PUBLIC FACILITIES LOCATION: CASE STUDY OF CATHOLIC CHURCHES IN BARRA DA TIJUCA AND RECREIO, RIO DE JANEIRO - BRAZIL

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RESUMO

Este artigo visa examinar aspectos da implementação de um estudo de localização de uma rede de equipamentos públicos. O estudo aborda as etapas de modelagem do problema, concepção e correção do banco de dados, utilização de funções geográficas, monitoramento dos resultados e comparação das soluções obtidas por diversos algoritmos. O objetivo é determinar a localização de equipamentos tal que minimizem a soma das distâncias percorridas pelos seus usuários. O trabalho baseia-se em outros estudos propostos para localizar serviços de saúde, ensino e atividades gerais de serviços baseados no modelo da *p*-mediana, e resolvidos por métodos heurísticos conhecidos, um proposto e a solução exata. O estudo de caso abordou a localização dos templos católicos existentes na Barra da Tijuca e Recreio dos Bandeirantes.

PALAVARAS CHAVE. Localização de equipamentos urbanos, Sistema de Informações Geográficas, modelo da *p*-mediana.

Área principal. L & T

ABSTRACT

This paper addresses the problem of locating a network of public facilities. The study examines all requirements such as: modeling the problem, conception and correction of the data base, use of geographic functions, monitoring of results, and comparison among different algorithms. The objective is to determine the facility locations such as to minimize the sum of distances traveled by the users. The work is based on other studies proposed to locate health services, school location and general service facilities, based on the use of the *p*-median model, and solved by known heuristics, one newly designed, and the exact method. The case study has examined the location of catholic churches in Barra da Tijuca and Recreio in Rio de Janeiro.

KEYWORDS. Location of urban equipment, Geographic information system, *p*-median model.

Main area. L&T

1. Introduction

This paper is dedicated to the problem of locating a network of public facilities. This topic is certainly important whenever one considers an efficient and sustainable urban planning, since in any large city the location of any public facility affects the movement of people, the transportation flows, the infrastructure planning, the logistics of products, and the generation of residues, among other activities.

Such topic grows in importance when the study is directed to urban settings that face large growth rates and demand constant changes in the equipment network. This is the case that affects the majority of the large Brazilian cities, that continue to see significant density increases resulting from speculation and from internal migration. In this way, this paper involves a research theme that draws the interest of several aspects of the urban and environmental engineering.

This paper considers the location of a network of facilities, a data base rooted on geoprocessing, transportation flows etc. The main objective of this research refers to the location of catholic churches in two districts of Rio de Janeiro, in order to reduce the collective displacement or inconvenience experienced by the faithful to reach the churches, and to contribute to find their best sizes.

The case study covers two important and newly occupied districts of Rio de Janeiro, the former capital state of Brazil, namely Barra da Tijuca and Recreio dos Bandeirantes, in which the population growth process is intense and the location of churches or any other kind of facility is a matter of interest. The methodology used in this study might be applied to other religious denominations, but the choice of Catholicism is quite natural for reasons that will be made explicit below. To the best of our recollection the chosen subject is new and it has not been found in our bibliographic search.

The evolution of the Catholic Church in Brazil has been directly connected to the colonial authorities since the year 1500, with the arrival of the Portuguese conquerors to Brazil. Since then, the Church had the control of civilian registers, and the planning of political and administrative units around the country, which were commonly shared by the State and by the Church. Until the fall of the kingdom in 1889, which succeeded National Independence in 1822, the Catholic Church remained the unique religion allowed in the country. The fall of the kingdom marks the start of the Republic of Brazil, which brought the separation between Church and State, and the permission to other religious denominations.

In the present days, there is an impressive growth of the Evangelical and Pentecostal Churches, subdivided in a large number of sects, which individually do not represent large fractions of the population. In 2010, according to the National Census Bureau which has collected religious beliefs, there were in Rio de Janeiro: 51.0% Catholics, 15.7% agnostic, 8.9% belonging to God`s Assembly, 8.0% to general evangelical groups, 4.1 belonging to Spiritism, 3.6% were Baptists, 1.7% to the Universal Church of the Reign of God, and the remainingt to a large number of Christians and non Christians denominations. In any case, the Catholic Church is the largely predominant faith in the country with 64.4% of the Brazilian population a proportion that justifies the focus of this study. In addition, contrary to most of the other denominations, the Catholic Church has a centralized control which directs its expansion plans (IBGE 2010).

