A GREEDOID ON THE VERTEX SET OF A VERY WELL-COVERED GRAPH

Vadim E. Levit
Ariel University Center of Samaria, ISRAEL
e-mail: levitv@ariel.ac.il

Eugen Mandrescu
Holon Institute of Technology, ISRAEL
e-mail: eugen_m@hit.ac.il

ABSTRACT

A *stable set* in a graph **G = (V,E)** is a set of pairwise non-adjacent vertices, and **α(G)** is the maximum size of a stable set. Recall that G is *well-covered* if all its maximal stable sets have the same cardinality (M. D. Plummer, *Journal of Combinatorial Theory*, 1970), and G is *very well-covered* if, in addition, it has no isolated vertices and **|V| = 2α(G)** (O. Favaron, *Discrete Mathematics*, 1982).

A set **S** is a *local maximum stable set* of **G** if **S** is a maximum stable set of the subgraph induced by its closed neighborhood. Let **Ψ(G)** denote the family of all local maximum stable sets. Nemhauser and Trotter Jr. (*Mathematical Programming*, 1975) proved that any **S∈Ψ(G)** is included in a maximum stable set of **G**, and interpreted this assertion as a sufficient local optimality condition for a binary integer programming formulation of the weighted maximum stable set problem.

Many problems in combinatorial optimization can be described using algebraic structures called matroids, which permit greedy algorithms to find optimal solutions. Introduced as generalizations of matroids, greedoids capture more problems whose solutions can be found by greedy methods. Further, it has been revealed that for many optimization problems, in order to run the greedy algorithm it was enough to have a structure satisfying a weaker than the hereditary property, namely, the accessibility property.

A *greedoid* (B. Korte, L. Lovász, *Lecture Notes in Computer Science*, 1981) is a pair **(V,ℑ)**, where $\mathfrak{I} \subseteq 2^V$ is a set system satisfying:

Accessibility: for every non-empty **X∈F** there is an **x∈X** such that **X–{x}∈ℑ**;

Exchange: for **X,Y∈ℑ, |X|=|Y|+1**, there is an **x∈X–Y** such that **Y∪{x}∈ℑ**.

It is known that **Ψ(G)** forms a greedoid for every forest **G**. Bipartite graphs, triangle-free graphs, unicyclic graphs **G** whose **Ψ(G)** is a greedoid were also characterized (by exhibiting a close relationship between some special matchings and local maximum stable sets of a graph) (V. E. Levit and E. Mandrescu, *Discrete Applied Mathematics*, 2002, 2004, 2007). Recently, it was also shown that **Ψ(G)** forms a greedoid (in fact, an interval greedoid), whenever it satisfies the accessibility property (V. E. Levit and E. Mandrescu, *Discrete Mathematics*, 2012).

In this work we combine topics of well-covered graphs and greedoids, by showing that if **G** is a very well-covered graph, then **Ψ(G)** forms a greedoid if and only if **G** has a unique perfect matching (V. E. Levit and E. Mandrescu, *Discrete Applied Mathematics*, 2012).

KEYWORDS. Very well-covered graph. Local maximum stable set. Unique perfect matching.