PREVENTIVE HEALTHCARE ASSISTANCE PLANNING: OPTIMAL LOCATION AND STAFF SIZING CONSIDERING SYSTEM CONGESTION

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RESUMO
A alocação de facilidades de saúde é um tema recorrente na literatura atual, entretanto o tema é raramente combinado com o dimensionamento da equipe, que em termos de orçamento e tempo de processo é um elemento crítico. Apresentamos um modelo de otimização de planejamento de logística de facilidade de saúde que foca na minimização do tempo de processo e considera a congestão sem comprometer a sua estrutura linear. O modelo foi aplicado no planejamento de vacinação em massa contra Influenza no Rio de Janeiro. Os resultados ajudaram a analisar as localizações das facilidades existentes.


Área principal SA – PO na área da Saúde

ABSTRACT
Although health facility allocation has been a highly discussed theme in current literature, it is rarely combined with staff sizing, which, in terms of budget and process times, is a critical variable. We present an optimization model to plan health facilities logistic that focuses on process time minimization and considers congestion without compromising its linear structure. The model was applied to plan the mass vaccination against Influenza in Rio de Janeiro. Results helped analyzing the localization of current facilities.


Main area SA – PO na área da Saúde
1. INTRODUCTION

It is well known that efficient preventive healthcare assistance can minimize deaths and serious disease contractions in addition to increase the recovery probability when executed properly. Besides, preventive healthcare might impact positively in money saving, once it is usually cheaper than treatment itself. The World Health Organization (WHO) claims that non-communicative diseases (non-infectious) kills more people in the world than all other combined. In 2008, two thirds of the 36 million worldwide deaths were caused by this type of diseases, many of which could have been avoided by an effective preventive healthcare plan.

Whilst health services are usually essential and can, in many situations, be considered as inelastic, preventive healthcare might have different characteristics. Participants of preventive healthcare campaigns respond elastically to such programs, depending on the quality of the service perceived by the patients, being this strongly related with a well planned facility (i.e., the service provider) configuration. Ringel et al. (2012) studied the price effects on the healthcare demand but there may be many other variables that influence the demand. If the wait times of a program are extensive, or the facility is too far, the willingness of participating in such program decreases, which makes facility planning a crucial matter to this area. Although healthcare planning has always been a well-discussed theme in the optimization field, due to having this unique characteristic, not all models that address healthcare are suitable for preventive healthcare.

The preventive healthcare theme has been insufficiently explored in the present literature. In a survey written by Rais and Viana (2010) that addressed operational research in healthcare, only three papers were concerned with problems related with preventive healthcare (Aaby et al., 2006; Verter et al., 2002; Zhang et al., 2009). Zhang et al. (2010, 2011) subsequently addressed the theme. In their papers, Verter et al. (2002) and Zhang et al. (2009, 2010, 2011) formulated models that aimed to maximize the number of participants and that were solved by heuristic methods. Verter et al. (2002) and Zhang et al. (2012) formulated a facility location model that assumes that participants would choose the closest facility to them. While Verter et al. (2002) formulation addresses the important matter of facility location, it does not consider congestion, that is, once the participants have decided to dislocate to the facility, it would make no difference for them to wait little or long for the service.

On the other hand, Zhang et al. (2009, 2010, 2012) consider in their work aspects related with congestion and the first two papers assume that participants would choose the facility that would allow them to minimize participation time (comprised by travel, queue, and service times). Zhang et al. (2009) formulates the congestion effect as a single server queue, which means that the staff size is not a decision to be made by the model. Parker and Srinivasan (1976) modeled waiting time as a linear function of the number of clients, which can be too simplistic depending on the problem.

Marianov (2003) also considered congestion in his non-essential service location model, but with multiple server queues. The article aimed to maximize expected participation considering demand elasticity in both travel time and queue size factors and the model using heuristics based on Langrangean relaxations. Similarly to Zhang et al. (2009), the staff size was not a variable to this model, that is, assuming that a facility was opened, the number of servers was fixed. Zhang et al (2010, 2012), on the other hand, considered the number of servers in each facility as a decision to be made.

User choice criteria are also a crucial aspect to preventive healthcare modeling. Choosing facilities that would allow participants to minimize participation time, as considered in by Zhang et al. (2009, 2010), might not be suitable for every preventive healthcare program once, provided that, the programs are quite often temporary and the participants may not have time to gather information
on which facility minimizes their time, such as in mass vaccination programs. Parker and Srinivasan (1976) modeled user choice by assuming that the participants would choose the facility that would maximize their utility. Many authors assumed that participants would choose their closest facility, such as Verter et al. (2002), Berman et al. (2006) and Zhang et al. (2012), which is a reasonable assumption for temporary preventive programs.

