A ROBUST APPROACH FOR THE BIODIESEL SUPPLY CHAIN BASED ON SMALL FARMERS

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ABSTRACT

The strategy adopted by the Brazilian Biodiesel Program is to base its fuel production on grains acquired from small, family-owned farms in the poorest regions of the country, thereby fostering social inclusion and transfer of income. The success of the program depends on the development of a robust supply chain logistic structure, appropriate distribution of crop production, and investments in new grain crushing units. This article presents the development of a mathematical model for optimizing the production arrangements for the supply of a biodiesel plant sourced from family farms, taking into account agricultural, logistic and industrial aspects and the uncertainties inherent. A computationally efficient approach is proposed in order to deal with the problem stochasticity and cope with the investor's risk tolerance profile. The model was successfully applied to a case study for the production chain for biodiesel fuel from castor oil in the semi-arid region of Brazil.

KEYWORDS. Biodiesel Supply Chain, Facility Location, Robust Optimization.

1. Introduction

The dissemination of the production and use of vegetable oils as a substitute for fossil fuels is a global phenomenon, yet there exist different motivations for such dissemination in different regions of the world. In the United States, the major concern is the establishment of an energy matrix that decreases US dependence on foreign petroleum. In Europe, the main motivation is to guarantee the environmental sustainability of economic growth by coping with strong fossil fuel substitutions and emission reduction targets. In Brazil, the federal government's focus has been concerned with the potential social benefits of biodiesel production. The main idea is that the adoption of a biodiesel production based on small farms would increase employment and therefore improve the income distribution in regions with lower degrees of development. Nevertheless, biodiesel production sustainability may become very fragile when it is based on suppliers that consist only of small family farms. In such cases, the supply chain might be composed of a large number of widely distributed suppliers, with low technology equipment and thus low productivity levels.

Family farming in Brazil plays a crucial economic role, accounting for around 20% of the national agricultural GDP. However, in most parts of the country, rural workers live and work in poor conditions and have low income levels and limited access to public resources. In response to this situation, the Brazilian government has singled out social benefits as the main strategic target of the Brazilian Biodiesel Program, seeking to integrate these small farmers into the production chain as suppliers of raw materials. This strategy can provide better conditions for the effective transfer of income and improved living conditions in rural regions.

In view of the forecasted growth of the biodiesel demand for the coming years, and the lack of adequate facilities, the continued success of the program requires significant investments in structuring the supply chain, to ensure efficient conditions for the production, transportation, and processing of raw materials. It is thus important to consider the agricultural, industrial, and logistic parameters of the system as a whole. For this reason, the supply chain must be carefully planned in order to reach the best logistic efficiency concerning transportation and raw material valuation. Such observation motivates the study of new industrial configurations in order to meet the projected growth for the national biodiesel production from vegetable oils.

The purpose of this article is to present an integrated analysis of the supply chain of vegetable oils for the production of biodiesel fuel, sourced from small farms, taking into account the production, transport and crushing of grains and the transport of vegetable oils to the biodiesel production units. Such analysis consists of decisions regarding production arrangements for supplying biodiesel producing plants, taking into account agricultural, industrial, and logistical aspects and inherent risks regard to these aspects. In order to help the decision making process, we propose an optimization model based on robust mathematical programming as a tool to support such decisions.

An adjustable robust approach is proposed to evaluate which of several risks tolerance profiles suits best the investor's expectations. We show that this model can be converted into a single level robust counterpart, which consists of a mixed-integer linear problem, and thus can be efficiently solved by off-the-shell branch-and-cut software. The uncertainty of the model is due to the productivity rate variability of the grain producers, mainly due to climatic conditions and rainfall. The decisions are the sizing, location, and processing technology of the crushing plants along the planning horizon, the land allocation for oilseed production, as well as the logistics of transportation among the supply chain in each period of the planning horizon. The model was applied to a real Brazilian case study for biodiesel production from castor oil.

2. Literature Review

Biodiesel is a biodegradable fuel derived from renewable sources. It can be produced from vegetable oils, residual oils, and fats. Conventionally, biodiesel is produced through a transesterification reaction of a natural oil triglyceride (animal fat or vegetable oil) with a short

chain alcohol (methanol or ethanol) in the presence of a catalyst, usually sodium or potassium hydroxide (Fernando et al., 2007). Glycerine and cakes are significant and valuable co-products of its production (Raneses, et al., 1999).

