Decision Making in a Fuzzy Environment as Applied to Analyzing and Prioritizing Industrial Districts

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RESUMO

O presente artigo reflete resultados da pesquisa relacionada com a aplicação de técnicas de tomada de decisão em um ambiente nebuloso para avaliar, comparar, escolher e/ou priorizar distritos industriais para apoiar a tomada de decisões relacionadas com a implementação do Programa de Revitalização e Modernização de Distritos Industriais no Estado de Minas Gerais. Os diversos critérios considerados são divididos em três classes: critérios socio-económicos, critérios de infraestrutura/logística e critérios ambientais. Como a representação dos critérios é realizada em diferentes formatos, a abordagem de Bellman-Zadeh para a tomada de decisões em um ambiente nebuloso combinada com o operador OWA (Ordered Weighted Average), bem como elementos de modelagem de preferências nebulosas, são aplicados no presente trabalho.

PALAVRAS CHAVE. Análise e Priorização de Distritos Industriais, Tomada de Decisão em Ambiente Nebuloso, Processamento de Informaçãoes Quantitativas e Qualitativas.

Tópicos (ADM – Apoio à Decisão Multicritério)

ABSTRACT

The present paper reflects results of research related to applying techniques of decision making in a fuzzy environment to evaluating, comparing, choosing, and/or prioritizing industrial districts to support decisions related to implementing the Program of Revitalization and Modernization of Industrial Districts in the State of Minas Gerais. Diverse criteria which are taken into account are divided in three classes: socio-economic criteria, infrastructure/logistics criteria, and environmental criteria. Since the measurement of criteria is realized in different formats, the Bellman-Zadeh approach to decision making in a fuzzy environment in combining with the OWA (Ordered Weighted Average) operator, as well as elements of fuzzy preference modeling, are applied in the present work.

KEYWORDS. Analysis and Prioritization of Industrial Districts, Decision Making in Fuzzy Environment, Quantitative and Qualitative Information Processing.

Paper topics (ADM – Multicriteria Decision Support)

1. Introduction

The analysis and choice of priority industrial districts (IDs), realized within the framework of the Program of Revitalization and Modernization of Industrial Districts of Minas Gerais (the Program is executed by the Federation of Industries of the State of Minas Gerais – FIEMG and the Company of Economic Development of the State of Minas Gerais – CODEMIG in 2015 and 2016), is a problem of a multicriteria character. Its solution requires the analysis of diverse types of initial data (quantitative as well as qualitative) of different municipalities with the IDs. From this analysis, it is possible to evaluate, compare, choose, and/or prioritize IDs to attend socio-economic, infrastructure/logistics, and environmental criteria.

The present work reflects result of research related to utilizing techniques of decision making in a fuzzy environment to evaluate, compare, choose, and/or prioritize IDs. Since the measurement of established criteria is realized in different formats, the use of the Bellman-Zadeh approach to decision making in a fuzzy environment is combined with the application of the Ordered Weighted Averaging (OWA) operator as well as techniques of fuzzy preference modeling, which are briefly discussed above.

2. Methodology

The fundamental methodological difficulty in solving multicriteria decision making problems is the lack of clarity in the concept of "optimal solution". In this context, the application of the Bellman-Zadeh approach to decision making in fuzzy environment [Bellman and Zadeh, 1970; Zimmermann, 1990] to multicriteria problems determines a valid concept, because the maximum "degree of implementing all objectives" serves as an optimality criterion, which is in accordance with the principle of guaranteed result, providing a constructive line in obtaining so-called harmonious solutions [Ekel, 2002; Pedrycz, Ekel, and Parreiras, 2011].

The Bellman-Zadeh approach preserves the natural measure of the uncertainty in multicriteria decision making. Moreover, it is necessary to indicate that the use of techniques for decision making in a fuzzy environment ensures the transparency in the decision making process, allowing to indicate the strengths and weaknesses of each solution alternative. Considering this, the Bellman-Zadeh approach to decision making in a fuzzy environment has been applied in the this work.