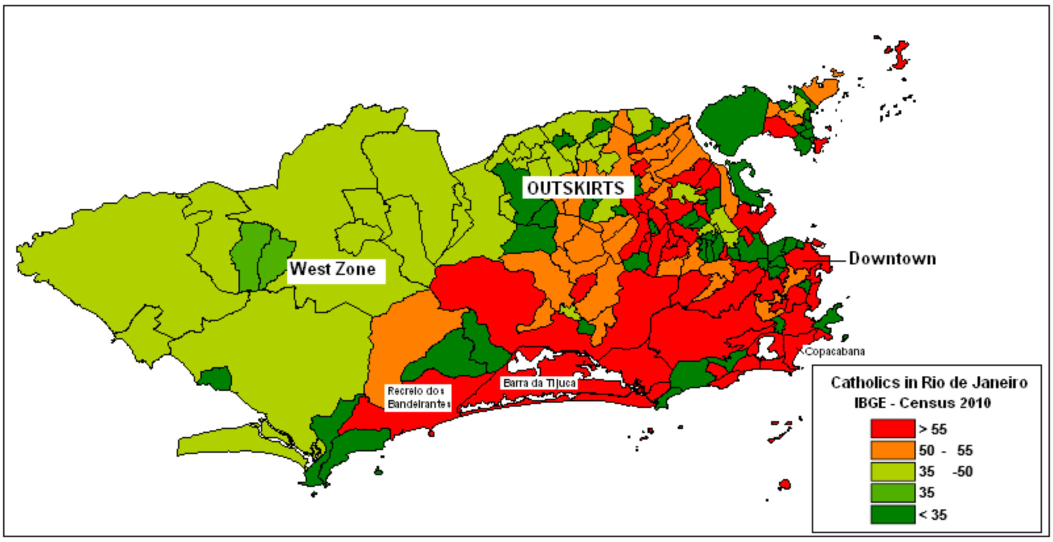
In an interview in Rio de Janeiro, during the Youth World Meeting 2003, the Pope Francis mentioned that the lack of physical proximity with the faithful might be a factor for loosing a large number of members.

In: "*Religião e Sociedades em Capitais Brasileiras*", Jacob *et* al. (2013) state three reasons for the continuous loss of ground of the Catholic Church in favor of the Evangelical Churches: (i) while the Catholic Church has a bureaucratic structure based in Rom and far from the local realities, the decentralized Evangelical Churches allow for a decision process strictly more connected to the claims and wishes of their herd; (ii) the education of the priests to serve the Catholic Church is sensibly longer and more demanding than the one required by the Evangelical pastors, restraining its evolution; (iii) the costs for constructing a Catholic parish are much higher than a similar construction for the Evangelical Churches; Guerra (2003), *apud*

Campos (2011), proposes some actions for the Catholic Church to revert the present declining trend, which in a free translation would be: "The growth of the Evangelical and loss of faithful has taken the Catholic Church to adopt new forms of actions such as the adoption of management models of private entrepreneurship style, centralization of administrative and financial management, the concentration of ecclesiastic power, the employment of professional staff, the use of marketing strategies, and the establishment of productivity goals for priests and bishops" .

We believe that, taking steps such as using an optimization method for locating a parish and defining the respective territory increases the proximity with the faithful, optimizes the occupation of spaces and introduces systematic procedures to evaluate and simulate is expansion, in agreement with the strategies proposed by Guerra (2003).

Map 1 presents the spatial proportion of Catholics in the municipality of Rio de Janeiro. They predominate in the Center and South Areas districts, and looses participation around several strongholds, in the West zone, and in the Metropolitan Area.



Map 1: Participation of Catholics in the city of Rio de Janeiro.
Source: Census Bureau Data (IBGE 2010).

In the rest of the paper we will describe the location problem (Section 2), case study description (Section 3), methodology and experiments (Section 4) and conclusions (Section 5).

2. Applied Location Methodology

Under varying formulations, location problems have continuously risen the interest of applied practitioners and academic professionals. Since remote times these problems have been object of studies by different sciences such as mathematics, physics, engineering, economy, operational research and, in general, resources management. The importance of this theme is being presently renewed with approaches that include environment factors.

If we take management science as the basis, or operations research as a more restricted term, the most popular studies are related to the location of public facilities, such as schools, maternities, health services, fireman, ambulances, post services and so on, inspired on direct needs of society, as well as radars, communication centers, oil platforms, distribution centers, etc., inspired on the needs of companies (White *et* al. 2011, Galvão *et* al. 2002, Yasenovsky and Hogdon 2007, Smith *et* al. 2009).

With the advent of Operations Research techniques, and progress in computing resources, the p-median model has become t a reference for locating problems. In analogy to the center of gravity which is the point that minimizes the weighted sum of distances to the central point, the *p*-median problem finds *p* points that minimize the sum of the weighted distance from all vertices to their closest point among these *p* vertices. In the latter case, the client may choose where to be served, which depends on *p* different choices or alternatives, and this favors the satisfaction of the geographically dispersed society's demand.