In recent years, the literature has continuously highlighted the importance of biodiesel (see Zhang, et al., 2003; Demirbas and Balat, 2006; Carrareto, et al., 2004) because of its special characteristics. Specifically, biodiesel can replace petroleum diesel in internal combustion engines without requiring major adjustments, its impact on engine performance is small, it releases fewer sulfates (almost zero emissions), and it has a small net contribution of CO_2 considering its whole life-cycle (cultivation, oil production, and conversion to biodiesel). For these reasons, there have been several campaigns designed to introduce and promote the use of biodiesel in many countries.

The European Union is the world leader in the development of biodiesel production (Figure 1). In Europe, biodiesel fuel represents around 80% of the total volume of biofuels produced.

After the European Union, the United States is the world's largest producer of biodiesel, with 176 plants currently in operation and a production capacity of around 2.61 billion gallons a year. There are a further 39 construction or expansion projects currently being built.
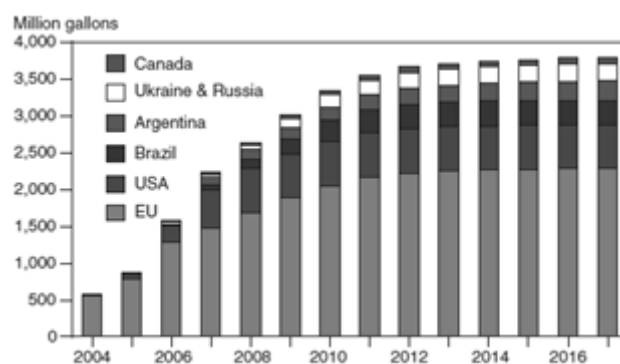

Figure 1 - Projections for biodiesel production worldwide (Source: USDA 2008)

This study aims to use the mathematical programming techniques to support decision making in the biodiesel supply chain from the perspective of optimization, taking into consideration the uncertainties inherent to the problem at hand and the desired characteristics of the optimal solution. To reach this objective, we used robust programming.

The first steps in the development of robust models appeared in the work of A. Soyster (1973), in the first half of the 1970s. Since then, several works were published regarding how to obtain solutions that remain feasible, regardless of the actual realizations of the uncertain variable conditions (Ben-Tal and Nemirovski, 1998; Ben-Tal and Nemirovski, 1999; Bertsimas and Sim, 2004; Bertsimas and Thiele, 2004; Bertsimas and Thiele, 2006). Another common approach in the literature consists of defining methodologies that deal with the robustness of the objective function value (Butler et al., 2004; Gutiérrez and Kouvelis, 1996; Kouvelis and Yu, 1997). Both approaches are commonly known in literature as robust optimization.

Specifically in supply chain network planning, robust optimization methods has become subject of interest for a number of researchers, since it become possible to address the uncertainties inherent of such systems. Critical parameters, such as prices, demands and resources availabilities, are quite uncertain. Unless the supply chain is designed to be robust with respect to its operating conditions, the impact of the uncertain parameters may cause significant damage to the system as a whole.

Daskin and Owen (1998) show in their literature review that a large number of works have addressed supply chain planning in a tactical level (i.e., involving decisions such as the aggregate quantities and material flows for purchasing, processing and production of products). However, research addressing the comprehensive design of supply chain networks under uncertainty is significantly smaller in number (MirHassani, et al., 2000).

When it comes to strategic (and tactical) supply chain planning applied to biodiesel production, an even smaller number of works address this theme. The studies found in the literature

that employ operational research techniques for planning the biodiesel chain are generally more focused on defining levels of subsidies, crops pricing and land allocation for oilseed production (see Bard, et al., 2000; Rozakis and Sourie, 2005). Such studies utilize economic models supported by simulation systems. Another line of research on biodiesel production is concerned with defining aspects of the production process itself, which covers questions involving the optimization of certain aspects of the chemical conversion processes applied (Vasudevan and Briggs, 2008).

Because it is a still very embryonic research field, we were not able to find in the literature any other studies addressing the same problem using either stochastic programming or robust optimization as decision support tools.