When using the Bellman-Zadeh approach to decision making in a fuzzy environment for solving multicriteria problems with the presence of q criteria, each pth criterion is represented by a membership function to a fuzzy set

$$A_p = \{X, \mu_{A_p}(X)\}, \quad p = 1,\ldots,q\,. \tag{1}$$

As shown in [Ekel, 2002; Pedrycz, Ekel, and Parreiras, 2011], the availability of fuzzy sets (1) reduces the problem of multicriteria decision making to search for

$$X^0 = \arg\max \min_{p=1,\ldots,q} \mu_{A_p}(X)\,. \tag{2}$$

However, the search for solutions based on constructing and solving *maxmin* problems represents a pessimistic view regarding the evaluation of satisfying the criteria levels, which is not adequate in certain decision making situations. In particular, solutions where a single criterion has a low satisfaction level and other criteria have high satisfaction levels are considered as bad solutions. Taking this into account, we complement the multicriteria analysis by the use of the Ordered Weighted Averaging (OWA) operator, proposed in [Yager, 1988]. Its use permits one to provide acceptable mutual compensation levels among the considered criteria in diverse decision making situations [Pereira Jr., 2014].

The OWA operator was originally proposed as follows:

$$\mathrm{OWA}_{\mathrm{w}}(a_1, a_2, \ldots, a_q) = \sum_{i=1}^{q} w_i b_i \ , \quad (3)$$

where b_i corresponds to the *i*-th largest value between $a_1, a_2, \ldots, a_q$ and the weights w_i satisfy the following conditions: $w_i \in [0,1]$ and $\sum_{i=1}^{q} w_i = 1$.

The application of the OWA operator includes the following steps:

1. Sort the arguments $a_1, a_2, \ldots, a_q$ in descending order;
2. Set the weights associated to the OWA operator using a suitable method;
3. Use the OWA operator applying (3) to add the arguments.

The key aspect in applying the OWA operator is the definition of its associated weights w_i. In particular, the weight w_i is not associated with an argument, but with the position *i* of the ordered arguments. The OWA operator can implement other operators as its specific cases through proper adjustment of its weights [Yager, 1988].

One of the existing techniques to define the OWA operator weights are associated with the construction of a fuzzy quantifier [Yager, 1988; Zadeh, 1983]. A fuzzy quantifier corresponds to a fuzzy set $Q(r)$, which reflects the portion $r \in [0,1]$ which satisfy the criteria represented by the term Q. For the analysis, realized in the present work, we used the following fuzzy quantifier [Zadeh, 1983]:

$$Q(x) = \begin{cases} 0 & \text{if } \ 0 \le x \le 0{,}5; \\ 2(x - 0{,}5) & \text{if } \ 0{,}5 < x \le 1, \end{cases} \quad (4)$$

where

- $Q(0)=0$;
- $Q(1)=1$;
- se $r_1 > r_2$, then $Q(r_1) > Q(r_2)$.

Then, OWA operator weights can be defined as [Yager, 1988]:

$$w_i = Q\left(\frac{i}{q}\right) - Q\left(\frac{i-1}{q}\right), \quad i = 1, \ldots, q \, . \quad (5)$$

The linguistic operator defined by (4) and (5), called “As many as possible”, considers only the second half of values after reordering. Although this operator still represents a pessimistic view regarding the criteria satisfaction levels, as only the “worst” half of the evaluated criteria is considered, it avoids that a single low satisfaction level criterion defines the solution quality.

Finally, the correlation (2) can be replaced by the following correlation:

$$X^0 = \arg \max_{p=1,\ldots,q} \mathrm{OWA}\, \mu_{A_p}(X) \, . \quad (6)$$

To apply (6), it is necessary to construct membership functions corresponding to fuzzy sets (1). In the presence of the alternatives $X_k, k = 1,\ldots,K$ with their estimates $C_p(X_k), k = 1,\ldots,K$, for the *p*th criterion, it is rational to apply the following correlation:

$$\mu_{A_p}(X_{k,p}) = \left[\frac{\max\limits_{1 \le k \le K} C_p(X_k) + \delta \ - \ C_p(X_k)}{\max\limits_{1 \le k \le K} C_p(X_k) + \delta - \min\limits_{1 \le k \le K} C_p(X_k)} \right]^{\lambda_p} \quad (7)$$

for the *p*th criterion, which is to be minimized, or

$$\mu_{A_p}(X_{k,p}) = \left[\frac{C_p(X_k) \; - \min\limits_{1\le k\le K} C_p(X_k) \text{-} \delta}{\max\limits_{1\le k\le K} C_p(X_k) - \min\limits_{1\le k\le K} C_p(X_k) \text{-} \delta} \right]^{\lambda_p} \quad (8)$$

for the *p*th criterion, which is to be maximized. In (7) and (8), λ_p, $p = 1,\ldots,q$ are the importance factors for the corresponding criteria and $\delta = 0.1 \times (\max\limits_{1\le k\le K} C_p(X_k) - \min\limits_{1\le k\le K} C_p(X_k))$ is a satisfaction level threshold.

