VNS BASED ALGORITHMS TO THE HIGH SCHOOL TIMETABLING PROBLEM

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ABSTRACT

The high school timetabling is a classical NP-Hard problem in combinatorial optimization. Since the use of exact methods for this problem is restricted, heuristics are usually employed. This paper presents an investigation on the development of a Variable Neighborhood Search (VNS) method which includes two powerful neighborhood operators to heuristically solve this problem. Two VNS based algorithms are presented and experimented on a benchmark data set from literature. The results have shown that the two algorithms are effective and efficient, as they have found proven optimal solutions (using pre-computed lower bounds) on a reasonable amount of time.

KEYWORDS. High School Timetabling Problem, Variable Neighborhood Search, Neighborhood Operators.

Main area. OR in Education, Metaheuristics, Combinatorial Optimization.
1. Introduction

The high school timetabling problem (HSTP) (SCHAERF, 1999; PILLAY, 2010) is a hard
combinatorial optimization problem and taking into account the computational complexity theory,
it is NP-Complete (EVEN et al., 1975). Besides the original, already complicating constraints, real
cases can include a multitude of different ones, as those collected in (POST et al., 2012). As the
best known algorithms to solve the problem to optimality are exponential time, their applicability to
solve real instances of the problem, become impracticable due to the large amount of computational
time required. For this reason, the problem is tackled by heuristic methods. Such methods do not
guarantee to solve the problem to optimality, but are capable to find good solutions in a feasible
computational time.

It is often the case that a timetabling problem is solved manually and in practice many
constraints cannot be solved. In addition, a handmade timetable can take several days to be
accomplished and due to frequent changes in the resources requirements, i.e. teachers who leave
the school in medical absences, it has to be remade causing inconveniences for the school staff. For
these reasons, this task is considered an onerous process.

Due to the above mentioned, more attention have been devoted to automatization of the
timetabling process during the last years by researchers. In special, in the last two decades a
large number of experimental papers tackling the problem by heuristics have been introduced at
the literature.

The most common methods used at the literature to solve the problem are: Genetic
Algorithms (SOUZA et al., 2002); Simulated Annealing (BRITO et al., 2012); Tabu Search
(SANTOS et al., 2005); Greedy Randomized Adaptive Search Procedures (SOUZA et al., 2003);
Variable Neighborhood Search (BRITO et al., 2012) and Iterated Local Search (SAVINIEC;
CONSTANTINO, 2012).

This paper is an extension of the research presented in (SAVINIEC et al., 2013). There, we
have proposed three iterated local search algorithms including two newly proposed neighborhood
operators to solve the HSTP. These algorithms were successfully experimented on a well known
benchmark data set of the problem (SOUZA et al., 2003) and the results have encouraged us to
test these same neighborhood operators with other metaheuristic. Taking this in mind, this paper
proposes two algorithms based on the VNS metaheuristic to solve the problem. These two VNS
algorithms are applied on the benchmark data set from (SOUZA et al., 2003) and to validate the
proposal, we contrast obtained results with lower bounds known for these instances (SANTOS et al.,
2012). The main findings are: the improved upper bounds obtained by our VNS algorithms have
allowed us to prove optimality for instances where strong lower bounds were previously known
and the statistical distribution of solutions of these two algorithms are very close to the optimal
solutions.

The paper is organized as follows: Section 2 defines the problem. Section 3 explains the
solution approach. Section 4 reports the experimental results and section 5 provides a summary and
future works.

2. Problem definition

The HSTP considered in this paper (SOUZA et al., 2003) is based on Brazilian high
schools. There is a set $P = \{p|1 \leq p \leq np, p \in \mathbb{N}\}$ of teachers who teach a set $T = \{t|1 \leq t \leq
nt, t \in \mathbb{N}\}$ of classes at school in a given shift, during a set $D = \{d|1 \leq d \leq nd, d \in \mathbb{N}\}$ of days,
with each day composed by a set $H = \{h|1 \leq h \leq nh, h \in \mathbb{N}\}$ of periods. Classes are disjoint
groups of students having the same subjects and no idle time periods during the week, and each
subject of a class is taught by only one teacher. Lessons between teachers and classes are previous
defined by the school. Classrooms are predefined and not considered in the scheduling. Most of
the teachers are not full time at school, thus teachers’ availability have to be considered and their
workload have to be concentrated in a minimum number of days during the week. In this way, an instance of the problem is according to definition 2.1.