The usual definition of a facility location problem in a network consists of making the choices of locations in such a way that the collective costs, discomfort, or inconvenience be minimized. In other words, the goal is to minimize the weighted sum of the shortest distances between those claiming for the service and their closest facility.

Pizzolato *et* al. (2012) show that the *p*-median problem may be formulated as the PPM model below, in which the decision variables are x_{ij} that represent the decision of clients that are in vertex *i* to be served by the facility that exists in vertex *j* (case x_{ij}=1) or, otherwise, be served somewhere else (case x_{ij}=0):

(PPM)

$$Min\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i d_{ij} x_{ij} \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} x_{ij} = 1; \quad i \in N \quad (2)$$

$$\sum_{j=1}^{n} x_{jj} = p \quad (3)$$

$$x_{ij} \le x_{jj}; \quad i, j \in N \quad (4)$$

$$x_{ij} \in \{0,1\}; \quad i, j \in N \quad (5)$$

where: $[d_{ij}]_{n \times n}$ is the symmetric distance matrix, in which $d_{ii} = 0$, $\forall i$; $[x_{ij}]_{n \times n}$ is the allocation matrix, with $x_{ij} = 1$ if vertex *i* is allocated to vertex *j*, and $x_{ij} = 0$, otherwise; and also: $x_{jj} = 1$ if vertex *j* is a median and $x_{jj} = 0$, otherwise; *p* is the number of facilities or medians to be located; *n* is the number of vertices in the network and N = {1, ..., *n*}; w_i represents the weight of vertex *i*.

The PPM objective function (1) to be minimized is the weighted sum of distances of each vertex to its closest median; restrictions (2) and (4) require that each vertex *i* be allocated to an unique vertex *j*, which ought to be a median; restriction (3) determines the exact number of *p* medians to be located, while restriction (5) establishes integrality.

The *p*-median problems generally present a very large search space and their exact solutions require the use of methods based on Integer Linear Programming, with advanced algorithms. Heuristic methods, otherwise, are based on a simple logic and are, in principle, of easy implementation. On the other hand, these last methods do not warrant an optimal solution but rather a good one. In the study case below, the following methods and algorithms have been used:

- Evaluation of the actual geographical locations, or the present collective inconvenience, in order to quantify how good or poor the present locations of the churches are. In addition, some tests and quantitative indicators are derived such as: (i) the maximum and the average distance travelled by one devoted to reach its closest church; (ii) if one church is to be eliminated, what would be its consequences; in particular, which is the church whose elimination has the least inconvenience effects; and (iii) if a new church is to be constructed what would be its proposed location. The other methods ignore the present locations, and evaluate where the churches should be located.
- The Greedy Algorithm, which progressively finds locations or medians, that reduce the collective inconvenience;
- The early Algorithm of Maranzana (1964) starts from an arbitrary solution with *p*-medians

and, using a number of steps, moves this solution towards a better one. This work has tried a number of strategies to define the values of the initial solution.

- A Mix Algorithm, proposed by the authors, tries to improve the solution found by the previous heuristic method. Basically, it considers the attraction areas defined by the Maranzana method and for each area it identifies the vertices that belong to its immediate neighborhood areas with which a complete enumeration is made to find the location of the locally optimal medians. Thanks to the Geographic Information System (GIS) methodology, which generates a neighborhood matrix, the vertices that belong to the neighborhood areas are identified and the complete enumeration is locally applied.
- The exact or optimal solution was calculated by a branch and cut method, as described by Alzamora (2013). The optimal solution is certainly non realizable since the existing churches cannot be relocated, but it serves to quantitatively evaluate the present locations and the quality of the applied heuristics.

3. The Case Study and a General Description of the Situation

The object of study are the catholic churches and chapels that exist in the districts of Barra da Tijuca and Recreio dos Bandeirantes, in the municipality of Rio de Janeiro. The entire area has been subject to an intense urbanization process that started in the 80's, in accordance with the developing plan of the urbanist Lúcio Costa, which was conceived with high buildings located in closed condominiums, and *shopping centers*, interconnected by express ways that favor the use of private cars.

The region has faced a strong demographic process during the last three decades, as shown in Table 1. According to the data, while in the 1980-2010 period the population grew by 24% in the whole municipality of Rio de Janeiro, in the districts under study this growth was over 700%. On the other hand, official projections estimate that the present rate of growth rate will be maintained at least along the present decade (Lins *et* al. 2013).