To process criteria which permit only qualitative estimates, the fuzzy set based qualitative scales [Pedrycz, Ekel, and Parreiras, 2011] have been utilized in the present work. Their application assumes the possibility to use qualitative estimates ("*very low*", "*low*", "*high*", etc.) to assign the corresponding fuzzy sets with membership functions $\mu[C_s(X_k)], k = 1,\ldots,K$ to the considered alternatives for the *s*th criterion. The processing of these estimates can be carried out on the basis of the results of [Ekel, Pedrycz, and Schinzinger, 1998; Ekel and Neto, 2011].

If $\mu[C_s(X_k)]$ and $\mu[C_s(X_l)]$ are the membership functions reflecting evaluations of the alternatives X_k and X_l, respectively, from the point of view of the *s*th criterion, the quantity $\eta\{\mu[C_s(X_k)], \mu[C_s(X_i)]\}$ is the degree of preference $\mu[C_s(X_k)] \succcurlyeq \mu[C_s(X_l)]$, while $\eta\{\mu[C_s(X_l)], \mu[C_s(X_k)]\}$ is the degree of preference $\mu[C_s(X_l)] \succcurlyeq \mu[C_s(X_k)]$. Then, the membership functions of the generalized preference relations $\eta\{\mu[C_s(X_k)], \mu[C_s(X_l)]\}$ and $\eta\{\mu[C_s(X_l)], \mu[C_s(X_k)]\}$ take the following forms:

$$\eta\{\mu[C_s(X_k)], \mu[C_s(X_l)]\} = \sup_{C_s(X_k), C_s(X_l) \in C_s} \min\{\mu[C_s(X_k)], \mu[C_s(X_k)], \mu_{R_s}\{[C_s(X_k)], [C_s(X_l)]\}\}; \quad (9)$$

$$\eta\{\mu[C_s(X_l)], \mu[C_s(X_k)]\} = \sup_{C_s(X_k), C_s(X_l) \in C_s} \min\{\mu[C_s(X_k)], \mu[C_s(X_k)], \mu_{R_s}\{[C_s(X_l)], [C_s(X_k)]\}\}, \quad (10)$$

where $\mu_{R_s}\{[C_s(X_k)], [C_s(X_l)]\}$ and $\mu_{R_s}\{[C_s(X_l)], [C_s(X_k)]\}$ are the membership functions of the corresponding fuzzy preference relations which, respectively, reflect the essence of the preferences of X_k over X_l and of X_l over X_k (for instance, "more attractive", "more flexible", etc.).

When the essence of preference behind relation R_s is coherent with the natural order (≤) along the axis of measured values of C_s, then (9) and (10), respectively, are reduced to the following expressions:

$$\eta\{\mu[C_s(X_k)], \mu[C_s(X_l)]\} = \sup_{\substack{C_s(X_k), C_s(X_l) \in C_s \\ C_s(X_k) \le C_s(X_l)}} \min\{\mu[C_s(X_k)], \mu[C_s(X_l)]\}; \quad (11)$$

$$\eta\{\mu[C_s(X_l)], \mu[C_s(X_k)]\} = \sup_{\substack{C_s(X_k), C_s(X_l) \in C_s \\ C_s(X_l) \le C_s(X_k)}} \min\{\mu[C_s(X_k)], \mu[C_s(X_l)]\}. \quad (12)$$

If C_s has a maximization character, the correlations (11) and (12) have to be written for $C_s(X_k) \ge C_s(X_l)$ and $C_s(X_l) \ge C_s(X_k)$, respectively.

Examples of the utilization of (11) and (12) are given in [Pedrycz, Ekel, and Parreiras, 2011].

The correlations (11) and (12) can serve for constructing a fuzzy preference relation R_s corresponding to the *s*th criterion. In particular, if X is a set of the considered alternatives $X_k, k = 1,\ldots,K$, then

$$R_s = [X \times X, \mu_{R_s}(X_k, X_l)], \quad X_k, X_l \in X \,, \tag{13}$$

where $\mu_{R_s}(X_k, X_l)$ is a membership function of the fuzzy preference relation corresponding to the *s*th criterion.

The fuzzy preference relation R_s (also called a nonstrict fuzzy preference relation or fuzzy weak preference relation in literature) is defined as a fuzzy set of all pairs of the Cartesian product $X \times X$, such that the membership function $\mu_{R_s}(X_k, X_l)$ represents the degree to which X_k weakly dominates X_l, i.e., the degree to which X_k is not worse than X_l for the *s*th criterion. In a somewhat loose sense, $\mu_{R_s}(X_k, X_l)$ also represents the degree of truth of the statement " X_k is preferred over X_l ".