Demand elasticity is also important once it has a direct impact on program participations. As it was previously stated, preventive healthcare is not considered essential and, therefore, presents an elasticity that cannot be disregarded. Some authors modeled the demand elasticity only for distance parameters, which means that the probability of attendance will only decrease with the distance between the participant and the facility, as considered in Verter et al. (2002). Therefore, no participants that arrive at the facility will abstain themselves from getting the service. Another way to model elasticity, used by Parker and Srinivasan (1976), is in terms of total utility, which is calculated as a function of some factors, such as accessibility and price and quality of care. Lastly, some authors chose to model elasticity as a function of total process time (Zhang et al., 2009; 2010; Marianov, 2003).

In this paper, we consider the preventive healthcare-planning problem through the use of operational research tools. We present an optimization model based on mixed integer programming that addresses the issues of locating health facilities and staff sizing, while taking into account the uniqueness of preventive healthcare characteristics, focusing on process time minimization and considering congestion without compromising the linear structure of the optimization problem. The elasticity of the demand was also considered as the expected attendance was calculated as a function of the total process time (travel, queue and service times). This paper, therefore, aims to extend Verter (2002) and Lapière’s (2002) model by adding congestion constraints and premises while focusing on preserving the linear structure and based on the use of off-the-shelf mixed-integer program (MIP) optimization tools.

To instance the model, we chose the process of mass vaccination against Influenza in Brazil. Although Brazil has been managing to achieve this number of vaccinations every year there might be still some concerns. First, the budget addressed for this cause might be unnecessarily high. Second, with the lack of an efficient planning, vaccines might get rotten since they possess considerably short expiring dates. Our main goal is to validate our model formulation with the help of a “toy case” and obtain relevant insights on the vaccination process in that area. In the second section, the model is discussed conceptually and in section 3 its formulation is presented. In section 4, the model results of our toy case are given as well as the discussion of the final results of Rio de Janeiro city. At last, we draw some conclusion in section 5.

2. PROBLEM DESCRIPTION

The central issue addressed in this paper is the location and staff sizing of preventive healthcare facilities, considering demand elasticity, characteristic in preventive healthcare and user choice. Facility localization and staff sizing will be defined with the support of a mixed-integer programming (MIP) model, while its congestion will be modeled considering queue theory.

The objective of the proposed model is to minimize the average process time (comprising travel, queue and service times) by locating facilities and allocating servers considering the total available budget and a maximum staff size. Figure 1 presents an illustrative example of the nature of the decisions that must be made in the present context. Our aim is to, given a set of possible facilities sites and a set of population centers, decide which facilities should be opened, which population centers should be assigned to which facility and, lastly, how many servers should be allocated in each facility. In the example given in Figure 1, the model have decided to open facilities 2 and 3, assign 4 population centers to facility 2 and 2 to facility 3, and allocate 4 servers to facility 3.
In order to represent the problem by means of a linear model, a few assumptions had to be made. First, the assumption that participants choose the closest facility is made. We also assumed that all facilities have the same size and provide services with the same quality. For simplicity purposes and aiming at modeling the problem as a MIP, population centers were also assumed to have the same size and a maximum number of servers per facility.

Due to complications in modeling queue time for non-allocated population centers, the premises that all centers should be allocated into facilities and that there is a maximum distance (or time period) that the participants are willing to accept in order to receive treatments, originally proposed by Verter et al. (2002), were incorporated in the proposed model.

To calculate queue time, a Poisson distribution was used to model the arrival rate and service rates. The average service rate was also assumed to be the same to all centers. The system was assumed to be an M/M/m/K queuing system, which means that the system has multiple servers and m servers and a maximum queue size of k.

The elastic demand has been modeled as a function of the total process time. For the sake of simplicity, the elasticity it has been modeled as a linear decaying function, such as in Verter and Lapierre (2002) and Zhang et al.(2008), although in our case, since it is not represented inside the model, it can be represented by much more complex functions, like the exponential curve, represented in Berman and Parkan (1981), Berman and Kaplan (1987), Berman (1995) and Berman and Drezner (2006).