3. Problem Description

The focus of this paper consists of planning the supply chain network for a biodiesel supply chain based on small farms. Biodiesel supply chains that use oilseeds as raw material are commonly composed of four major layers, namely raw material producers, crushing plants, biodiesel plants and biodiesel distribution.

The first layer represents the raw material producers, which, in the case of this study, are typically small farms, widely spread with low production capacity. These characteristics are the main difficulties faced during the development of a logistic plan for the biodiesel supply chain. The transportation costs between the farms and the crushing plants might be prohibitive if the production could not be consolidated somehow in certain strategic points. For the sake of simplicity, in this paper we assume that each production zone has a predefined consolidation point from which the seeds are transported to the crushing plants.

The next layer consists of the crushing plants, where the raw material is submitted to a first transformation process. Such a crushing process can be carried out by means of different technologies, each one with specific production yields and operational costs. The most common techniques applied for oilseed crushing are mechanical crushing, which uses primarily mechanical compression to extract the oil from the seeds, and solvent based extraction, which employs chemical solvents in the process and enables improved production yields at the expense of increased operational costs.

The following layer is composed of the biodiesel plants, where the oil obtained from the seeds at the crushing plants are converted into the biodiesel itself. Diesel can also be mixed with the biodiesel in such plants, in order to cope with legal specifications for the diesel commercialized over the country.

A fourth layer is composed by the facilities related to the distribution of the biodiesel are located. Such facilities are not taken into consideration in this study. Figure 2 shows a schematic representation of the layers that compose the supply chain arrangement, as well as the links among them.
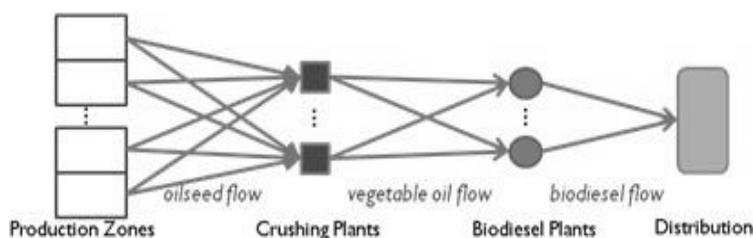

Figure 2 - Oilseed-based biodiesel supply chain

The planning framework proposed in this paper is based on the assumption that the biodiesel plants are already located and it is not possible to change their locations or sizes. On the other hand, there are still several decisions that must be made concerning the location of the crushing plants, the crushing technology applied, and its size and - i.e., the processing capacity of the crushing plant. Another important decision that must be made here is related to the sizes of the production zones. The productivity of each farm is directly related to its size, which is directly

proportional to the number of families allocated to it. Thus, the decisions concerning the number of families allocated to each farm and the total size of each production zone are strongly correlated.

Finally, after defining the sizes of the farms and the configuration of the crushing plants, it is necessary to determine the flows of raw material and intermediate products - namely seeds and vegetable oil - along the supply chain.

4. Notation

For the sake of clarity, this section presents the notation to be used hereafter in this paper. The additional elements specific to each model will be fully detailed in their respective sections.

Sets	Index
Time period	t
Crushing plant project	k
Production zone	p
Crushing plant possible location	c
Biodiesel plant	u
Scenario	s

Parameters	Description	Unit
α	oil percentage in the oilseed	unitless
β_k	crushing efficiency of crushing plant project k	unitless
γ_p^t	oilseed productivity at production zone p at period t	ton/hectare
AZ_p^t	average land size at production zone p at period t	hectare
CC_k	unitary crushing cost of the crushing plant project k	\$/ton
D_u^t	biodiesel plant u vegetable oil demand at period t	m³
$GTC_{p,c}$	unitary oilseed transportation cost between the production zone p and the crushing plant c	\$/ton
IC_k^t	crushing plant project k installation cost at period t	\$
$OTC_{c,u}$	unitary vegetable oil transportation cost between crushing plant c and the biodiesel plant u	\$/m³
PC_p	unitary oilseed production cost at production zone p	\$/ton
TF^t	minimum number of families to be allocated, and at period t	dimensionless
$\overline{W}_k$	annual capacity of the crushing plant project k	ton/year
$\bar{Z}_p^t$	production zone p total available area at period t	hectare