**Definition 2.1 (HSTP instance)** An instance of the HSTP is the data entry to the timetable construction process in a given shift and it is represented by the following sets:

- A set \( L = \{ \langle t, p, \theta, \lambda, \mu \rangle \mid t \in T, \ p \in P, \ \theta \in \mathbb{N}, \ \lambda \in \mathbb{N}, \ \mu \in \mathbb{N} \} \) of quintuples, named as lessons requirement set. Where \( \theta \) is the number of lessons, \( \lambda \) is the maximum number of permitted lessons per day and \( \mu \) is the minimum number of double lessons requested by teacher \( p \) with class \( t \).
- A set \( U = \{ \langle p, d, h \rangle \mid p \in P, \ d \in D, \ h \in H \} \) of triples, named as set of teachers’ unavailable periods. Where exists a triple \( \langle p, d, h \rangle \) if teacher \( p \) is unavailable at period \( h \) of day \( d \).

Then, the problem consists in the scheduling of a weekly timetable \( Z \), composed by five days with five periods each, for the lessons in \( L \), satisfying the hard constraints (definition 2.2) and minimizing the soft constraints (definition 2.3).

**Definition 2.2 (Hard constraints)** The hard constraints are represented by the set \( A = \{ a_i \mid 1 \leq i \leq 5 \} \) of constraints:

- \( a_1 \): every \( \theta \) lessons required for class \( t \) and teacher \( p \) must be scheduled;
- \( a_2 \): a class must attend a lesson with only one teacher by period;
- \( a_3 \): a teacher must teach only one class by period;
- \( a_4 \): teachers must not be scheduled in periods they are not available;
- \( a_5 \): a class \( t \) must not be scheduled to attend more than \( \lambda \) lessons with a same teacher \( p \) per day.

**Definition 2.3 (Soft constraints)** The soft constraints are represented by the set \( B = \{ b_j \mid 1 \leq j \leq 3 \} \) of constraints:

- \( b_1 \): the number \( \mu \) of double lessons requested by teacher \( p \) with class \( t \) has to be attended;
- \( b_2 \): idle times in the scheduling of teachers should be avoided;
- \( b_3 \): the scheduling for each teacher should encompass the least possible number of days.

### 3. Heuristic approach

This section discusses some fundamental concepts for building heuristic approaches and defines the proposed approach to solve the HSTP. In the following, we present each component that composes our approach: solution representation structure (section 3.2), objective function (section 3.3), the heuristic used to build initial solutions (section 3.4), the local search technique applied (section 3.5), the neighborhood operators (section 3.6) and the ILS algorithms employed to solve the problem (section 3.7).

#### 3.1. Concepts

On the context of combinatorial optimization (CO) problems, all possible solutions for a given instance of a problem, feasible or not, define the solution (or search) space \( S \), and each solution in \( S \) can be seen as a candidate solution. Thus, solving a CO problem requires to formulate it as a maximization or minimization problem. In this type of formulation there is an objective function \( f : S \rightarrow \mathbb{R} \) and the problem consists in finding solutions that maximize or minimize \( f \).

On the context of the high school timetabling, the problem is generally formulated as minimization and \( f \) is measured by weighting the number of violations for each constraint of the problem and the aim is to satisfy the hard constraints and minimize the soft constraints. Then, to solve the problem, one has to find a solution \( Z^* \in S \) with minimum objective function, that is,
\[ f(Z^*) \leq f(Z), \forall Z \in S, \] where \( Z^* \) is called **global minimum** in \( S \) and the set \( S^* \subseteq S \) of all solutions \( Z^* \) is the **set of global minimum**.

A powerful class of algorithms to solve CO problems, in which no polynomial time algorithm is known, are heuristic algorithms based on the concept of local search.

A local search heuristic starts from an initial solution \( Z_0 \) and iteratively replaces the current solution \( Z \) by a better solution \( Z' \) in an appropriately defined neighborhood \( N(Z) \) of the current solution, until no more improvements are possible and the heuristic gets stuck in a **local minimum**.

Neighborhoods are generated by neighborhood operators (definition 3.1) and they enable to define the concept of **local minimum** (definition 3.2).