The fuzzy preference relation R_s can be processed to construct a fuzzy strict preference relation as follows:

$$\vec{R}_s = R_s \setminus R_s^{-1}, \tag{14}$$

where R_s^{-1} is the inverse fuzzy preference relation.

The membership function corresponding to (14) is the following:

$$\mu_{\vec{R}_s}(X_k, X_l) = \max\{\mu_{R_s}(X_k, X_l) - \mu_{R_s}(X_l, X_k), 0\}\,. \tag{15}$$

It serves as the basis for the choice procedure introduced in (Orlovski, 1978). Its properties as well as questions of its axiomatic characterization are discussed, for instance, in (Sengupta, 1998).

The utilization of (15) permits one to construct the set of nondominated alternatives with the membership function

$$\mu_{R_s}^{ND}(X_k) = \inf_{X_l \in X}[1 - \mu_{\vec{R}_s}(X_l, X_k)] = 1 - \sup_{X_l \in X} \mu_{\vec{R}_s}(X_l, X_k)\,, \tag{16}$$

which allows one to evaluate the levels of nondominance of the considered alternative $X_k, k = 1,..., K$. These levels can serve as the result of transforming the qualitative estimates in the quantitative ones.

3. Prioritizing Industrial Districts

The IDs have been evaluated on the basis of 20 criteria. These criteria have been divided into three groups: socio-economic criteria, infrastructure/logistics criteria, and environmental criteria. Socioeconomic data reflects the availability of skilled labor for the industry development and the impact of each ID in the state's economy. The infrastructure/logistics group aim to evaluate the expansion capability of the ID, as it evaluates the total area of the ID and its accessibility to production transportation. Finally, the environmental group evaluates if the ID has a good natural condition to development, also it evaluates if there is a good infrastructure for waste disposal and, consequently, the reduction the environmental impact. The initial data related to the considered criteria $C_p, p = 1,..,20$ is presented in Tables 1-3, which include the description of these criteria, their goal (Maximize or Minimize), and the type of the applied estimates.

Table 1 – Socio-economic Criteria

Criterion	Description	Goal	Values
C_1	Percentage of total state Gross Domestic Product.	Maximize	[0,100] %
C_2	Percentage of total state industrial Gross Domestic Product.	Maximize	[0,100] %
C_3	Percentage of total state tax revenue.	Maximize	[0,100] %
C_4	Percentage of total state export value.	Maximize	[0,100] %
C_5	Percentage of total state import value.	Maximize	[0,100] %
C_6	Percentage of total state number of importers.	Maximize	[0,100] %
C_7	Percentage of total state number of exporters.	Maximize	[0,100] %
C_8	Percentage of the total state education rate (complete high school, college, graduate).	Maximize	[0,100] %

Table 2 – Infrastructure/Logistics Criteria

Criterion	Description	Goal	Values
C_9	ID área in m^2.	Maximize	Real
C_{10}	Criterion reflecting the logistical access to the city, considering three types: airport, road, and rail. Good has 3 accesses, Medium has 2 accesses, Bad has one access.	Maximize	{Good, Medium, Bad}

Table 3 – Environmental Criteria

Criterion	Description	Goal	Values
C_{11}	Biome in the ID region (Good: Atlantic forest; Bad: Cerrado).	Maximize	{Good, Bad}
C_{12}	Distance from the ID to a conservation unit.	Maximize	{Good, Medium, Bad}
C_{13}	Availability of surface water in the ID.	Maximize	{Very High, High, Medium, Low, Very Low}
C_{14}	Availability of underground water in the ID.	Maximize	{Very High, High, Medium, Low, Very Low}
C_{15}	Distance in km from the ID to a landfill of the class I.	Minimize	Real
C_{16}	Distance in km from the ID to a landfill of the class II.	Minimize	Real
C_{17}	Distance in km from the ID to a waste incineration plant.	Minimize	Real
C_{18}	Distance in km from the ID to a waste co-processing unit.	Minimize	Real
C_{19}	The availability of a regularized landfill in the city (Good: exists, Bad: does not exist).	Minimize	{Good, Bad}
C_{20}	Criterion reflecting the environmental licensing of the ID (Good: licensed, Medium: Licensing in progress, Bad: No licensing).	Minimize	{Good, Medium, Bad}

4. Simulation and Results

The criteria listed in the Tables 1-3 have been used to evaluate, compare, and prioritize 53 IDs. However, for the visibility of the most important results, we provide information (Tables 4-7) on $K = 10$ IDs, which have been ordered as the top ones. With the availability of initial data, the process of the analysis includes the four main steps:

1. Survey of initial data;
2. Transform the existing qualitative estimates into quantitative estimates, applying (11), (12), (15) and (16);
3. Transform all estimates into values of the membership functions, using (7) or (8);
4. Generate the aggregated evaluations of the IDs, utilizing (6).

