



SOLVING THE ARC ROUTING PROBLEM WITH A HYBRID ELECTROMAGNETIC MECHANISM ALGORITHM

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ABSTRACT

Over the last decade the interest on Hybrid Electromagnetic-like Mechanism (HEM) algorithms has grown considerably due to the promising results obtained when it is applied to different problems from several areas. This paper presents an algorithm based on the HEM to solve the Capacitated Arc Routing Problem (CARP), a well-known NP-hard combinatorial optimization problem. The goal in this problem is to find a set of minimum cost routes that serve a given set of customers in a network. The proposed HEM algorithm is based on two key concepts: the first one is inspired on the theory of electromagnetism and its action and repulsion effects, the second one relies on the use of biased randomization and a divide-and-conquer strategy. The efficiency of the proposed method is tested by comparison with other similar approaches in the literature.

KEYWORDS: Capacitated Arc Routing Problem, Metaheuristics, Hybrid Electromagnetic Mechanism

1. Introduction

The Capacitated Arc Routing Problem (CARP), a NP-hard problem (Lenstra & Rinnooy, 1981), is a well-known combinatorial optimization problem in which a fleet of homogeneous vehicles, initially located at a single depot, has to be routed so all customers' demands are served at a minimum cost without violating the loading capacity of each individual vehicle. In the CARP, the road network is a non-complete graph, so not all pairs of nodes have a direct connection. Also, it is assumed that customer's demands are located in some of the arcs connecting these nodes. The capacity restriction refers to the maximum load that each vehicle can carry.

This paper introduces a new hybrid metaheuristic, based on a Hybrid Electromagnetic Mechanism (HEM) to solve the CARP. The electromagnetic mechanism (EM) was proposed and developed by Birbil & Fang (2003), being initially used for global optimization. The mechanism stimulates the points of convergence to an attractive valley in the solution space, and discourages the points to keep moving away from the steeper, corresponding to the optimal problem hills. This principle is analogous to the mechanism of action and repulsion of the theory of electromagnetism, and has been implemented in combinatorial optimization problems such as: Single-machine scheduling (Sels & Vanhoucke, 2014), p-hub Median problem (Kratka, 2013), Knapsack problem (Bonyadi & Li, 2012), Periodic job-shop scheduling problem (Jamili et al., 2011), Traveling salesman problem (Javadian et al., 2008; Peitsang et al., 2012), Vehicle routing problem (Peitsang et al., 2007), and Multi -depot vehicle routing problem (Huang, 2006).

The article is structured as follows: in Section 2 introduces a review of the existing literature; Section 3 describes in detail the HEM algorithm adapted to the CARP; Section 4 presents the results obtained for different problem sets extracted from the literature; finally, Section 5 presents the main conclusions of this article as well as some future work.

2. Literature Review

The CARP was formally proposed by Golden & Wong (1981). This problem has presented a number of variations and applications during the last decades: garbage collection (Candida & Amado, 2005; Zuhaimy & Mohammad, 2011), distribution of salt on the streets in order to avoid large amounts of snow (Muyldermans et al., 2002) and mobile units assignment (Vansteenwegen et al., 2010). Some of these examples incorporate CARP variants, for example: CARP defined on a digraph (DCARP), Maniezzo & Roffilli (2008); CARP defined over a mixed graph or Mixed CARP (MCARP), Belenguer et al. (2006), CARP-like problem with several vehicles but without capacity constraints (k-CPP), Ahr & Reinelt (2000) and CARP with vehicle/site dependencies (CARP-VSD), Sniezek & Bodin (2006). More insight can be found in Wohlk (2005) and Mei et al. (2010).

Considering exact methods, which not only find the total optimal solutions but also proving its optimality, the most common approaches in CARP are: Branch and Bound Hirabayashi et al. (1992); Cutting Planes, Belenguer & Benavent (2003); Branch and Cut, Baldacci & Maniezzo (2006); Cutting Plane and Column Generation, Belenguer et al. (2005); Branch and Cut and Price, Longo et al. (2006) and Letchford & Oukil (2009), Column Generation, Martinelli et al. (2009); Branch and Price, Christiansen et al. (2009) and Cut and Branch and Price, Bode & Irnich (2012). Some used transformations to graph, others are based on a simple ILP formulation. During the early 1980s, problem specific heuristics were the most common method for solving the CARP. Classical ARP heuristics include the Construct Strike procedure, the Path Scanning method and the Augment Merge method. The methods are designed for fast generation of 'good' solutions – usually not even pseudo-optimal – for the given problem. See Dror (2000) for a survey.