Variables	Description	Unit	Domain
$y_{c,k}^t$	decision of implementing the crushing plant project k at location c, and at period t	unitless	{0,1}
z_p^t	size of area p allocated for oilseed production at period t	hectare	$\mathbb{R}^+$
$gx_{p,c}^t$	amount of oilseed transported from the production zone p to the crushing plant c at period t	ton	$\mathbb{R}^+$
$ox_{c,u}^t$	amount of vegetable oil transported from the crushing plant p to the biodiesel plant u at period t	m³	$\mathbb{R}^+$
$w_{c,k}^t$	amount of oilseed crushed by the crushing plant project k at location c, and at period t	ton	$\mathbb{R}^+$

5. Deterministic Linear Problem

The problem considered in this paper can be modeled as a mixed integer linear program, disconsidering the stochastic nature of the land productivity γ_p^t. Thus, the deterministic model can be stated as follows:

$$\min_{\substack{y \in \{0,1\} \\ gx \geq 0 \\ ox \geq 0 \\ z \geq 0 \\ w \geq 0}} \sum_{c,k,t} IC_k^t y_{c,k}^t + \sum_{p,c,t} GTC_{p,c} \, gx_{p,c}^t + \sum_{c,u,t} OTC_{c,u} \, ox_{c,u}^t + \sum_{p,t} PC_p z_p^t + \sum_{c,k,t} CC_k w_{c,k}^t \quad (1)$$

$$s.t. \quad \sum_c x_{c,u}^t \geq D_u^t \qquad\qquad \forall \, u, t \quad (2)$$

$$z_p^t \leq \bar{Z}_p^t \qquad\qquad \forall \, p, t \quad (3)$$

$$\sum_p z_p^t / AZ_p^t \geq TF^t \qquad\qquad \forall \, t \quad (4)$$

$$\sum_c gx_{p,c}^t = \gamma_p^t z_p^t \qquad\qquad \forall \, p, t \quad (5)$$

$$\sum_p gx_{p,c}^t = \sum_k w_{c,k}^t \qquad\qquad \forall c, t \quad (6)$$

$$\sum_u ox_{c,u}^t = \sum_k \alpha \beta_k w_{c,k}^t \qquad\qquad \forall \, c, t \quad (7)$$

$$w_{c,k}^t \leq \bar{W}_k \sum_{t' \leq t} y_{c,k}^{t'} \qquad\qquad \forall c, k, t \quad (8)$$

The model (1) - (8) stated above is the deterministic optimization model for biodiesel supply chain planning. The integer decisions are the investment profile along the time horizon, regarding the location, technology and size of the crushing plants. There is no restriction on the number of plants to be installed (i.e., the number of investments done all over the planning horizon).

The other decisions are those made regarding logistics aspects of the biodiesel production, regarding product flows, and production allocation as well. In (1) - (8) there is a source uncertainties, exclusively related to the land production yield Γ_p^t, which is assumed to be a well-known deterministic value in the above model.

The objective function (1) considers the total cost of the investments, oilseed and vegetable oil transportation, land production, and oilseed crushing. Constraint (2) states that the vegetal oil demand D_u^t of each biodiesel plant u must be satisfied by the vegetable oil produced among the crushing plants; constraint (3) consists of an upper bound on the total area z_p^t to be allocated for the production of the oilseed in each production zone p; constraint (4) states that the total of families allocated among all the production zones p must meet a target defined by federal social policies, which is greater or equal than a minimum family allocation requirement TF^t. It is worthwhile to point out that the number of the families to be allocated at the production zone p is defined by the term z_p^t / AZ_p^t, considering that each family must be allocated to a farm with average size $\$AZ_p^t$; constraint (5) states that the total amount of oilseeds that leaves a production zone p is equal to the total area z_p^t dedicated to the production of oilseed times the production yield per area unit γ_p^t; constraint (6) requires that the total amount of oilseeds allocated to a certain crushing plant located at location c must be equal to the total amount of oilseeds crushed at the same crushing plant; constraint (7) states that the total amount of vegetable oil produced in a certain crushing plant c is equal to the total amount of oilseeds crushed times the oil percentage α of the oilseed and the efficiency β_k of the crushing process defined in project k used by the crushing plant located at c; constraint (8) states that the total available capacity of a crushing plant located at c is defined by the

decision to implement a certain crushing plant project k since the time period t times the total crushing capacity $\overline{W}_k$ specified in this project.