3. SOLUTION APPROACH

3.1. QUEUE PROPERTIES

Unlike original maximal covering location problems (MCLP), the facility service capacity is considered in this problem, as the process time increases with the augment of the queue. The queue time is calculated as a function of the number of servers made available in the center and number of population centers assigned to that facility. Before obtaining the expected participation time, the probability of having k people in the queue is calculated as shown in model 1.

\[
P_k = \begin{cases} 
\frac{\rho^k}{k!} \cdot p_0 & \text{k } \leq n \\
\frac{\rho^n}{n!} \left( \frac{\rho}{n} \right)^{k-n} \cdot p_0 & \text{n } < k \leq n + \text{Tr} 
\end{cases}
\]
Where $n$ is the number of servers, $p_0$ is the probability of having 0 customers in the system, $\rho$ is the offered load, calculated by $\frac{\lambda}{\mu}$ where $\lambda$ is the arrival rate and $\mu$ is the service rate. To simplify the arrival rate calculation, we considered it homogeneous throughout the mass vaccination campaign, that is, we divided the amount of possible participants by the total time of the vaccination campaign. The M/M/m/K model considers the following premises: (1) $n$ servers; (2) Poisson input; (3) Exponential service times; (4) Finite number of waiting positions ($Tr$); (5) Finite storage; (6) Total number of system places is equal to $Tr + n$.

As it can be noticed, all probabilities are described as a function of $p_0$ and, therefore, can only be found if the value of $p_0$ is calculated. Since the sum of all probabilities has to be equal to 1, we simply state that $p_0 = 1 - \sum_k p_k$. Here, we consider the size of the queue to be finite (represented by $Tr$) in order to simplify our model formulation.

After the calculation of $p_0$ is done, the probabilities of having $k$ people in the system $p_k$ can be found. The average number of customers in the system represented by $N$ is calculated as $\sum_k kp_k$.

The effective arrival rate, which is the arrival rate ($\lambda_{eff}$) that considers the fact that, when the maximum system queue capacity is achieved, no arriving people would be allowed to stay in line, is calculated by diminishing from the original arrival rate $\lambda$ the proportion of time the system stays full, that is $P(Tr)$. Therefore, $\lambda_{eff} = 1 - P(Tr)$ and, thus, the average queue time is obtained through $\frac{N}{\lambda_{eff}}$.

### 3.2. OPTIMIZATION MODEL

First, we propose a non-linear integer model based on the premises and objectives stated above. The mathematical formulation of the model is represented below:

<table>
<thead>
<tr>
<th>Sets</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Population centers</td>
</tr>
<tr>
<td>$B$</td>
<td>Number of population centers allocated on a vaccination center</td>
</tr>
<tr>
<td>$J$</td>
<td>Vaccination centers</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of employees on a vaccination center</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{ij}$</td>
<td>Variable that equals 1 if population $i$ is allocated in vaccination center $j$</td>
</tr>
<tr>
<td>$y_j$</td>
<td>Binary variable that equals 1 when the vaccination center $j$ is open</td>
</tr>
<tr>
<td>$w_{nj}$</td>
<td>Binary variable that equals one when vaccination center $j$ has $w$ employees</td>
</tr>
<tr>
<td>$\theta_{bij}$</td>
<td>Variable that represents the product $w_{ni} \times z_{bj}$</td>
</tr>
<tr>
<td>$z_{bj}$</td>
<td>Binary variable that equals one when $b$ populations are allocated in vaccination center $j$</td>
</tr>
</tbody>
</table>

The model is formulated as follows:

$$
\max \sum_i \sum_j x_{ij} \left[ p - \frac{p}{T} \left( \frac{d_{ij}}{v} + \sum_b \sum_n t_{bn} w_{nj} z_{bj} \right) \right]
$$

s.t. $\sum_j x_{ij} = 1, \forall (i,j): i \in I, j \in J$, $d_{ij} \leq D_{\text{max}}$.  \hspace{1cm} (1)

$x_{ij} \leq y_j, \forall (i,j): i \in I, j \in J$ \hspace{1cm} (2)
Constraints (2) to (4) are similar to the ones in Verter and Lapiere’s (2002) model. Constraint (2) state that every population center has to be allocated into exactly one facility whose distance from the population center is smaller than the maximum distance stated and constraint (3) states that no population center can be allocated into a closed facility.