In recent years, most new approaches for solving the CARP are based on metaheuristics: Genetic Algorithm Hybrid by Lacomme et al. (2001), Simulated Annealing by Wøhlk (2005), Tabu Search by Brandão & Eglese (2008), Memetic Algorithm by Tang et al. (2009), Ant Colony by Santos et al. (2010), Adaptive Large Neighbourhood Search by Laporte et al. (2010) and Grasp

with Evolutionary Path-Relinking by Usberti et al. (2013).

As regards as the *Electromagnetic Mechanism* (EM), this was designed to solve continuous optimization problems. However, the approach was also extended to (discrete) combinatorial optimization problems. When the algorithm is extended, the first step is the representation of the solution, as illustrated in the work of Bean (1994). EM has been applied to various combinatorial optimization problems (Norman & Bean, 1999; Snyder & Daskin, 2006). There even some recent applications of this approach to routing problems, e.g.: TSP (Peitsang et al., 2012), VRP (Peitsang et al., 2007) and Multi-depot VRP (Huang, 2006). Even when the aforementioned works are interesting in the sense that they show how EM principles can be used to solve combinatorial optimization problems, none of them has been able to generate state-of-the-art solutions to this problems.

3. The Hybrid Electromagnetic Algorithm

Considering the principle of action and repulsion in EM (Birbil & Fang, 2003), each solution corresponds to a charged point that is free in space, this charge being related to the value of the objective function. The charged point determines the attraction or repulsion of the points on the sample population. While better the value of the objective function is greater the value of attraction.

In its simplest version, the metaheuristic based on EM consists on 4 phases: (i) initialization; (ii) local search; (iii) calculation of forces; and finally (iv), movements. Particularly, in this work is proposed a hybridization of the EM (HEM), where the local search procedure is modified, a new step called *splitting* is added just after the local search for solution refinement purposes, and also, includes a rule for movement of the particles. Further details of every phase of the algorithm will follow in this section. Table 1 summarizes the parameters required by the HEM proposal.

Table 1: Parameters of HEM

m : n° of sampling points	l_k : lower limit on coordinate k
$MAXITER$: maximum number of iterations	$nSplittings$: n° of iterations in splitting method
$LSITER$: n° of iterations in local search	$nRoutes$: n° maximum routes in subset
u_k : upper limit on coordinate k	$wImprov$: iterations without improvement

Algorithm 1 presents the general outline of HEM used to solve CARP.

Algorithm 1: General scheme of HEM for the CARP

HEM Algorithm
1: Initialize_CARP ()
2: $time \leftarrow 0$
3: While $time < MAXTIME$ do
4: $localSearch$ ()
5: $splittingMethod$ ()
6: $F \leftarrow CalcF$ ()
7: $Move$ (F)
8: $time \leftarrow elapsedTime$ ()
9: End While

3.1. Representation of the solution

Considering the representation based on the concept of Random Key (Bean, 1994), and in the context of vehicle routing, TSP (Peitsang et al. (2012)) and VRP (Peitsang et al. (2007)), it is

possible to represent a solution for the CARP. An example is shown in Table 2, a problem with 8 required edges, with an unit demand for each edge and a maximum capacity of 3 units per vehicle.

Table 2: Representation of the CARP solution

		Route 2											
Required edges	Depot	1	11	4	Depot	2	14	Depot	5	15	8	Depot	
Position		3.5	5.4	2.1		1.5	5.0		5.8	6.7	7.6		
		Route 1					Route 3						

In accordance to this representation of the CARP solution, the example shows three routes that do not exceed the vehicle capacity on the route. These routes start and terminate in a node called depot. In the first row required edges, the connection of these edges to form a route is determined by a new way problem called *Shortest Path Problem* (in this case using the Floyd-Warshall algorithm). Following in the first row, for the example, the numbers 1, 11 and 4 represent directed edges. Finally, in row 2, it is shown that each edge has an associated quantity called "position", which is determined in the initialization of EM and subsequently modified during the remaining phases. Noting that the positions for the required edge correspond to coordinates in space for a solution or particle.