Table 1: Evolution of the population in the area of study and in the city of Rio de Janeiro

YEAR	*Study Area*	*Rio de Janeiro*
1980	29,127	5,090,700
1991	77,836	5,336,179
2000	129,805	5,851,914
2010	218,063	6,323,037
2020 (*)	285,748	6,661,359

Source IBGE, Census data since 1980;
(*) Projection - Instituto Pereira Passos for the RA XXIV – January 2013(Lins *et* al. 2013).

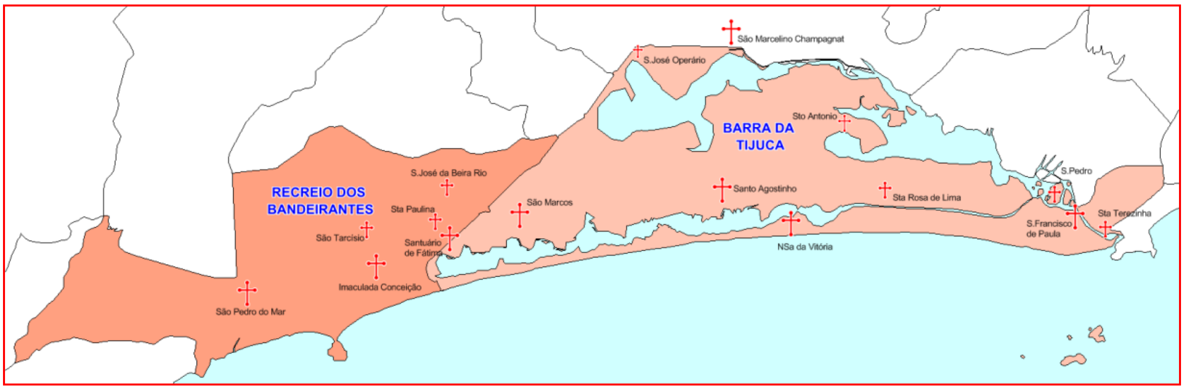
Quite often, this growth was anarchical and not in conformance with the original planning. This has brought several kinds of congestion, specially on the public highways, with a resulting loss of mobility and poor quality of life for its inhabitants.

Similarly, the network of catholic churches has not followed the population growth in the area, promoting excess of participants in the cults, very long displacements to reach a temple, and, consequently, evasion of many that have to relay on cars to reach the churches.

In this way, a study on the optimal location of the churches shall minimize the distances covered by the devoted, reduce the number of those that relay on private cars, reduce the air pollution, and the congestion. Benefits are certainly widespread and include better air quality, and increased attractivity for the Catholics.

4. The Methodology and Experiments

The fundamental assumption of the study is that users of the public facility prefer the one closest to their residence in a way to reduce their displacement or their inconvenience. For the case study, the catholic churches selected for the study are those that offer at least one Sunday service in the given districts of Barra da Tijuca and Recreio dos Bandeirantes. Those churches are shown in Map 2.

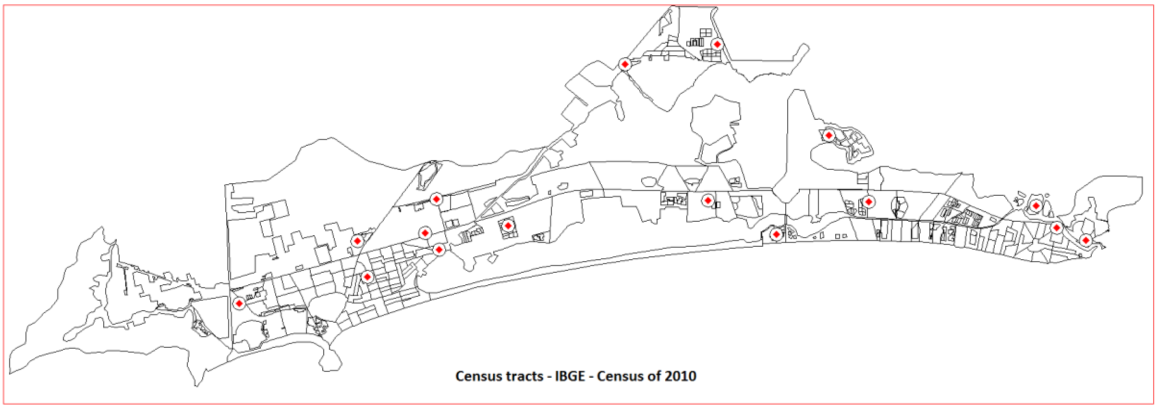


Map 2: Map of the region under study and the location of 13 churches and 3 chapels.