**Definition 3.1 (Neighborhood operator)** A neighborhood operator is a function \( N : S \rightarrow P(S) \) that assigns to every solution \( Z \in S \) a set of neighbors \( N(Z) \subseteq S \). \( P(S) \) is the power set of \( S \) and \( N(Z) \) is called neighborhood of \( Z \).

**Definition 3.2 (Local minimum)** A local minimum solution with respect to a neighborhood operator \( N \) is a solution \( Z^* \), such that \( \forall Z \in N(Z^*) \Rightarrow f(Z^*) \leq f(Z) \).

### 3.2. Solution representation

A solution of the HSTP is represented according to definition 3.3.

**Definition 3.3 (HSTP solution)** A HSTP solution is stored in a non-negative integer three-dimensional matrix \( Z_{|T| \times |D| \times |H|} \), where \( z_{t,d,h} \in \{1,2,\ldots,np\} \) stores the teacher scheduled to teach for class \( t \) on period \( h \) of day \( d \).

Note that using this structure, constraints \( a_1 \) and \( a_2 \) are automatically satisfied and they are not included on the objective function.

### 3.3. Objective function

In order to solve the HSTP, it is treated as an optimization problem in which an objective function \( f : S \rightarrow \mathbb{R} \) has to be minimized. The objective function \( f \) associates each solution \( Z \) in the solution space \( S \) to a real number and this is defined to measure the violation degree on the HSTP constraints. Thus, a timetable solution \( Z \) is evaluated according to the objective function in definition 3.4.

**Definition 3.4 (Objective function)** A HSTP solution \( Z \) is evaluated by the following function:

\[
 f(Z) = f_A(Z) + f_B(Z) \tag{1}
\]

Such that:

\[
 f_A(Z) = \sum_{i=3}^{5} \alpha_{ai} \times \beta_{ai} \tag{2}
\]

\[
 f_B(Z) = \sum_{j=1}^{3} \alpha_{bj} \times \beta_{bj} \tag{3}
\]

Where equations 2 and 3, respectively, measure the feasibility and quality of a timetable solution and the weight \( \alpha_{ai} \) (resp. \( \alpha_{bj} \)) reflects the relative importance of minimizing the amount of violation \( \beta_{ai} \) (resp. \( \beta_{bj} \)) at constraint \( a_i \in A \) (resp. \( b_j \in B \)).

From definition 3.4 a timetable is feasible if \( f_A(Z) = 0 \) and the variables \( \beta_{ai} \) and \( \beta_{bj} \) are computed as below.

\[
 \beta_{ai} = \sum_{p \in P} \sum_{d \in D} \sum_{h \in H} (\pi_{p,d,h} - 1), \forall (\pi_{p,d,h} > 1). \tag{4}
\]

Where \( \pi_{p,d,h} \) is the total number of lessons allocated for teacher \( p \) on period \( h \) of day \( d \);
\[ \beta_{a_1} = \sum_{p \in P} \sum_{d \in D} \sum_{h \in H} \rho_{p,d,h}. \] Where \( \rho_{p,d,h} = 1 \) if teacher \( p \) has been scheduled to teach at an unavailable period \( h \) on day \( d \), and \( \rho_{p,d,h} = 0 \) otherwise;

\[ \beta_{a_2} = \sum_{t \in T} \sum_{p \in P} \sum_{d \in D} (\sigma_{t,p,d} - \lambda_{t,p}), \forall (\sigma_{t,p,d} > \lambda_{t,p}). \] Where \( \sigma_{t,p,d} \) is the total number of lessons allocated for class \( t \) with teacher \( p \) on day \( d \) and \( \lambda_{t,p} \) is the maximum of permitted lessons per day from definition 2.1;

\[ \hat{\beta}_1 = \sum_{t \in T} \sum_{p \in P} (\mu_{t,p} - \phi_{t,p}), \forall (\mu_{t,p} > \phi_{t,p}). \] Where \( \mu_{t,p} \) is the minimum number of double lessons requested by teacher \( p \) with class \( t \) (definition 2.1) and \( \phi_{t,p} \) is the effective number of allocated double lessons;

\[ \hat{\beta}_2 = \sum_{p \in P} \sum_{d \in D} \eta_{p,d}. \] Where \( \eta_{p,d} \) is the number of idle times at the agenda of teacher \( p \) on day \( d \). For example, if a teacher has been scheduled to teach at the first and fourth periods and is free at the second and third ones, then he has two idle times on this day;

\[ \hat{\beta}_3 = \sum_{p \in P} \chi_{p}. \] Where \( \chi_{p} \) is the total number of scheduled days for teacher \( p \) on the timetable.