3.2. Initialization

Since the behavior of the EM algorithm is strongly influenced by the quality of the initial solution (Jamili et al. (2011)), we considered an initial solution given by the metaheuristic RandSHARP (González et al. (2012)). The RandSHARP procedure is based on the Savings Heuristic for the Arc Routing Problem (SHARP). The SHARP is an adaption to the CARP of the well known Clarke and Wright Saving (CWS) heuristic introduced by Clarke & Wright (1964) for the CVRP. This solution is further improved by applying a splitting method repeatedly (with a limit number of iterations without improvements, wImprov). This solution is then differentiated, an amount equal to the number of points in the problem (subtracting the first), through the acceptance of changes that do not necessarily reflect an improvement of splitting method. Already having determined the initial population of points for EM, subsequently, each coordinate of the points is initialized assuming a uniform distribution between the corresponding upper limit and lower limit. Then the points are presented in the space, with a similar value in the objective function and identifying the best point.

3.3. Local search

This simple procedure is used in order to gather local information for the point x^i . The single parameter that help this process is LSITER, indicating the number of iterations in the local search. The procedure takes point by point of the population ($\forall i \in m$), for each of these, the first route and a new randomly route are selected (completing a set of iterations LSITER, later the second, third, etc. route), for both routes are generated four random numbers; the first indicates the edge required in the first route, the second the other required edge present in the new route, the third determines the value of the position of the edge of the first route and analogously the fourth, determines the value of the position of the edge required in the new route. Subsequently, the edges are exchanged under all possible combinations, if that is feasible in terms of capacity, determining also new values in positions that do not affect the ordering of positions. This method is effective if there is a reduction in the cost of the route, representing a reduction. If this is the case, the modification is done and this solution is used as the base for future improvements (the point x^i is replaced by x^0), otherwise, modification is not performed.

3.4. Splitting method

The splitting method exploits the concept of divide and conquers, breaking iteratively in

subproblems that are themselves smaller instances of the same type of problem, Juan et al. (2011). Basically, the problem is randomly divided in different subproblems, each of them corresponding to a subset of required arc that is solved with the heuristic RandSHARP. This principle exploits the properties of RandSHARP: the heuristic has proven to be a very good choice for small problems and low computation time.

The above method considers two parameters: the first with the number of times that is applied RandSHARP (nSplittings) and the second parameter associated with the maximum size of the sub problem (nRoutes).

3.5. Calculation of forces

The principle of superposition (Figure 1), in the theory of electromagnetism states that "the force exerted on a point via other points is inversely proportional to the distance between the two points squared and directly proportional to the product of the two charges, Cowan (1968), thereby determining a key concept for calculating the forces between points.

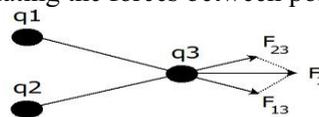


Figure 2: Principle of superposition. $\vec{F}_{i3} = \frac{q_3 q_i}{4\pi\epsilon_0\epsilon_r r^2} \vec{e}_r$, $i = 1, 2$.

In (Peitsang et al., 2012), shows the procedure for calculating the force. It can ensure that between two particles, the particle which has the greatest value attracts the other. Conversely, the point which has the worst value of FO repels the other. Now, x^{best} has the minimum value of FO, acting as an absolute point of attraction, i.e., can attract all other points of the population. Note also, that determine a direction through force vectors resembles the statistical estimation of the slope of f . However, the estimate obtained by the algorithm is calculated in a different way, because the direction depends on the Euclidean distance among the points.

Algorithm 2: Move of force

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Move (F)
1:  $A \leftarrow \text{repeatedSubset}()$ 
2: for  $i = 1$  to  $m$  do
3:   If  $i \in A$  then
4:      $\lambda \leftarrow U(0,1)$ 
5:      $F^i \leftarrow \frac{F^i}{\|F^i\|}$ 
6:     for  $k = 1$  to  $n$  do
7:       If  $F_k^i > 0$  then
8:          $x_k^i \leftarrow x_k^i + \lambda F_k^i (u_k - x_k^i)$ 
9:       else
10:         $x_k^i \leftarrow x_k^i + \lambda F_k^i (x_k^i - l_k)$ 
11:      end if
12:    end for
13:     $x^i \leftarrow \text{tuningSolution}(x^i)$ 
14:  end if
15: end for
    