Field visits and interviews with some responsible at all parishes have been made and all sorts of tabulated data and maps have been collected using the IBGE Census Bureau and the Instituto Pereira Passos of the municipality of Rio de Janeiro.

Data have been compiled in tables and maps using the MapInfo Professional version 6 *software*, with programmed applications on the MapBasic 6.0 *software*, and images from the Google Earth applicative.

The population of those catholic declared residents have been collected, according to the IBGE Census Bureau 2010, by using census tracts, which are small units for controlling the space and collecting the data. Map 3 shows the existing churches and the 350 census tracts defined for the region. On average, each tract has 216 family units and 628 residents.



Map 3: Churches and Census tracts at Barra da Tijuca and Recreio dos Bandeirantes. Source: (IBGE, 2010).

Since every census tract informs the total population, the proportion of catholics in each tract was derived using an average evaluation, i.e., the number of Catholics were estimated using sample data made by the same Census Bureau for 2010.

The population of each census tract was allocated to its mass center, called its centroid. The location of every mass center was important for subsequently measuring distances between centroids. For this effect, ortophoto maps have been employed which aggregate tracts inside condominiums, as shown in Figure 1, and placing its centroids at points of larger concentration of houses, as shown in Figure 2.

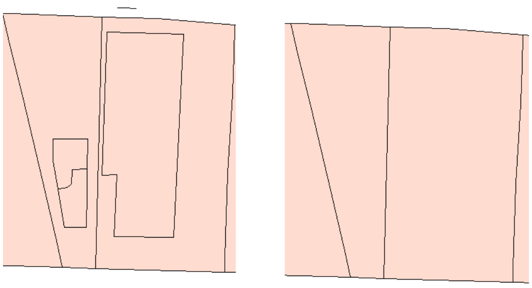


Figure 1

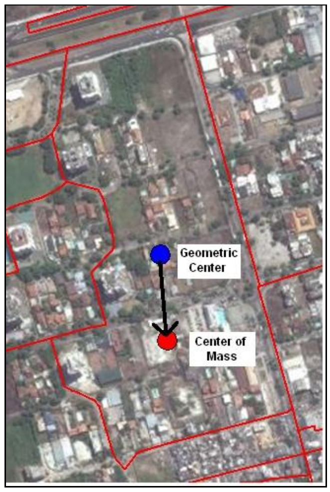


Figure 2

Computation of distances between every pair of vertices among the 238 centroids was made over the most important highways of the region. Using a network of auxiliary symmetric arcs, Figure 03 presents the connection of every centroid to the highway network. These connection points play the role of fictitious vertices without any population, but required for a precise measurement of distances between two vertices. The length of every arc was calculated with the use of the GIS.

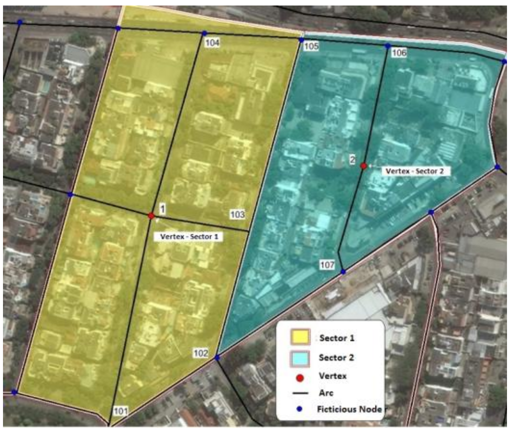


Figure 3: Auxiliary network of symmetric arcs.

Once the length of each arc of the network was determined, the Floyd Algorithm was employed to calculate distances between every pair of centroids, producing a symmetric distance matrix between all 238 centroids.