6. Robust Approach

As mentioned before, the problem approached in this paper has inherent uncertainties regarding the productivity rate variability of the grain producers, mainly due to climatic conditions and rainfall. One suitable approach that might be taken into consideration here is the appliance of stochastic programming, in order to evaluate scenario-wisely the impacts of such uncertainties.

However, such an approach requires a considerably volume of data and a complete understanding of the random phenomenon, which is not the case for the problem taken into consideration.

Considering the aforementioned characteristics of the problem, especially the fact that such projects are typically financed by the government and involve large amounts of money and bureaucratic effort, one can observe that a robust approach to the problem might be reasonable. It might be interesting to have some certainty regarding what should the investor do in order to protect himself from the uncertainties of the problem and thus, guarantee the project accomplishment.

The first trial for the development of robust models in this fashion reports to the work of Soyster (1973), which proposes a model where the solution must be feasible for all possible realizations within a convex set of uncertainty. This approach may be considered too conservative, since it renounces a large portion of optimality in exchange for the guarantee of viability.

The next milestone in the development of such models comes with the works of Ben-Tal and Nemirovski (1998) and Ben-Tal and Nemirovski (1999), where models are proposed considering less conservative criteria, while using ellipsoidal uncertainty sets. The adoption of this sort of uncertainty set involves the solution of robust counterparts of the nominal problem in the form of conic quadratic problems. One difficulty of such an approach is that, for practical purposes, this class of problems consists of nonlinear problems, which more computational demanding when compared to the previous approach of Soyster (1973)

Afterward, Bertsimas and Sim (2004) propose an approach based on Soyster (1973), but with a parameter to adjust the level of conservatism to be adopted. The main idea of Bertsimas and Sim (2004) is to control the conservatism of the robust solution by introducing a parameter that can be defined by the decision-maker. Since in practice it is unlikely that all the uncertain coefficient are equal to their worst case value (such as Soyster's method), the authors propose a less conservative approach where the decision-maker can choose the number of uncertain factors on which he wishes to be protected.

In order to develop a robust model for the problem taken into consideration, it is necessary to generate a constraint that relates the unknown parameter $\tilde{\gamma}_p^t$ (land productivity) with the demand for vegetable oil D_u^t. Straightforwardly, one can extract from constraints $(2) - (7)$ the relation:

$$\sum_p \tilde{\gamma}_p^t z_p^t \geq \frac{\sum_u D_u^t}{\alpha \beta^*} \quad \forall t \quad (9)$$

where $\tilde{\gamma}_p^t \in [\gamma_p^t - \hat{\gamma}_p^t ; \gamma_p^t + \hat{\gamma}_p^t]$ consists of the uncertain parameter on the land productivity, which are defined in a uncertainty set limited by a maximal up and down variation of $\hat{\gamma}_p^t$ from its nominal known value γ_p^t, and $\beta^* = \inf\{\beta_k\}$ represents the worst oil extraction coefficient among the crushing plant projects considered. At a first glance, one could point out that such constraint might introduce a greater degree of conservatism when compared with the original formulation, as it requires an oilseed production over the necessary, in case the model decides to install crushing plants which the oil extraction coefficient β is such that $\beta > \beta^*$. However, in real-life applications such coefficients do not present large variations among the techniques applied for the oil extraction, and therefore, does not compromises the amount of oilseed necessary for fulfilling the vegetable oil demand. Thus, one can define a protection function $\Omega_t(z, \Gamma_t)$ as:

$$\Omega_t(z, \Gamma_t) = \max_{S_t | S_t \subseteq \mathcal{G}_t, |S_t| = \Gamma_t} \left\{ \sum_{p \in S_t} \hat{\gamma}_p^t z_p^t \right\} \quad (10)$$

where S_t represents the set of Γ_t most constraining coefficient variation selected from the set $\mathcal{G}_t$, which contains all parameters that can vary, for each period t. Moreover, it is straightforward to see that $\Omega_t(z, \Gamma_t)$ can be stated as linear optimization problem as follows. (see proof at [13]).