```

3.6. Movement

This work has considered a fairly simple methodology to perform the movement. Basically, we consider that if more than one point has the same target value, for the repeated points, the algorithm development movement that is adjusted for $\text{tuningSolution}(x^i)$. The procedure adjusted

the solution without changing the order of edges, i.e., iteratively modifies the direction of each edge selected randomly. Then, after evaluating the total force of the vector F_i , the point is moved respect to the force by a random step length. Here the random step length λ , assumes a uniform distribution between 0 and 1 (due to ease of calculation). Then, the length is randomly selected to ensure that the points have a non-zero probability to move to regions unvisited. The algorithm 2 defines the phase of move in EM, whereas the best point and no repeated points, is not moved and is transported to subsequent iterations, thus avoiding the calculation of the total force.

3.7. The stopping criterion

The stopping criterion of HEM is given by a time limit equal to 180 seconds. This is in order to compare the results obtained with the solutions obtained by González et al. (2012).

4. Computational Experiments and Analysis of Results

To assess the performance of the proposed algorithm, four sets of classical instances were considered (Table 3), all of them obtained from http://www.wohlc-son.dk/sanne/research_carp.html.

Table 3: Four sets of benchmarks with a total of 87 instances

<i>Instance</i>	<i># of instances</i>	<i>Nodes</i>	<i>Arcs</i>	<i>Density</i>	<i>Source</i>
<i>gdb</i>	23	12	29	53.77%	Golden et al. (1983)
<i>val</i>	34	36	63	10.59%	Benavent et al. (1992)
<i>kshs</i>	6	8	15	55.95%	Kiuchi et al. (1995)
<i>egl</i>	24	109	144	2.65%	Li (1992) and Li & Eglese (1996)

It is worth to mention that egl instances come from real problems are considered one of the most difficult sets to solve. These dataset refers to winter gritting in the county of Lancashire, UK. The egl instances contain both required and not required arcs. These instances are grouped in two different network configurations with very low arc density. On the other hand, instances gdb and kshs are artificially generated and all the instances, in both sets, contain only required edges. Finally, the val instances are considered relatively difficult due to their large size (Wohlc, 2005). Table 3 shows averages of nodes, arcs and density for each set of instances.

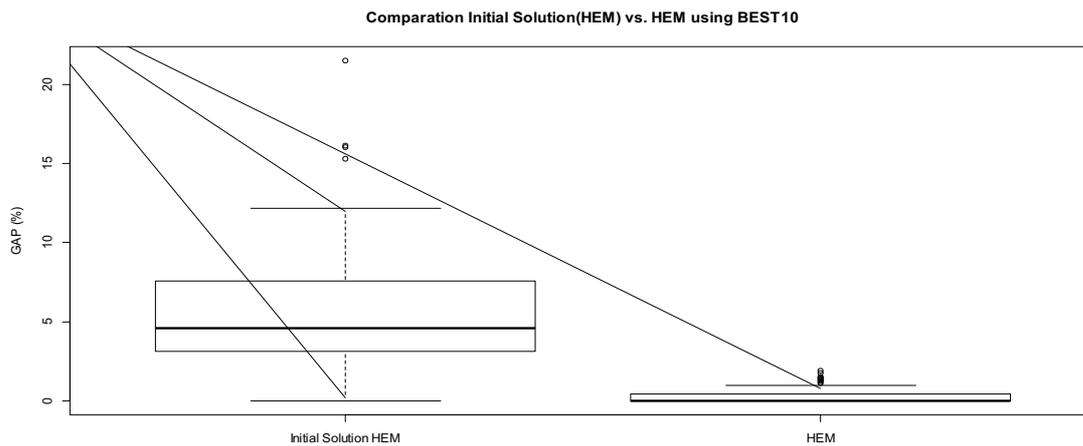
4.1. A comparison using the BEST10 and AVG10 metrics