### 3.4. Algorithm for building initial solutions

In this work, initial solutions of the HSTP are constructed by means of a randomized algorithm (see algorithm 3.1). This algorithm gets the lessons requirement set \( L \) from definition 2.1 as input and builds an initial solution by selecting and scheduling lessons randomly.

**Algorithm 3.1 Algorithm for building initial solutions**

```
GENERATE-RANDOM-SOLUTION(L)
1    Initialize Z
2    for each e \in L do
3        t = e.t
4        p = e.p
5        NumberOfLessons = e.\#T
6        while NumberOfLessons > 0 do
7            Put p in a randomly selected free cell \( z \in Z \)
8            NumberOfLessons = NumberOfLessons - 1
9    return Z
```

### 3.5. Local search

In summary, for CO problems, given an initial solution \( Z_0 \) as input, a local search heuristic moves from \( Z_0 \) to a local minimum \( Z' \) by exploring neighborhoods. At the literature, the most used techniques to perform local search are: best improvement and first improvement (HANSEN et al., 2010):

**Best improvement**: the heuristic start at an initial solution \( Z' = Z_0 \), and at each iteration, replaces \( Z' \) by \( Z = \min \{ Z'' \in N(Z') \} \) while \( f(Z) < f(Z') \). This technique explores the whole neighborhood and moves to the best solution.

**First improvement**: this technique is an alternative to the first when the neighborhood is large to be entirely explored. This is similar to the first, but at each iteration it moves to the first solution \( Z_i \in N(Z') \) found, if it improves the current solution \( Z' \).

In this paper the first improvement technique is employed as local search (algorithm 3.2).

### 3.6. Neighborhood operators

Neighborhood operators are the key ingredient to develop powerful local search algorithms. Some researches (LAARHOVEN et al., 1992; DELL’AMICO; TRUBIAN, 1993;
Algorithm 3.2 First improvement heuristic

\[
\text{FIH}(Z_0, N)
\]

\[
1 \quad Z = Z_0
\]

\[
2 \quad \text{repeat}
\]

\[
3 \quad Z' = Z
4 \quad i = 0
5 \quad \text{repeat}
\]

\[
6 \quad i = i + 1
7 \quad Z = \min\{Z, Z_i\}, Z_i \in N(Z')
8 \quad \text{until } (f(Z) < f(Z') \text{ or } i = |N(Z')|)
9 \quad \text{until } (f(Z) \geq f(Z'))
10 \quad \text{return } Z'
\]

OSOGAMI; IMAI, 2000) have demonstrated, for some CO problems, that it is possible to define neighborhood operators that reduce the search space. Such operators exclude out of the search process, a large set of non-feasible solutions and the local search algorithm can efficiently search the restricted solution space.

In this paper, two neighborhood operators, named as \textbf{Matching operator (MT)} and \textbf{Torque operator (TQ)}, are employed. These operators exclude out of the search process a large set of undesirable solutions. These two operators are proposed and detailed described in (SAVINIEC \textit{et al.}, 2013). By this reason, this paper will only comment them.

\textbf{Matching operator:} this is based on the resolution of Assignment Problems (AP). A neighbor of a solution \( Z \) is obtained by selecting a random class, and from this, a random set \( \hat{Y} \) of non-repeated teachers. This set of teachers are moved out of the timetable and we solve an assignment problem to re-schedule them. Figure 1 illustrates an operation of MT. To simplify, in this example only constraint \( a_3 \) with weight \( \alpha_{a_3} = 1 \) is taken into account. The MT operator is applied on lessons of class \( t_3 \) at the solution \( Z \) in figure 1(a), where \( t_3d_1h_1, t_3d_1h_2, t_3d_1h_3, t_3d_1h_5 \)
\[
\hat{Y} = \{2, 9, 1, 6\}
\]
is the non-repeated set of teachers. Figures 1(b)-1(e) illustrate how to construct and solve an AP for the set \( \hat{Y} = \{2, 9, 1, 6\} \) and figure 1(f) shows the obtained neighbor \( Z' \).