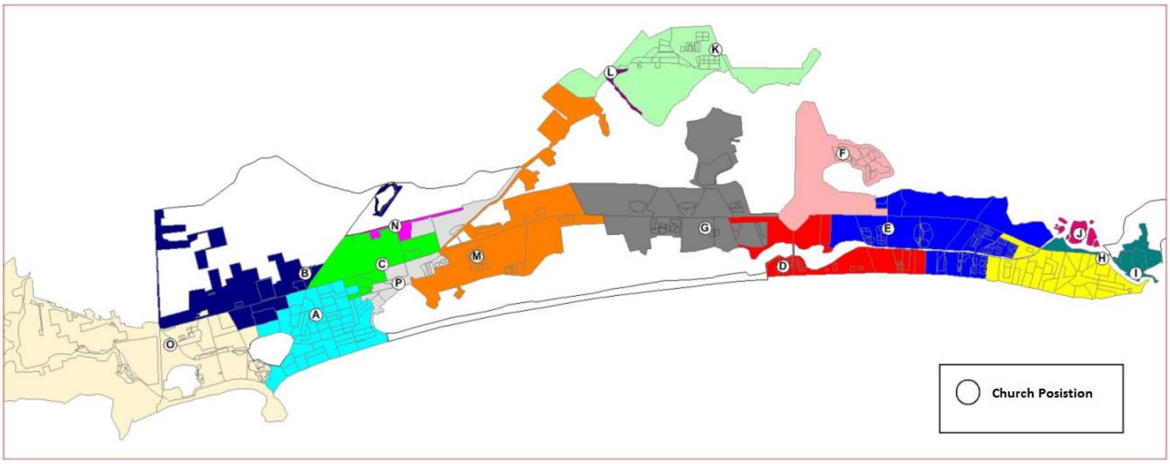
Next, an asymmetric matrix using the relation: $C_{ij} = w_i \times d_{ij}$ was generated. The elements of the 238 x 238 matrix indicate the weighted cost, or inconvenience, for the population that lives in vertex i which number w_i to reach the vertex j, which is at distance d_{ij}.

About nine different experiments have been made, described in Magalhães (2013), and summarized on Table 3. Note that the most relevant information is the value Z given on the second column of Table 3. The first three experiments refer to the existing churches network and rely on shortest distances calculations rather than models; the next five experiments apply heuristic methods; and the last experiment calculates the optimal solution. In all the experiments the following information have been collected: the population of Catholics and the total population; the church attraction area in square km, the maximum distance; the average distance; and the collective inconvenience Z/1,000.

In Experiment 1 the characteristics of the present church locations have been determined. Using different colors, Map 4 indicates the attraction areas for each church over the studied area. Table 2 lists the existing churches and indicates the relevant quantitative indicators. Note that the three small chapels, having a suffix (FIX) on Table 2 are considered to serve a small and specific local population, and have been excluded from the remaining experiments and computations.

Table 2: Qualitative indicators of the Attraction Areas of each existing church, experiment 1.

ID	Color	Church	Catholics	Population	Area (sqkm)	Max.Dist (m)	Mean Dist. (m)	Cost Z/1,000
A		Imaculada Conceição	20,497	33,983	2.95	2,278	804	16,483
B		São Tarcísio	6,400	11,453	3.00	2,959	1,438	9,205
C		Santa Paulínia	3,236	5,425	1.85	1,233	688	2,226
D		Nossa Senhora da Vitória	8,994	12,812	2.18	2,942	1,347	12,114
E		Santa Rosa de Lima	28,596	44,277	4.56	3,022	2,010	57,464
F		Santo Antonio	3,727	5,949	2.82	659	85	316
G		Santo Agostinho	14,075	22,215	5.26	1,815	935	13,157
H		São Francisco de Paula	15,176	22,954	2.24	2,638	1,252	19,002
I		Santa Terezinha	1,326	2,018	0.70	1,781	324	430
J		São Pedro (FIX)	1,065	1,670	0.24	0	0	0
K		São Marcelino Champagnat	11,762	20,088	3.90	2,822	1,075	12,645
L		São José Operário (FIX)	799	1,252	0.10	0	0	0
M		São Marcos	8,869	14,024	4.90	2,288	771	6,841
N		São José do Beira Rio (FIX)	3,064	5,349	0.22	0	0	0
O		Paróquia São Pedro do Mar	16,450	30,303	12.24	1,942	1,285	21,144
P		Santuário de Fátima	5,744	9,103	1.42	1,270	611	3,512
		TOTAL	**149,780**	**242,875**	**48.58**		**1,165**	**174,538**



Map 4: Attraction areas according to actual location.

Experiment 2 has evaluated the ideal location for constructing a new church (p=13+1). The result is shown in Map 5. The new solution has brought a 22 % reduction in the collective inconvenience, *Z*.

Experiment 3 has simulated the elimination of one church (p=13-1), in order to find the one whose elimination promotes the least increase in the collective inconvenience. The solution requires an increase of only 1 % in the value of *Z*.