For each instance we compare the performance of HEM with the results of RandSHARP and with the Best Known Solution (BKS). To evaluate the performance of the proposed algorithm, we implemented it in Java over NetBeans IDE 7.4 using Windows XP. A standard personal computer with an Intel® Core2® Duo 2.4 GHz processor and 2 gigabytes of memory was used in all the tests. 10 independent iterations (replicas) were run and the following parameters for HEM: number of particles = 10; MAXITER = 500, LSITER = 1000, u_k = number of requested edges, $l_k = 1$, nSplittings = 50, nRoutes < 9 and wImprov = 200 for all requested edges. These values determine better convergence of the hybrid approach, in a limit time of 180s. The consideration of a time limit was with the objective to compare the performance of RandSHARP with HEM. Both studies were developed in similar environments, RandSHARP with a standard personal computer Intel® Core2® Quad CPU Q9300@2.50 GHz and 8 GB RAM running the Windows®7 Pro operating system.

In order to adjust the parameters for the algorithm, various experiments were performed using instances egl. They consisted on changing parameters and observing the behavior of the algorithm. The formula used for quantifying the solution quality was the error percentage with respect to the BKS: $GAP = \left(\frac{BKS - Value_{HEM}}{BKS} \right) \times 100$, and $Value_{HEM}$ the cost for the solution obtained by

HEM. Thus, the smaller the GAP is obtained, the better the HEM solution will be. 10 executions were executed for every instance and taking two different values: (i) AVG10 with the average of the solutions found during the 10 executions, for measuring the variance of the solutions found; and (ii) BEST10, with the best solution found among the 10 executions, for measuring quality solution.

Figure 3: Best HEM initial solution vs best HEM solution (after 10 runs)



In order to evaluate the contribution of the initial solution HEM, Figure 3 shows a box plot comparing both GAPs: the one between the initial solution of HEM and BKS; and the one between the best solution obtained with HEM (after 10 runs) and BKS. HEM greatly improves the average quality of the solution and the variability of this. Furthermore, the GAP distribution is skewed and there are upper outliers, mainly by instances of the group val that have high GAP (see Table 5 HEM).

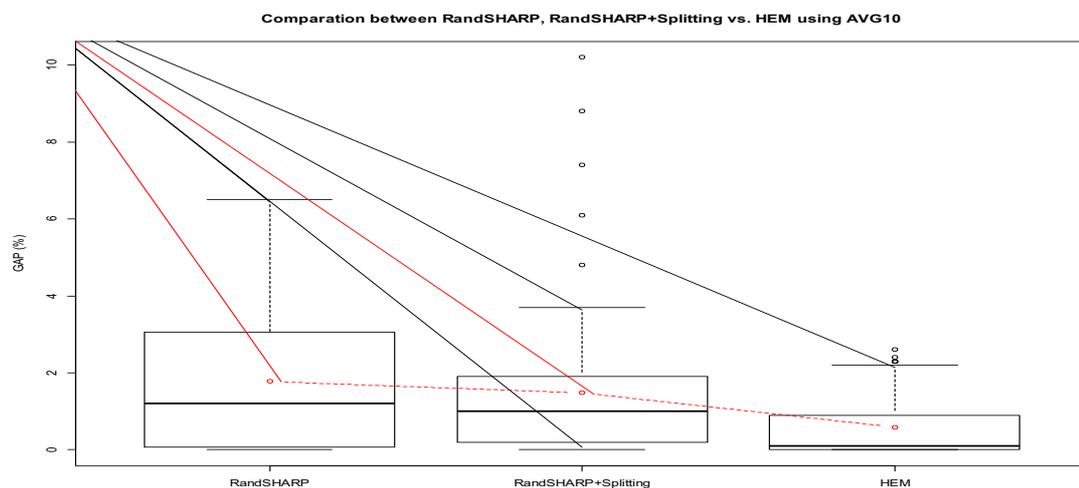


Figure 4: Comparison between RandSHARP, RandSHARP+Splitting and HEM using AVG10

Additionally, in order to evaluate the contribution of the splitting phase to the final solution, we performed a comparative study for all instances. We obtain an initial solution from RandSHARP and then we iteratively applied the splitting method, with a time limit of 180s., in order to find improvements. This proposal incorporates only one extra parameter (nRoutes) with respect to the

original RandSHARP. The box plot in Fig. 4 shows a comparison in terms of average gaps (AVG10), among RandSHARP, the modified RandSHARP with a splitting phase and HEM.