$$\max_{\lambda_p^t} \quad \sum_{p \in \mathcal{G}_t} \hat{\gamma}_p^t z_p^t \lambda_p^t \quad (11)$$

$$s.t. \quad \sum_{p \in \mathcal{G}_t} \lambda_p^t \leq \Gamma_t \quad (12)$$

$$0 \leq \lambda_p^t \leq 1 \quad \forall p \in \mathcal{G}_t \quad (13)$$

In order to prevent that the problem become nonlinear once the protection function is inserted into the formulation, one should consider the dual formulation of problem $(11) - (13)$, which can be stated as follows:

$$\min_{\zeta_t, \rho_p^t} \quad \sum_p \rho_p^t + \Gamma_t \zeta_t \quad (14)$$

$$s.t. \quad \sum_p \rho_p^t + \zeta_t \geq \hat{\gamma}_p^t z_p^t \quad \forall p \quad (15)$$

$$\rho_p^t, \zeta_t \geq 0 \quad \forall p, t \quad (16)$$

Where ζ_t and ρ_p^t represent the dual variables from constraints (12) and (13) respectively. By strong duality, if $(11) - (13)$ is feasible and bounded for all $\Gamma_t \in [0, |\Gamma_t|]$, then problem $(14) - (16)$ is also feasible and bound and their optimal objective coincide. Thus, one can directly insert the protection function $\Omega_t(z, \Gamma_t)$ in (9). Therefore, we have that:

$$\sum_p \gamma_p^t z_p^t - \Omega_t(z, \Gamma_t) \geq \frac{\sum_u D_u^t}{\alpha \beta^*} \quad \forall t \quad (17)$$

At last, inserting the constraint (17) into the original problem $(1) - (8)$, we can finally state the robust counterpart of the nominal problem $(1) - (8)$ as follows:

$$(1) - (8)$$

$$\sum_p \gamma_p^t z_p^t - \sum_p \rho_p^t - \Gamma_t \zeta_t \geq \frac{\sum_u D_u^t}{\alpha \beta^*} \quad \forall t \quad (18)$$

$$\zeta_t + \sum_p \rho_p^t \geq \hat{\gamma}_p^t z_p^t \quad \forall p, t \quad (19)$$

$$\rho_p^t, \zeta_t \geq 0 \quad \forall p, t \quad (20)$$

7. Probability Bounds

Bertsimas and Sim (2004) show that it is possible to establish bounds for the probability of the solution become unfeasible once the uncertainty unveils. In fact, the results obtained in this paper show that even if more than **Γ** coefficients change, the solution is still feasible with high probability. The parameter **Γ_t** controls the tradeoff between the probability of violation and the effect to the objective function of the nominal problem, which is often called "the price of robustness". Such an "price" can be understood as the amount of optimality the decision-maker are giving-up in order to be more sure about the feasibility of his decision once the uncertainty unveils.

One important result from Bertsimas and Sim (2004) that can be used in this paper is the following: let

$$\eta_p^t = \frac{\tilde{\gamma}_p^t - \gamma_p^t}{\hat{\gamma}_p^t}, \quad \delta_p^t = \begin{cases} 1, \text{if } p \in S_i^* \\ \frac{\hat{\gamma}_p^t z_p^t}{\hat{\gamma}_{r^*}^t z_{r^*}^t}, \text{if } p \in \mathcal{G}_t \backslash S_i^* \end{cases}$$

where $\eta_p^t \in [-1,1], \forall p, t$, and $r^* = \arg\min_{r \in S_t^*}\{\hat{\gamma}_r^t z_r^t\}$. If η_p^t obey an unknown, but symmetric distribution, and are independent, the violation probability $P\left(\sum_p \tilde{\gamma}_p^t z_p^t \leq \frac{\sum_u D_u^t}{\alpha\beta^*}\right)$ obeys the following relation:

$$P\left(\sum_p \tilde{\gamma}_p^t z_p^t \leq \frac{\sum_u D_u^t}{\alpha\beta^*}\right) \leq P\left(\sum_{p \in \mathcal{G}_t} \eta_p^t \delta_p^t \geq \Gamma_t\right) \leq B(|\mathcal{G}_t|, \Gamma)$$

Where $B(|\mathcal{G}_t|, \Gamma_t)$ is the bound for the violation probability. Furthermore, it is shown that, under such conditions, $B(|\mathcal{G}_t|, \Gamma_t)$ can be reasonably approximated by:

$$B(|\mathcal{G}_t|, \Gamma_t) \approx 1 - \Phi\left(\frac{\Gamma_t - 1}{\sqrt{|\mathcal{G}_t|}}\right)$$

where

$$\Phi(\theta) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\theta} exp\left(-\frac{y^2}{2}\right)dy$$

is the cumulative distribution function of a standard normal.