\textbf{Torque operator:} this is a generalization for the well known double move operator (DM) generally used to solve the HSTP. The DM operator consists in swapping two lessons of a class that are scheduled in two different periods \( h_i \) and \( h_j \), for example. But when applying moves using DM operator, new clashes between lessons can occur and \( a_3 \) constraint is violated. Thus, the TQ operator is developed to prevent this disadvantage using the idea of \textit{Kempe chain interchange} (LÜ \textit{et al.}, 2011). In this operator, a graph \( G \) is built with lessons from two distinct periods \( h_i \) and \( h_j \), where nodes are pairs of lessons and edges are added between nodes having conflicted lessons. A neighbor of a solution \( Z \) is obtained by swapping the lessons in each node of a connected component of \( G \). Figure 2 illustrates this operator.

\subsection{3.7. VNS based algorithms to the HSTP}

The proposed approach is composed by two algorithms based on the VNS metaheuristic (HANSEN \textit{et al.}, 2010).

\textbf{VNS-MT-TQ:} this VNS (algorithm 3.6) performs local search by exploring firstly, six neighborhoods \( MT(Z) \) of different sizes and \( TQ(Z) \) in the sequence.

\textbf{VNS-TQ-MT:} this algorithm is analogous to the first one, but in this case \( TQ(Z) \) is explored before \( MT(Z) \).
These algorithms make use of the N-RANDOM-PERTURBATION procedure (algorithm 3.3), that applies a random move by using the TQ operator to perform perturbation and escape from local minimum.

**Algorithm 3.3 Perturbation procedure**

```plaintext
N-RANDOM-PERTURBATION(Z, N, n)
1    while (n > 0) do
2        Z = Random Z' ∈ N(Z)
3        n = n − 1
4    return Z
```

4. Experimental results

This section reports the experimental results of running the proposed algorithms on the high school timetabling benchmark from (SOUZA et al., 2003). This benchmark has seven instances where timetables have five days \( (nd = 5) \) with five periods each \( (nh = 5) \). For the three largest instances in this benchmark, the optimal solutions are not known, however, lower bounds were computed using an extended Integer Linear Programming Formulation (SANTOS et al., 2012).

The proposed approach was coded using MS Visual Basic 6. The experiment was performed on Windows Server 2008-R2 running on the KVM virtual machine set to work with 30GB of RAM and 50 cores of a server with 4 CPU Intel Xeon E7-4860 (24MB of Cache -
(a) Solution $Z$ without clashes at $a_3$ (b) Graph $G$ for two distinct periods $h_i = h_1$ and $h_j = h_2$

(c) The neighbor $Z'$ without clashes in $a_3$, after swapping the lessons at the blue color connected component

**Figure 2. The torque operator**

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**Algorithm 3.4** Randomized first improvement heuristic

```plaintext
RANDOMIZED-FIH(\(Z, n\))
1 repeat
2 \(Z' = Z\)
3 \(i = n \times |T|\)
4 while \((i > 0)\) do
5 \(t = \text{Random } t' \in T\) // select a random class $t$
6 \(\hat{Y} = \text{Random } \hat{Y}' \in U_t\) // select a random set $\hat{Y}$ in $U_t$
7 \(Z = \text{MT}(Z, \hat{Y})\) // solve the AP to the set $\hat{Y}$ and return the neighbor of $Z$
8 \(i = i - 1\)
9 until \((f(Z) \geq f(Z'))\)
10 return \(Z'\)
```