Experiments 4 through 8 ignore the present location of the churches and using heuristic methods try to find their ideal location. Experiment 4 was called Mrz_Atu and has applied the

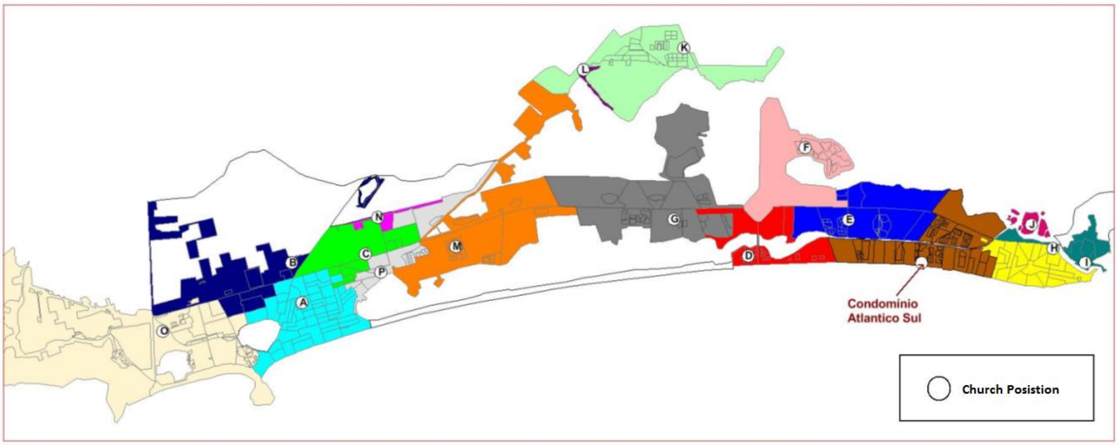
Maranzana Method, using as its initial arbitrary solution, *p=13*, the actual position of the existing churches.

Experiment 5 was called Misto_Mrz_Atu and has executed what we have called the Mix Method, after using the Method of Maranzana, and identified the neighborhood areas and vertices (Experiment 4).

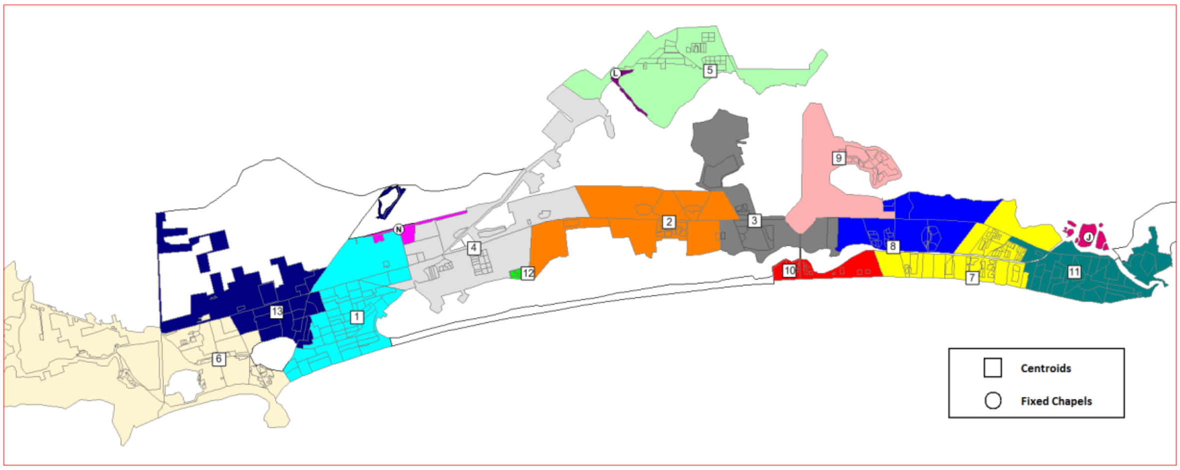
Experiment 6 has used the Greedy Method to identify the *p=13* points.

Experiment 7 was called Mrz_Gul and has applied the Maranzana Method using as its initial arbitrary solution, *p=13*, the solution obtained by the Greedy Method.

Experiment 8 was called Misto_Mrz_Gul and has applied the Mix Method to the result found by the Maranzana Method (Experiment 5).Incidentally, this result is presented on Map 6, and it is the best heuristic solution found. The collective inconvenience is 33 % below the present church distribution.