Constraint (4) formulation has an ordering purpose. Given a population center $i$, the facilities are cardinally organized as a function of their distance from that population center, receiving a coefficient $i_{m}$. In other words, the facility $j$ that is the closest to population center $i$, for example, would receive the coefficient $i_{1}$.

Constraint (5) limits server and facility investment to a predefined budget, while constraints (6) and (7) state that servers can only be hired to an opened facility and limits the amount of servers per facility respectively. Constraints (8) and (9) model the variable $z_{b,j}$, stating that can only be one number of population centers allocated to each facility (8) and that this number has to be exactly the number of allocated population centers modeled by $\sigma x_{i,j}$ for each center $i$. Variable $z_{b,j}$ is, in fact, necessary once the linearization of a product of variables would not be possible if one of them was represented by a sum ($\sigma x_{i,j}$).

Constraint (10) limits the total of people assigned to facility $j$ as a percentage $\alpha$ of the total facility capacity. As the queue represented in this problem is finite, if this constraint did not exist, the problem would be able to assign population centers to a facility without considering the consequences of not serving the amount of people that would exceed the maximum queue size, which would decrease the average process time. The integrality requirements on the decision variables are expressed by (11) and (12).

As the queue time depends on both staff number and number of allocated population centers, its formulation in the objective function (1) resulted in a nonlinear model. Since both variables are binary, it is possible to linearize the product $w_{n,j}z_{b,j}$ in order to be able to use integer optimization approaches. It is given below the exact linearization of the proposed model.

$$\min \frac{1}{2} \sum_{i} x_{i,j} \theta_{i,j} + \sum_{b} \sum_{n} \sum_{t_{b,n}} \theta_{b,n,j}$$

$$(1)$$

$$(2) \cdot (10)$$

$w_{n,j} \geq \theta_{b,n,j}, \forall (b,j): b \in B, j \in J$$

$$(11)$$
\[ z_{bj} \geq \theta_{bnj}, \forall (b, n, j): b \in B, n \in N, j \in J \]  \hspace{1cm} (12)

\[ \theta_{bnj} \geq w_{nj} + z_{bj} - 1, \forall (b, n, j): b \in B, n \in N, j \in J \]  \hspace{1cm} (13)

\[ y_j, w_{nj}, z_{bj} \in \{0, 1\}, \forall (b, n, j): b \in B, n \in N, j \in J \]  \hspace{1cm} (14)

\[ \theta_{bnj}, x_{ij} \in [0, 1], \forall (i, n, j): i \in I, n \in N, j \in J \]  \hspace{1cm} (15)

Constraints (11) to (13) represent the linearization of the product \( w_{nj} \times z_{bj} \). Since both of them are binary variables, it was possible to transform their product into a linear variable \( \theta_{bnj} \) that has the same results as the product would (1 if both variables \( w_{nj} \) and \( z_{bj} \) had value of 1 and 0 otherwise). Unfortunately, this linearization has some implications. As a new variable had to be created, the problem size was increased, which might jeopardize computational feasibility. Constraints (14) and (15) express the integrality requirements on the decision variables.

Instead of modeling the expected participation, in order to transform the problem into a MIP, we chose to minimize the average process time in (1). Note that, although these objective functions do not have necessarily the same optimal results, it is possible to show that they are strongly related, once the expected participation decreases with the increase of process time as shown in next session example.

4. COMPUTATIONAL EXPERIMENTS

4.1. SMALL EXAMPLE

In this section, we present the case study that was used to validate the in terms of decision and obtained results. The data used to test is originally from Zhang et al. (2008). In this example, four location candidates for the facilities and ten population centers were considered. The computer used had the following configuration: Intel i5, 21.6 GHz, 4Gb of RAM.