**Algorithm 3.5** Acceptance criterion and operator change procedure

```plaintext
NEIGHBORHOOD-CHANGE(\(Z^*, Z^{**}, k\))
1 if \(f(Z^{**}) < f(Z^*)\) then
2 \(Z^* = Z^{**}\)
3 \(k = 1\)
4 else
5 \(k = k + 1\)
```
Algorithm 3.6 VNS-MT-TQ algorithm

\begin{algorithm}
\STATE $k_{max} = 7$
\STATE $Z^* = Z_0$
\STATE \textbf{repeat}
\STATE \hspace{1em} $k = 1$
\STATE \hspace{2em} \textbf{repeat}
\STATE \hspace{3em} $Z' = \text{N-RANDOM-PERTURBATION}(Z, TQ, 1)$
\STATE \hspace{3em} \textbf{if} $k <= 6$ \textbf{then}
\STATE \hspace{4em} $Z^* = \text{RANDOMIZED-FIH}(Z', k)$ \hfill // local search using MT operator
\STATE \hspace{2em} \textbf{else}
\STATE \hspace{3em} $Z^* = \text{FIH}(Z', TQ)$ \hfill // local search using TQ operator
\STATE \hspace{2em} $\text{NEIGHBORHOOD-CHANGE}(Z^*, Z'^*, k)$ \hfill // acceptance criterion and operator change
\STATE \hspace{1em} \textbf{until} $(k > k_{max})$
\STATE $t = \text{CPU TIME}()$
\STATE \textbf{until} $(t > t_{max}$ or $f(Z) = 0)$
\STATE \textbf{return} $Z^*$
\end{algorithm}

2.26 GHz) with Linux CentOS 6 operating system. In this experiment 50 tests of 900 seconds were carried out for each instance. The whole experiment was performed in two phases, at each phase an algorithm was experimented by executing 50 simultaneous processes. The constraints were penalized with the follow weights on the objective function: $\beta_{a_3} = 100.000$, $\beta_{a_4} = 5.000$, $\beta_{a_5} = 100$, $\beta_{b_1} = 1$, $\beta_{b_2} = 3$, $\beta_{b_3} = 9$.

| Instance | $|T|$ | $|P|$ | Lessons | VNS-MT-TQ | VNS-TQ-MT | LB | TS | IP |
|----------|------|------|---------|-----------|-----------|----|----|----|
| 1        | 3    | 8    | 75      | *         | *         | 202 | *  | *  |
| 2        | 6    | 14   | 150     | *         | *         | 333 | *  | *  |
| 3        | 8    | 16   | 200     | *         | 426       | 423 | *  | *  |
| 4        | 12   | 23   | 300     | *         | *         | 652 | 653| *  |
| 5        | 13   | 31   | 325     | *         | *         | 762 | 766| 764|
| 6        | 14   | 30   | 350     | *         | *         | 756 | 760| 765|
| 7        | 20   | 33   | 500     | *         | *         | 1017| 1029| 1028|

Figure 3 shows the statistical distribution of solutions and table 1 the best solutions found by the two algorithms. Column LB tabulates the lower bounds found by the cut and column generation algorithm from (SANTOS et al., 2012). The distributions on the two boxplot graphics are based on the concept of relative distance in definition 4.1. By this concept we compare the results found by our algorithms with the lower bounds. For the open instances, 5 to 7, our algorithms have reached the lower bounds and it helps to prove the optimality for these instances. In addition, optimal solutions were found for all instances and according to the boxplot in figure 3(b) the statistical distribution of solutions, for these algorithms, are very close to the optimal solutions, less than 7% far from the optimum.

**Definition 4.1 (Relative distance)** Given an instance of the HSTP. Let $Z$ be an arbitrary solution and $Z_{best}$ the best known solution for this instance. The relative distance from $Z$ to $Z_{best}$ is denoted by:

$$rd = \frac{f(Z)}{f(Z_{best})} \quad (4)$$

As additional information, columns TS\(^1\) and IP\(^2\) (table 1) show the best known results

\(^1\)Tabu Search (SANTOS et al., 2005)
\(^2\)Integer Programming (DORNELES et al., 2012)
found in previous studies for these instances. The “∗” symbol in cells of table 1 means that the algorithm was able to reach the lower bound in column LB.

![Figure 3. Statistical distribution of solutions](image-url)

(a) By instance

(b) By algorithm
5. Conclusions and future works

In this paper we have proposed two VNS algorithms to solve the high school timetabling benchmark from (SOUZA et al., 2003). These algorithms have shown to be effective and efficient to solve the problem, as their statistical distribution of solutions are very close to the optimal solutions and global optimum were found for all instances. Furthermore, we believe that the high performance of our algorithms are due to the use of the two neighborhood operators employed and the main contribution of this work was help to prove the optimality for the three open instances.

As future works we intend to test these algorithms with additional set of instances.

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