Such results presents a simplified way to estimate such bounds and, as many results in the literature suggest, this bound approximation turned out to be tight enough for most applications (Bertsimas and Sim, 2004; Bertsimas and Thiele, 2004; Bertsimas and Thiele, 2006).

8. Case Study

The case study presented here evaluates an arrangement for supplying a single biodiesel plant, located in the district of Montes Claros in the state of Minas Gerais, over a time horizon of 5 years.

The model elaborated in this study was applied to a group of production zones, and considers vegetable oil produced from castor seeds. The case study focuses on the supply of vegetable oil to a single biodiesel plant. Nonetheless, the structure of the model was designed in such a way that it can be applied to any other region, regardless of its geographic size; the mathematical formulation does not change, and the scale of the model can be expanded as required.

The production zones were determined according to the discretization of the universe to which the study is restricted; they can be as small as the desired level of granularity for the model. In the present study, districts located in the northern area of the state of Minas Gerais, which comprise around 260 thousand square kilometers, were defined as production zones. Two criteria were observed in selecting the production zones: proximity to the district where a biodiesel plant is located, and the geographic boundaries of the Brazilian semi-arid region. A total of 11 production zones were selected.

Based on its agricultural potential in the region of interest, castor oil was selected as the oilseed for the development of this work. The strategic alignment between incentives offered by biodiesel producers and by the Brazilian government was also taken into account.

Among the existing grain crushing technologies, mechanical press and solvent extraction were selected. Moreover different capacities were selected for each of these technologies, in order to represent small and large units.

For the possible locations of the crushers, 18 districts were chosen that satisfy the minimum required conditions for the sustainable installation of an oilseed crushing unit. The selected districts should have good infrastructure, be relatively close to the production zones, and be strategically

located close to paved highways in good condition. In the region under study, the districts considered suitable for the installation of a crushing unit were those with a population of over 40,000 inhabitants.

9. Results

In order to illustrate the applicability of the models presented in this paper, tests were carried out considering the case study previously presented. The models were implemented in AIMMS 3.10 and solved with CPLEX 11.2. The tests were performed on an Intel Core 2 Duo P8600 2.4 GHz with 4 Gb RAM, in 64-bit platform. Table 2 summarizes the main characteristics of the proposed model.

# Constraints	1,186
# Variables	2,331(360 integers)
FO($)	from 89,871,041 to 139,809,483
Average Solving Time(s)	1.6

Table 1-Model Summary

For the model presented, Table 2 shows the objective function value range for $\Gamma_t \in [0,11], \forall t$, as well as the average solution time for these values of Γ_t. Table 3 illustrates the investment decision for the different models. For reasons of space saving, only the investment profiles will be analyzed further.

	Investment	Tech. Type	Size(ton/year)	Location	Year
$\Gamma_t \in [0,1]$	1	Mech. Crushing	25,000	Montes Claros	2009
	2	Mech. Crushing	25,000	Unaí	2012
$\Gamma_t \in [2,7]$	1	Mech. Crushing	25,000	Unaí	2009
	2	Mech. Crushing	25,000	Montes Claros	2011
$\Gamma_t \in [8,9]$	1	Mech. Crushing	25,000	Montes Claros	2009
	2	Mech. Crushing	25,000	Unaí	2011
$\Gamma_t \geq 10$	1	Mech. Crushing	25,000	Montes Claros	2009
	2	Mech. Crushing	25,000	Unaí	2011
	3	Mech. Crushing	12,500	Capelinha	2012

Table 2 - Investment Decisions

As can be seen, the model decides to invest in a single type of mechanical crushing in different places (Unaí and Montes Claros) at two different years (2009 and 2012), allocating a crushing capacity of 25,000 ton of grain per year. As the level of robustness increases, the model gradually tends to focus on the investments in the Unaí district where, despite the less attractive logistics costs, the production rates have historically been higher. As the security level Γ_t continues to rise - i.e., it is assumed that a higher number of places might have the worst possible production rate - the model decides to install an additional crushing plant in the location of Capelinha.