From the results in Figure 4: (i) the 50% of the solutions found by HEM are very close to BKS; (ii) the 75% of the solutions found by HEM are under a limit GAP that approximately reflects 50% of the solutions for the other proposals; and (iii) the variance of the GAPs for RandSHARP is greater than RandSHARP + splitting, analogously, the variance of RandSHARP + splitting method is greater than HEM. Considering also that the new HEM has an average error of 0.58% (AVG10), for the 87 instances, surpassing the results obtained for the RandSHARP and RandSHARP + splitting, with average results of 1.78% and 1.47% respectively. The proposed HEM presents a favorable structure that incorporates the help from splitting method with concepts based on an electromagnetic analogy to improve results of RandSHARP.

Table 4: Comparison between BKS, HEM and RandSHARP for the egl instances

Max. Time = 180 s												
					BEST10				AVG10			
					RandSHARP		HEM		RandSHARP		HEM	
Instance	V	E/Er	K	BKS	Value	GAP	Value	GAP	Value	GAP	Value	GAP
egl-e1-A	77	98/51	5	3548	3548	0.0%	3548	0.0%	3548	0.0%	3548	0.0%
egl-e1-B	77	98/51	7	4498	4498	0.0%	4498	0.0%	4512	0.3%	4508	0.2%
egl-e1-C	77	98/51	10	5595	5632	0.7%	5613	0.3%	5632	0.7%	5613	0.3%
egl-e2-A	77	98/72	7	5018	5022	0.1%	5018	0.0%	5043	0.5%	5018	0.0%
egl-e2-B	77	98/72	10	6317	6344	0.4%	6342	0.4%	6366	0.8%	6357	0.6%
egl-e2-C	77	98/72	14	8335	8477	1.7%	8335	0.0%	8518	2.2%	8406	0.9%
egl-e3-A	77	98/87	8	5898	5924	0.4%	5898	0.0%	5941	0.7%	5909	0.2%
egl-e3-B	77	98/87	12	7775	7847	0.9%	7799	0.3%	7868	1.2%	7824	0.6%
egl-e3-C	77	98/87	17	10292	10386	0.9%	10338	0.4%	10494	2.0%	10366	0.7%
egl-e4-A	77	98/98	9	6444	6504	0.9%	6476	0.5%	6530	1.3%	6490	0.7%
egl-e4-B	77	98/98	14	8983	9120	1.5%	9020	0.4%	9185	2.2%	9068	0.9%
egl-e4-C	77	98/98	19	11559	11886	2.8%	11649	0.8%	11907	3.0%	11736	1.5%
egl-s1-A	140	190/75	7	5018	5018	0.0%	5018	0.0%	5025	0.1%	5018	0.0%
egl-s1-B	140	190/75	10	6388	6435	0.7%	6416	0.4%	6450	1.0%	6437	0.8%
egl-s1-C	140	190/75	14	8518	8518	0.0%	8518	0.0%	8529	0.1%	8518	0.0%
egl-s2-A	140	190/147	14	9884	10076	1.9%	9962	0.8%	10140	2.6%	10021	1.4%
egl-s2-B	140	190/147	20	13088	13356	2.0%	13273	1.4%	13457	2.8%	13347	2.0%
egl-s2-C	140	190/147	27	16425	16752	2.0%	16609	1.1%	16803	2.3%	16712	1.7%
egl-s3-A	140	190/159	15	10220	10478	2.5%	10368	1.4%	10519	2.9%	10447	2.2%
egl-s3-B	140	190/159	22	13682	13986	2.2%	13863	1.3%	14081	2.9%	13988	2.2%
egl-s3-C	140	190/159	29	17189	17538	2.0%	17360	1.0%	17653	2.7%	17439	1.5%
egl-s4-A	140	190/190	19	12268	12647	3.1%	12469	1.6%	12737	3.8%	12547	2.3%
egl-s4-B	140	190/190	27	16267	16693	2.6%	16475	1.3%	16776	3.1%	16596	2.0%
egl-s4-C	140	190/190	35	20484	21071	2.9%	20791	1.5%	21149	3.2%	20900	2.0%
<i>Avg:</i>					<i>1.4%</i>		<i>0.6%</i>		<i>1.8%</i>		<i>1.0%</i>	

Table 4 shows a comparison for instances egl, between HEM and RandSHARP, both compared with BKS. The first column identifies the instance name. The column V refers to the number of nodes in the instance, the column E refers to the total number of arcs, the column E_r refers to the subset of required edges, the column K is the total number of vehicles for BKS. The values under the BEST10 column refer to the best solution obtained by HEM or RandSHARP after 10 independent runs. The values under the AVG10 column refer to the average solution obtained after 10 independent runs. In particular, for each tested instance, 10 independent replicas were run. Each replica was run for a maximum time of 180 s. Then, for each set of replicas, the best solution found (BEST10) as well as the average value of the replicas (AVG10) were registered.