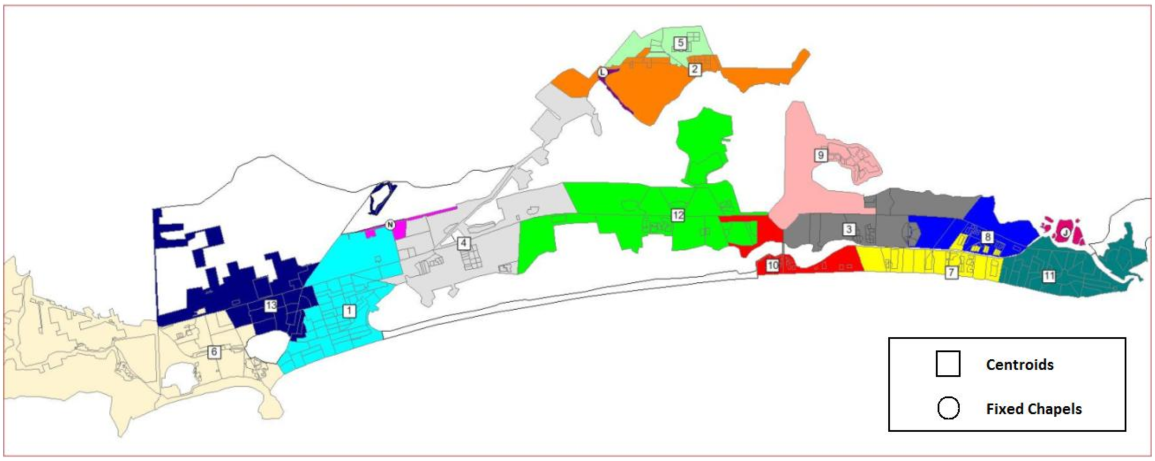


Map 5: Attraction Areas IF a new church is constructed in the South Atlantic Condominium.



Map 6: Solution resulted from the use of Misto_Mrz_Gul, the best heuristic solution found.

Finally, Experiment 9 finds the optimal solution using the branch and cut algorithm. Since the present locations are not subject to any change in their location the optimal location of the 13 points of the network had the objective to evaluate the relative quality of the heuristics used. The results are displayed on Map 7 and, interestingly, the collective inconvenience is 35 % lower than the value of *Z* in the present setting. Table 3 summarizes the results of the nine experiments.



Map 7: Attraction Areas: Optimal Solution

We have stated that the most important efficiency indicator is the value *Z*, or the collective inconvenience. However, Table 3 suggests other appropriate indicators such as the maximal distances, the average maximal distance, the homogeneity among regions, and so on. The homogeneity may be expressed by the dispersion coefficient of *Z*, which is calculated by the standard deviations of the costs *Z* divided by the average *Z*. Concluding the evaluation of the results, we may state that:

- With solutions not too far from the optimal one, the results obtained suggest that concerning the collective inconvenience the Mix Method has offered better proposals. The development of a neighborhood matrix has favored the identification of improved solutions with computational advantages since unnecessary searches might be avoided.

Table 3: Synthesis of the Indicators

Solution	Costs Z/1,000	Relative Variation In Costs %	Dispersion	Variation Dispersion %	Medium Distance (m.)	Maximun Average Distance (m.)	Variation Max.Dist Average %
Present Network	174,538		1.038		1,165	1,975	
p = 14	136,326	- 22 %	0.770	- 26 %	910	1,890	- 4 %
p = 12	175,751	+ 1 %	0.965	- 7 %	1,173	2,273	+ 15 %
Mrz_Atual	127,393	- 27 %	0.757	- 27 %	851	1,969	- 0 %
Misto_Mrz_Atu	121,322	- 30%	0.730	- 30 %	810	1,865	- 6 %
Greedy Method	144,209	- 17 %	1.042	+ 0 %	963	2,057	+ 4 %
Mrz_Gul	142,053	- 19 %	1.008	- 3 %	948	2,164	+ 10 %
Misto_Mrz_Gul	117,109	- 33 %	0.741	- 29 %	782	1,927	- 2 %
Exact	112,711	- 35%	0.655	- 37 %	753	2,033	+ 3 %

- The economies brought by the Maranzana and Mix Method are dependent of the quality of the initially arbitrated solution.
- The gains obtained regarding the collective inconvenience, *Z*, do not imply more homogeneous results when other indicators are considered.
- The averages of the maximum distances are uncorrelated to the collective inconveniences, *Z*.

5. Conclusions

According to the simulations performed, the additional procedure proposed in the present research – the Mix Method, which considers neighborhood areas of influence - was able to improve the current heuristic solutions. According to the results found, it was possible to conceive a solution which reduces by 33 % the total cost of displacement of the devoted, value only 4 % worst than the one obtained by the optimal solution. The result might suggest that the procedure is valuable for small size problem instances.

The use of geoprocessing methodologies brought an important contribution to the study, since it has identified distortions noted from the direct utilization of the data base with census tracts. This has allowed the refinement of the results through the use of automatized corrections on local problems and to determine the vertices of neighboring regions candidates to interact on the Mix Method. The results might be interpreted as distance cost indicators which, conjointly with other indicators, may offer elements for the decision making process based on quantitative methods.

It is certainly clear that this study do not consider any kind of displacement of churches or chapels from their present location. However, a detailed analysis of data and maps might contribute to dimension the capacity of the temples and the frequency of the masses, according to the estimated demands given by the models. The dimensioning is certainly dependent on the capacity of devoted per mess in each church.

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