The parameter \( \Gamma_{\text{prod}} \) adjusts the demand robustness and \( \text{prod}_{jt} \) represents the demand deviation and take values in the interval \([0,1]\).

Model (5) has a robust formulation as follows:
Maximize
\[ \sum_{n \in T} \sum_{j \in J} \sum_{j \in J} (p_j x_{jt} - c_j x_{jt}) - \lambda \sum_{n \in T} \sum_{j \in J} \mu_{\text{price}} \sum_{j \in J} x_{jt} - \lambda \sum_{n \in T} \sum_{j \in J} \mu_{\text{cost}} \sum_{j \in J} x_{jt} \]
Subject to
\[ (6.1) \quad x_{jt} = x_{jt} + \lambda_{\text{prod}} x_{jt} + \mu_{\text{prod}} \mu_{\text{prod}} x_{jt} - \lambda_{\text{prod}} x_{jt} - \mu_{\text{prod}} \mu_{\text{prod}} x_{jt} \leq 0 \quad \forall j \in J_{\text{prod}}, i = |J_{\text{prod}}| + 1, t \in T, \]
\[ (6.2) \quad \sum_{i} \eta x_{jt} \geq x_{jt} + \lambda_{\text{prod}} \mu_{\text{prod}} x_{jt} + \mu_{\text{prod}} \mu_{\text{prod}} x_{jt} \quad \forall j \in J_{\text{prod}}, t \in T, \]
\[ (6.3) \quad \lambda_{\text{prod}} + \mu_{\text{prod}} \mu_{\text{prod}} \geq \prod_{\text{prod}} x_{jt} \quad \forall j \in J_{\text{prod}}, t \in T, \]
\[ (6.4) \quad \lambda_{\text{prod}} + \mu_{\text{prod}} \mu_{\text{prod}} \geq \prod_{\text{prod}} x_{jt} \quad \forall j \in J_{\text{prod}}, t \in T, \]
\[ (6.5) \quad \lambda_{\text{price}} + \mu_{\text{price}} \mu_{\text{price}} \geq \prod_{\text{price}} x_{jt} \quad \forall j \in J_{\text{price}}, t \in T, \]
\[ (6.6) \quad \lambda_{\text{cost}} + \mu_{\text{cost}} \mu_{\text{cost}} \geq \prod_{\text{cost}} x_{jt} \quad \forall j \in J_{\text{cost}}, t \in T, \]
\[ (6.7) \quad x_{jt} \geq 0 \quad \forall j \in J_{\text{prod}}, t \in T, \]
\[ (6.8) \quad x_{jt} = 1 \quad i = |J_{\text{prod}}| + 1, \]
\[ (6.9) \quad \mu_{\text{price}} \geq 0 \quad \forall j \in J_{\text{price}}, t \in T, \]
\[ (6.10) \quad \mu_{\text{cost}} \geq 0 \quad \forall j \in J_{\text{cost}}, t \in T, \]
\[ (6.11) \quad \mu_{\text{prod}} \geq 0 \quad \forall j \in J_{\text{prod}}, t \in T, \]
\[ (6.12) \quad \mu_{\text{prod}} \geq 0 \quad \forall j \in J_{\text{prod}}, t \in T, \]
\[ (6.13) \quad \lambda_{\text{price}}, \lambda_{\text{cost}}, \lambda_{\text{prod}}, \mu_{\text{prod}} \mu_{\text{prod}} \geq 0 \]

Given constraints (5.1), (5.3), (5.4), and (5.6).

4. Numerical Example
In this work, a numerical study based on a representative example drawn from the literature (Allen, 1971) is presented and solved in the optimality to demonstrate the effectiveness of implementing the robust model. This base model was also adopted in the work of Khor et al. (2008) and Ravi and Reddy (1998) which employed, respectively, robust stochastic programming and fuzzy programming to account for uncertainty.

The refinery topology in study is presented in Figure 1. The refinery begins with the Crude Distillation Unit (CDU) for fractionation of crude oils into separate hydrocarbon groups. The resultant products are directly related to the characteristics of the crude processed. Most distillation products are further converted into more usable products by changing the size and structure of the hydrocarbon molecules through cracking.