From Table 5, the results of HEM outperform those of the RandSHARP and, in fact, they are able to obtain almost negligible gaps with the best-known results in the literature. Taking into account the limited computational time allowed for each test, this shows that the HEM can be successfully implemented for CARP in order to generate state-of-the-art results.

Table 5: Comparison between HEM and RandSHARP for the four instances studied

Instance Set	# of instances	Nodes	Arcs	Max. Time = 180 s			
				AVG (BEST10)		AVG (AVG10)	
				RandSHARP	HEM	RandSHARP	HEM
<i>gdb</i>	23	12	29	0.3%	0.0%	0.5%	0.1%
<i>val</i>	34	36	63	1.6%	0.2%	2.8%	0.7%
<i>kshs</i>	6	8	15	0.6%	0.0%	0.6%	0.0%
<i>egl</i>	24	109	144	1.4%	0.6%	1.8%	1.0%

4.2. A comparison using time-evolution of the GAP

In order to discuss the effect of computing time, several instances have been randomly selected and then solved using each of the discussed approaches. Note the execution time limit is 180 s., so that is interesting compare the evolution of both algorithms in one instance (see Fig. 5).

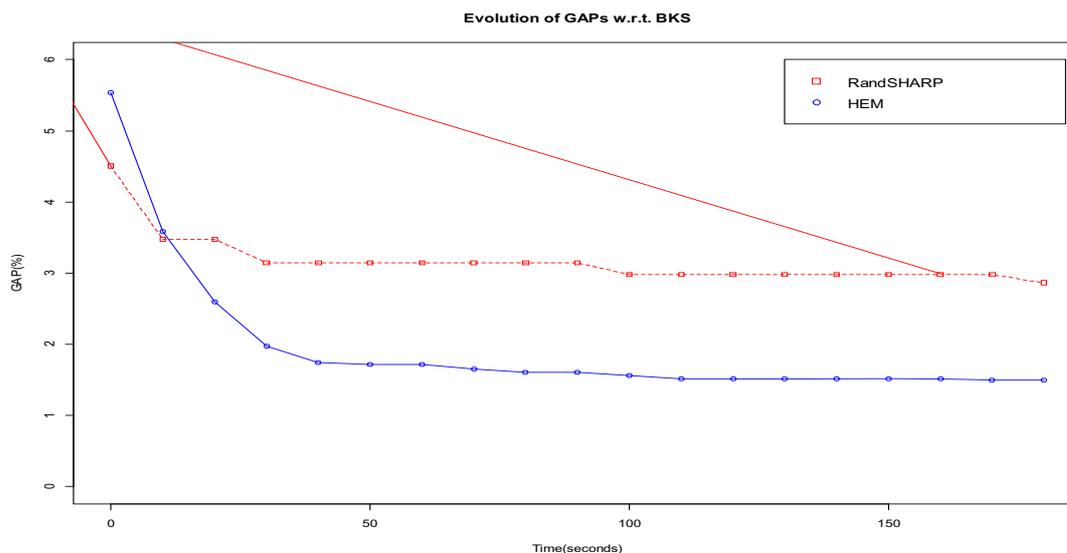


Figure 5: GAP time-evolution for the egl-s4-C instance

Figure 5 shows the evolution for the egl-s4-C instance, defined with 190 required edges and 140 nodes, where each proposal differs by a line type and point. Despite considering similar initial solutions between RandSHARP and HEM, being even worse the starting point for our HEM, it easily outperforms after 10 seconds the RandSHARP. After executing for 40 seconds it maintained a significant difference approximately constant. On the other hand, it is usually very difficult to beat a good solution in a very short time. However, our HEM reached a gap of approximately 1.8% within 40 seconds, showing that it is a very good option in terms of solution quality and time.

5. Conclusions

In this work we have developed a new HEM algorithm to solve the CARP. HEM has shown itself to be able to generate state-of-the-art solutions in short computing times, and it is even able to overcome other recent algorithms for the CARP. As far as we know, this is the first time that an algorithm inspired in the Electromagnetism-like Method has been able to provide state-of-the-art solutions in combinatorial optimization problems, which might reactive the interest of other researchers in this approach.

Particularly, the hybridization proposed consists in a mix between EM and two improvement methods (local search and splitting). Regarding the EM, we have made a modification to the local search and movement phases, for other side, the splitting method helps improving the solutions. The results showed finally the good performance of HEM, where the proposed algorithm is competitive compared to BKS and outperforming RandSHARP.

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