As Zhang et al. (2008) uses travel time as parameter and this model uses distance over time, the travel time was converted into distance by assuming an average speed of 14km/h (based on the Geographical Information System used to obtain distance information). Facility candidates locations where named A, B, C and D and population centers where numbered from 1 to 10. The results of the time vs. distance conversion, in kilometers (km), are represented in Table 1.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.88</td>
<td>94.5</td>
<td>78.82</td>
<td>84.42</td>
</tr>
<tr>
<td>2</td>
<td>62.3</td>
<td>100.8</td>
<td>107.94</td>
<td>83.02</td>
</tr>
<tr>
<td>3</td>
<td>77.56</td>
<td>92.12</td>
<td>74.06</td>
<td>33.04</td>
</tr>
<tr>
<td>4</td>
<td>114.52</td>
<td>69.58</td>
<td>94.78</td>
<td>116.34</td>
</tr>
<tr>
<td>5</td>
<td>32.62</td>
<td>19.04</td>
<td>16.24</td>
<td>115.22</td>
</tr>
<tr>
<td>6</td>
<td>45.5</td>
<td>39.2</td>
<td>21.42</td>
<td>12.32</td>
</tr>
<tr>
<td>7</td>
<td>100.66</td>
<td>80.36</td>
<td>51.1</td>
<td>25.76</td>
</tr>
<tr>
<td>8</td>
<td>27.44</td>
<td>18.9</td>
<td>53.9</td>
<td>68.74</td>
</tr>
<tr>
<td>9</td>
<td>114.94</td>
<td>38.5</td>
<td>68.88</td>
<td>108.5</td>
</tr>
<tr>
<td>10</td>
<td>85.82</td>
<td>59.36</td>
<td>60.06</td>
<td>95.76</td>
</tr>
</tbody>
</table>

Table 1 – Travel distance from population centers to facilities

Some parameters were not present in Zhang et al. (2008) and had to be estimated. They are presented in Table 2.
It is important to notice that either budgets or center and server costs have arbitrary values. We chose, therefore, to use arbitrary proportions and numbers instead of actual currencies. In addition, depending on the purpose of modeling, the proportions between facility and staff costs and the budget might drastically change. For example, this model can be used for both planning a campaign and, consequently, preparing existent facilities to it or to choose the location of a new facility. The proportion of costs and budgets in these examples might be completely different from each other.

The results for different budget values are shown in Table 3, each problem took less than one minute of CPU time. Constraint (10) was relaxed, otherwise case #2 (budget = 18) and case #3 (budget = 20) would not be feasible. In this case, some people did not get served due to the fact that there were more people allocated than the facility capacity in those problems was considered in the calculation of the expected time. Each line represents the optimal result of the model calculated with the budget parameter set at the value presented in the table’s column “Budget”. The second column named “Average Time” is the objective function value and represents the average time one might spend at the vaccination process (travel, queue and service times combined). The column named “Num Centers” shows the number of opened facilities, followed by the number of allocated servers in each center (“Serves in”). The last four columns named “Alloc in” represents the number of population centers allocated in each facility.

Moreover, it is possible to see that the best option is always to balance the amount of allocated people in each opened facility. In some cases, some facilities are not very well centralized in the area, which means only a few population centers would consider travelling to that center, as happened to facility C. Only a single population considered facility C as its first option, therefore the model preferred to increase staff in the other facilities than opening this facility. As it can be seen in column 6 and 10, no server or population center have being allocated in facility C, which means it was never opened.

Additionally, it is also clear that, at the beginning, the average time decreases quite fast as the budget increases. Case #2, for example, provides to the vaccination campaign participants an average process time of 11 hours and 30 minutes approximately, which infeasible in real world. Although Case # 3 still leads to the unreasonable scenario of 7 hours of process time, with an increase of 10%, the model was able to reduce time by 36%.

<table>
<thead>
<tr>
<th>Budget</th>
<th>Average Time</th>
<th>Num centers</th>
<th>Servers in A</th>
<th>Servers in B</th>
<th>Servers in C</th>
<th>Servers in D</th>
<th>Alloc in A</th>
<th>Alloc in B</th>
<th>Alloc in C</th>
<th>Alloc in D</th>
</tr>
</thead>
<tbody>
<tr>
<td>O=15</td>
<td>Inf.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>O=18</td>
<td>11.520</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>O=20</td>
<td>7.364</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>O=25</td>
<td>3.196</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2 – Example parameters
Furthermore, by plotting the resultant optimum average time for each budget, it is possible to see that, at some point, increasing the budget will not be relevant in reducing the average process time, as it is shown in Figure 2. This observation reinforces the notion that an intelligent planning might mean a considerable budget reduction. These results suggest that the proposed model can be helpful in the process of stipulating a reasonable budget to be spent in a vaccination campaign or even a preventive healthcare plan. In this case, the best option would be to spend between 25 and 35 in the campaign, depending on the elasticity of the demand.

After obtaining the optimum average time, we can finally calculate the expected attendance of the vaccination program. Since we are optimizing the average time, the model does not guarantee that the time for each individual will be minimum.