As shown in the Figure 1, the linear model case study represents a refinery that consists of three units: crude distillation (CDU), cracking and blending. The refinery processes crude oil (x_1) to produce gasoline (x_2), naphtha (x_3), jet fuel (x_4), heating oil (x_5), and fuel oil (x_6). The base case model has an objective function (OF) to total daily profit maximization. Besides of purchasing cost of crude oil and refinery products prices, the OF considers operating costs of CDU and cracker unit. There is no cost associated with blending. The deterministic objective function of the model is given by:

Maximize \[ z = -8.0 x_1 + 18.5 x_2 + 8.0 x_3 + 12.5 x_4 + 14.5 x_5 + 6.0 x_6 - 1.5 x_{14} \] (7)

The optimal objective function value (maximum profit) is equal to $23,387.50/ day.
The robust formulation to account for uncertainty in the objective function coefficients considers 10% of positive deviation in the costs coefficients and 10% of negative deviation in the prices coefficients from the expected value. The parameters $\Gamma_{\text{price}}$ and $\Gamma_{\text{cost}}$ take values in the intervals $[0, 5]$ and $[0, 2]$, respectively.

In addition, the demand and yield uncertainty robust dimensions are constructed based on, respectively, 5% and 1% of negative deviation from the nominal value. For these approaches, $|J|$ equals to 1.

In this work, only the yields of products from distillation unit were controlled. To ensure that material balances are satisfied, yield for residuum is determined by subtracting the sum of yields for the other four products from distillation unit.

### 4.1. Computational Results and Discussion

The model was implemented in AIMMS (Advanced Integrated Multidimensional Modeling Software - Bisschop and Roelofs, 2007) and solved with CPLEX 11.1.

First, the experiments were conducted for each of the uncertain parameters (yield, demand, cost, and price) in a separately way. The robust parameter $\Gamma$, was varied in integer values into the interval $[0, |J|]$. After, the four uncertain parameters were combined.

From Table (1), comparing the objective function value of the four types of uncertainties in the Soyster case ($\Gamma_j = |J_j|$), it can be concluded that price deviation has the highest impact in the profit, followed by cost, yield and demand, respectively. Computational results also showed that yield deviation has higher impact in the objective function (17.44% lower than the deterministic solution) than demand deviation (only 3.09%). This also can be seen in the figures below.

**Table 1. Objective function value for four types of uncertainties in the Soyster case ($\Gamma_j = |J_j|$)**

<table>
<thead>
<tr>
<th>Uncertain Parameter</th>
<th>Yield</th>
<th>Demand</th>
<th>Cost</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective function value</strong></td>
<td>19,308.55</td>
<td>22,665.0</td>
<td>13,783.52</td>
<td>11,924.15</td>
</tr>
</tbody>
</table>

The Figures 2a and 2b show the computational results for price and cost deviations only and combined with yield and demand uncertainties. From these figures, it can be seen that prices deviations impacted more the objective function than costs deviations. In fact, considering only
price and cost uncertainties, the solution for \( \Gamma_{\text{price}} = 5 \) is 49.01\% worst than the deterministic solution, while for \( \Gamma_{\text{cost}} = 2 \) is 41\% worst.

![Figure 2a. Objective function variation with \( \Gamma_{\text{price}} \)](image)

The solutions for \( \Gamma_{\text{cost}} = 1 \) and \( \Gamma_{\text{cost}} = 2 \) are almost constant, because the costs related to the crude distillation unit impact more the objective function than the ones related to cracker unit, as can be seen in the deterministic objective function (expression 7).

The results also considering yield and demand randomness combine the results for these uncertainties with the solutions for price and cost deviation. As a consequence, it gradually deteriorates the robust solution. Moreover, it is interesting to note that the objective function value is marginally affected by the increase in the level of protection, i.e., the increments decrease each time \( \Gamma \) increases. This is a feature of robust formulation and independent of the problem treated.