Moreover, the maximum deviation of $\hat{\gamma}_{p,o}^t$ was set to one standard deviation, considering the historical data available. The model obtains solutions that vary from $89,871,041 to $139,809,483, for values of $\Gamma_t \in [1,11]$, remaining stable for greater values of Γ_t. Such behavior can be easily explained, since for greater values of Γ_t all the uncertainties inherent to the problem have been already considered - remember that are only 11 production zones considered in the case study - which makes the model equivalent to the formulation in Soyster (1973). Figure 3 shows the behavior of the objective function as a function of Γ_t and Figure 4 illustrates the probability bounds for each value of Γ_t.
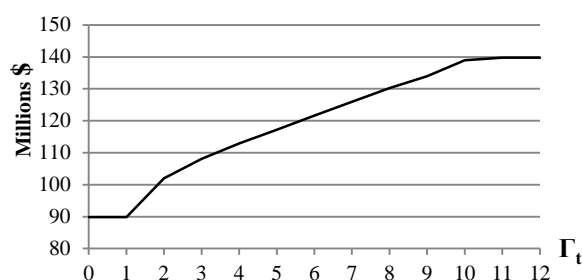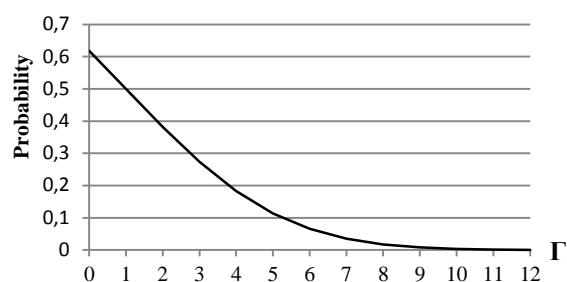
Figure 3 - Objective Function Value vs. Γ_t

Figure 4 - Violation probability bounds

The charts illustrate the tradeoff between the degree of robustness desired by the decision maker and the level of optimality that is compromised by this choice. Another point to highlight is related to the flexibility of the approach. Depending on the profile of the investor, and therefore the level of robustness adopted, the model can provide solutions ranging from \$89,871,041 to \$139,809,483 and the associated probability of such a solution becomes unfeasible once the uncertainty unveils.

10. Conclusion

In this paper we presented an optimization model to the problem of planning the supply network for biodiesel chain based on small farms, taking into considerations uncertainties regarding the agricultural production yield. To this end, an adjustable robust model is offered to deal with such uncertainty. The approach is based on Bertsimas and Sim (2004), which is characterized by the possibility of fitting the robustness criteria to the risk profile of the investor.

The proposed model was tested in a real case study for the development of a biodiesel supply chain based on small farmers, as presented in this paper. The case study is to establish a supply chain based on small farmers capable to supply an existing biodiesel plant in the district of Montes Claros, considering 11 production zones, 18 possible locations for the crushing plants and a planning horizon of 5 years.

The results show that the proposed model proves to be a valuable tool for adoption of robust criteria for several reasons. First, the flexibility of the tool allows the decision making process to be done in a much more adjustable fashion, which makes the approach suitable for different risk tolerance profiles when it comes to the adoption of robust solutions. Second, since the approach consists of a fully-free scenario formulation, the size of the model is completely independent of the scenario tree generation process and thus much more efficient regarding solution times. Another important advantage of being independent of scenario tree generation is that the model is thus also free of quality issues regarding the scenario tree generation process itself. Finally, it is worth noting that the proposed approach does not require full knowledge of the random phenomenon probability distribution. Although it is common to assume in theoretical papers that a distribution is fully known, very often in real-life cases the stochastic process is not completely understood. In such cases, the application of sophisticated sample processes can be debated, since it will only guarantee convergence toward a probability distribution that might not be the real one.

Finally, we encourage other academics to work on numerical and quantitative approaches to production chains based on the activities of small farmers. Especially bearing in mind the recent introduction of the Brazilian Biodiesel Program, any idea that that economic feasibility and family farming are unrelated in terms of agro-energy must be firmly rejected. Thus, academic work on this subject can play a fundamental role in helping to ensure that the country's strategic goals for social inclusion and income transfer to rural areas will be achieved.

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