To calculate the expected attendance, we first defined the probability curve. As Verter et al. (2002), we have decided to use a decreasing linear curve as well, with the exception that we have considered that the probability should not start decreasing before the process time of one hour. We have used the value of 0.95 for $p$, as established by Verter et al. (2002). This value represents the proportion of population that would participate in the campaign if the distance from population center and facility were negligible. In our model, $p$ represents the proportion of population that would participate in the campaign if the vaccination process time is less than 1 hour.

The expected attendance results are shown in Table 4. The second column ("EA") represents the expected attendance represented as percentages. For example, for Case #3, the expected population attendance is 12.2%. The third column ("Time 1") represents the process time for population center 1 and so on.

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>EA (%)</th>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
<th>Time 4</th>
<th>Time 5</th>
<th>Time 6</th>
<th>Time 7</th>
<th>Time 8</th>
<th>Time 9</th>
<th>Time 10</th>
</tr>
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<tbody>
<tr>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>18</td>
<td>0.0</td>
<td>22.728</td>
<td>22.628</td>
<td>19.058</td>
<td>21.668</td>
<td>18.058</td>
<td>17.578</td>
<td>18.538</td>
<td>18.048</td>
<td>19.448</td>
<td>20.938</td>
</tr>
<tr>
<td>20</td>
<td>12.2</td>
<td>22.728</td>
<td>22.628</td>
<td>19.058</td>
<td>5.042</td>
<td>1.432</td>
<td>17.578</td>
<td>18.538</td>
<td>1.422</td>
<td>2.822</td>
<td>4.312</td>
</tr>
<tr>
<td>25</td>
<td>47.1</td>
<td>6.102</td>
<td>6.002</td>
<td>2.432</td>
<td>4.999</td>
<td>1.389</td>
<td>0.952</td>
<td>1.912</td>
<td>1.379</td>
<td>2.779</td>
<td>4.269</td>
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<tr>
<td>30</td>
<td>47.1</td>
<td>6.102</td>
<td>6.002</td>
<td>2.432</td>
<td>4.999</td>
<td>1.389</td>
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<tr>
<td>35</td>
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<td>0.969</td>
<td>4.499</td>
<td>2.409</td>
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<td>1.417</td>
<td>0.929</td>
<td>1.889</td>
<td>1.407</td>
<td>2.807</td>
<td>6.179</td>
</tr>
<tr>
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<td>56.2</td>
<td>0.948</td>
<td>4.478</td>
<td>2.409</td>
<td>4.998</td>
<td>1.388</td>
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<td>1.378</td>
<td>2.778</td>
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<tr>
<td>45</td>
<td>56.4</td>
<td>0.948</td>
<td>4.478</td>
<td>2.388</td>
<td>4.991</td>
<td>1.381</td>
<td>0.908</td>
<td>1.868</td>
<td>1.371</td>
<td>2.771</td>
<td>6.158</td>
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<tr>
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<td>0.941</td>
<td>4.471</td>
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<td>4.991</td>
<td>1.381</td>
<td>0.908</td>
<td>1.868</td>
<td>1.371</td>
<td>2.771</td>
<td>6.151</td>
</tr>
</tbody>
</table>

Table 4 – Expected attendance and average process times

There are some remarkable results in this table. First, there are some cases in which even though the budget was increased, the process time in some population centers also increased. For example as we increased the total budget from 30 to 35 the process time in center 4 increased from 4.99 hours...
to 5.02. As the model aims to decrease only the average time between all population centers, it does not necessarily minimizes each process time, causing such results. It is important to note that these fluctuations have minor impact in the resulted expected attendance.

Second, it is clear that as the average time decreases, the expected attendance increases. This suggests that minimizing average process time and maximizing expected attendance are strongly related and, in consequence, our initial hypothesis used to replace the non-linear objective function into a linear one seems reasonable. The comparison between average process time and expected attendance can be seen in Figure 2.