Additionally, when \( \xi_{\text{yield}} = 0 \) the additional production to protect against the largest deviation in the uncertain parameters is attributed to \( \lambda_{\text{yield}} \). Since \( \lambda_{\text{yield}} \) is multiplied by zero in yield robust constraint (6.2), this variable can take any value to satisfy the minimum deviation constraint (6.4). Similar analysis can be done for demand, price and cost uncertainties.
Tests were also done for uncertainties in both costs and prices. Figure 3 presents the computational results for $\Gamma_{\text{cost}} = 2$. The solutions with $\Gamma_{\text{cost}} = 1$ were around 1% closer to the nominal solution. Results considering the four types of randomness varied from 62% to 80% of deterministic solution. The worst case solution for price, yield, and demand uncertainties, shown in the Figure 2a, led to a deviation of 59% from the nominal value. This indicates a high impact of the combination of costs with other uncertainties in the objective function.

![Figure 3. Objective function variation with $\Gamma_{\text{price}}$ ($\Gamma_{\text{cost}} = 2$)](image)

Figure 4 shows the computational results for variations in price deviations for the case study considering the four types of randomness - case (m) of Figure 3 that considers 10% of price deviation. These results proved the model sensitivity to deviations from nominal value.

![Figure 4. Objective function variation with different price deviations ($\Gamma_{\text{cost}} = 2$)](image)

From the results above, it can be concluded that the consideration of uncertain parameters gradually deteriorates the objective function value. On the other hand, ignoring uncertainty of parameters in planning problems can yield suboptimal or infeasible decisions.
Thus, in order to properly evaluate the added-value of including uncertainty to the model, the robust solution was applied to the deterministic objective function to calculate the total profit. The difference between the deterministic solution considering the worst case value of the uncertain parameters and the robust solution using the deterministic objective function quantify the benefit of incorporating uncertainty in the different model parameters for the operational planning problem. It also can be evaluated as the cost of ignoring uncertainty in the problem and it takes place. In this work, these benefits have reached $1,747.03 for the case including four types of uncertainty (case m). It represents 7.47% of the deterministic solution of the original problem, which indicates that the robust model can provide good gains to the refinery operational planning.

**Probability Bounds of Constraint Violation**

The robust model does not account for a decision-maker risk behavior. For this reason, a more realistic approach should include a measure of the degree of solution conservatism/reliability. An interesting measure is the probability of constraint violation, which helps in a more appropriate choice for $\Gamma_i$, such as showed in the expression (4).

Table (2) presents the results for choice of $\Gamma_{\text{price}}$ as a function of the probability bound. A range of probability levels was evaluated to provide the decision maker the tradeoff between robustness and profit. Such as the maximum profit decreases when the degree of conservatism increase, the problem is solved for the different integer values of the price parameter ($\Gamma_{\text{price}} \in [0,5]$) and then, the decision-maker can judge the tradeoff between the conservatism and total profit in order to choose $\Gamma_{\text{price}}$ value.

**Table 2. Choice of $\Gamma_{\text{price}}$ value as a function of the probability of constraint violation**

<table>
<thead>
<tr>
<th>Probability of constraint violation</th>
<th>0.02</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{\text{price}}$</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

From Table (2), if the decision-maker is only able to accept 2% of probability of constraint violation, then he has to use $\Gamma_{\text{price}} = 5$ to protect himself against price parameter deviations. On the other hand, if the decision-maker accepts up to 60% probability, there is no necessity of any additional protection from the nominal problem and $\Gamma_{\text{price}}$ can be set in zero.

**5. Conclusions**

This work focused on a systematic methodology for developing robust programming models for the operational refinery planning problem by simultaneously accounting for uncertainties in costs, prices, product demands, and product yields. It was shown that the Bertsimas and Sim approach can be a useful tool for modeling planning problems without introducing additional computational complexity. In addition, probability bounds of constraint violation were calculated to help the decision-maker in a more appropriate choice of the parameter to control robustness. A numerical example was utilized to demonstrate the implementation of the proposed approach. The results showed the effectiveness of the proposed approach.

While in this paper the robust method was applied to hypothetical conditions, extensions to real world systems with similar structure are straightforward. For future works, this research could be extended to actual refineries, including discrete decisions and nonlinearity present in the influence of the unit's operational variables on the properties and product flow rates.
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