A concerning aspect of the results was that the expected attendance only reached the value of 56%. Due to the fact that many population centers were far from the possible facilities, the increase of the budget could do little to increase the expected attendance. This emphasizes the importance of efficiently selecting facility location candidates. As it can be seen in Table 5, as the dislocation speed increases (column “km/h”) the process average time (column “Fobj”) decreases and expected attendance (column “EA”) increases. For reasonable moving speed values such as 45km/h, we observe that the probability of attendance reaches 95%, which confirms that mobility issues, which also play an important role in the campaign attendance, can be also considered in the proposed model.

$$\begin{array}{|c|c|c|}
\hline
V\text{\ (KM/H)} & FOBJ\text{\ (H)} & EA\text{\ (%)} \\
\hline
16 & 2.4 & 0.616 \\
18 & 2.118 & 0.67 \\
20 & 1.908 & 0.715 \\
25 & 1.53 & 0.79 \\
30 & 1.278 & 0.837 \\
35 & 1.098 & 0.869 \\
40 & 0.963 & 0.89 \\
45 & 0.858 & 0.905 \\
\hline
\end{array}$$

Table 5 – Dislocation speed vs. expected attendance

4.2. CASE STUDY

With the purpose of validating exemplify the use of the proposed methodology in a realistic context, we have applied the model to the city of Rio de Janeiro. Since Rio de Janeiro city is the
capital among Brazilian cities with the lowest scores in the National Health Service rates (IDSUS, 2011), we decided that it would be a suitable option for our experiment.

To subdivide the city, we used a pre-established governmental division. There are 33 administrative regions in the city, but they are aggregated and form 7 local city halls. In order to evaluate the effectiveness of the proposed approach, we have chose to consider the neighborhood of Santa Cruz, which is a representative portion of one of those local city halls.

To obtain the set of population centers in that area, we only considered the population centers that were within 6 km or less of any facility in the area. Although by this division many people from other administrative regions will be considered in this problem, we believe it is reasonable once in reality people are able to choose freely which facility to use and, following our premise, they would choose the closest one even if it is located in a different region.

The arrival rate was calculated by dividing the total amount of people assigned to a facility by the number of open hours and the number of days the campaign lasted. In this case, due to the lack of accessible data about the available budget and facility and staff costs, they were not well estimated. Nevertheless, the purpose of this case study is to illustrate the applicability of the proposed approach, rather than providing solutions to the actual problem of planning the next vaccination campaign. The parameters used can be seen in Table 6.

![Table 6 – Study case parameters](image)

Santa Cruz neighborhood has around 750 thousand habitants, over 190 thousand people in the vaccination target groups (divided in 1214 population centers) and 9 health facilities that are used as vaccination center. Considering the current parameters, the problem is infeasible for whichever budget, once the maximum capacity to serve of the area would be of around 170 thousand people (number of vaccination days x opening hours divided by the time it takes to vaccine one person). This might indicate that the number of facilities in this area is not sufficient to serve its entire population. Although in mass vaccination campaigns, it is possible to open temporary centers, in other preventive healthcare fields permanent facilities are needed. Therefore, the service in these facilities might be compromised by the excessive amount of participants.

The results are shown in Table 7, similarly to the results presented in the previous example. The computer used to solve this problem was an Intel i7, 3.3 GHz, 64Gb of RAM. The solver used was Xpress and the total CPU time was of 2.49 hours. Only the data of facilities 1, 2, 7 and 8 were included once those were the only facilities selected. As it was expected, the total capacity of the facilities in the area is much smaller than the demand (total capacity is around 75 thousand people). If we did not consider the fact many people that would be willing to participate would not be served due to the size of the queue, the expected attendance would be the maximum possible, 95%, as the average process time is less than one hour for every population center. Considering the capacity issue, only 43.37% of the target group would be able to get assistance.
5. CONCLUSION

In this paper, we address the preventive healthcare-planning problem by means of an optimization model based on MIP that addresses the issues of locating health facilities and staff sizing, while taking into account the particular characteristics of preventive healthcare campaigns, focusing on process time minimization and considering congestion.

As was stated before, preventive healthcare is an insufficiently explored area. Once it is clear the importance of facility accessibility, facility planning becomes crucial in this field. We propose a model that aims to augment participation in preventive healthcare programs through the optimal location and staff sizing, taking into consideration important aspects of this type of service, such as user choice and demand elasticity.

As these characteristics are relevant for achieving a realistic and consistent model, we believe that a deep study toward them is also necessary. In addition, it is also important to address the matter of population center division and the geographical division of the problems. Administrative regions such as the ones used in this paper might not correspond to a proper division.

The case study results demonstrate the concerning conditions of the Rio de Janeiro preventive healthcare system. This matter deserves to be addressed more carefully, once the logistic of this system is crucial to granting the attendance of the population in preventive healthcare programs.

REFERENCES


Marianov, V. (2003), Location of multiple-server congestible facilities for maximizing expected demand, when services are non-essential, Annals of Operations Research, 123, 125–141.