Table 4 – Socio-economic Initial Data

ID	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
X_1	2.81	1.88	0.39	7.28	5.04	2.86	1.60	1.45
X_2	4.69	9.21	0.55	23.93	1.99	5.31	1.28	2.47
X_3	2.89	2.95	0.38	3.92	0.22	2.04	0.53	1.83
X_4	6.83	7.60	2.45	3.97	2.36	9.39	5.88	7.31
X_5	6.94	21.69	2.06	83.82	3.73	2.04	0.53	13.19
X_6	3.66	4.52	0.64	10.58	0.15	2.04	0.43	1.67
X_7	9.14	9.59	2.00	27.98	13.64	3.27	2.46	7.74
X_8	3.27	1.92	0.16	15.55	0.14	4.08	0.64	1.30
X_9	6.51	7.57	2.84	2.59	29.70	9.80	4.92	7.69
X_{10}	0.56	0.84	0.37	0.12	0.00	0.82	0.32	0.97

Table 5 – Infrastructure/Logistics Initial Data

ID	C_9	C_{10}
X_1	1174375.00	Medium
X_2	868080.00	Good
X_3	275347.00	Good
X_4	346800.00	Good
X_5	856232.00	Medium
X_6	1527916.00	Medium
X_7	5410917.00	Good
X_8	790785.00	Good
X_9	1736025.00	Medium
X_{10}	635780.00	Medium

Table 6 – Environmental Initial Data – Part 1

ID	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
X_1	Bad	Medium	Low	High	138.00
X_2	Bad	Medium	Medium	High	507.00
X_3	Bad	Good	Low	Very Low	102.00
X_4	Bad	Good	Low	Low	88.70
X_5	Bad	Bad	Medium	Medium	135.00
X_6	Medium	Bad	High	Medium	610.00
X_7	Bad	Medium	Very low	Very Low	431.00
X_8	Medium	Good	High	Medium	352.00
X_9	Bad	Bad	Low	High	186.00
X_{10}	Bad	Bad	High	Medium	261.00

Table 7 – Environmental Initial Data – Part 2

ID	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
X_1	315.00	138.00	153.00	Good	Bad
X_2	37.20	37.20	369.00	Good	Medium
X_3	192.00	125.00	28.60	Good	Medium
X_4	88.70	104.00	95.10	Bad	Bad
X_5	107.00	103.00	123.00	Good	Bad
X_6	136.00	136.00	472.00	Good	Bad
X_7	431.00	413.00	1.00	Bad	Bad
X_8	153.00	153.00	267.00	Bad	Bad
X_9	363.00	186.00	201.00	Good	Bad
X_{10}	233.00	43.20	249.00	Good	Bad

The initial data were obtained with the help of experts from various fields of knowledge and these data are available in many different formats, as we can see on Tables 4-7. To correctly evaluate the IDs using the initial data provided, it is necessary to transform these data to the same measure. Using (11), (12), (15) and (16), it is possible to transform the qualitative information in criteria C_{10} to C_{14}, C_{19} and C_{20} into fuzzy membership values representing the criteria satisfaction levels. Using (7) or (8), it is possible to transform the quantitative information into fuzzy membership values as well, in order that we have all the criteria on the same measure. Finally, working with fuzzy membership values allow us to apply (3) to aggregate all the information of the criteria.

The results presented in Tables 8-11 reflect the values of the membership functions (criteria satisfaction levels) for the IDs.

Table 8 – Levels of Satisfying Socio-economic criteria

ID	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
X_1	0.3296	0.1363	0.1680	0.1687	0.2450	0.2975	0.3007	0.1270
X_2	0.5289	0.4560	0.2234	0.3495	0.1518	0.5455	0.2483	0.2031
X_3	0.3379	0.1827	0.1652	0.1322	0.0976	0.2149	0.1259	0.1555
X_4	0.7553	0.3857	0.8698	0.1327	0.1631	0.9587	1.0000	0.5628
X_5	0.7671	1.0000	0.7363	1.0000	0.2051	0.2149	0.1259	1.0000
X_6	0.4200	0.2513	0.2559	0.2045	0.0953	0.2149	0.1084	0.1431
X_7	1.0000	0.4725	0.7151	0.3935	0.5083	0.3388	0.4406	0.5944
X_8	0.3779	0.1380	0.0909	0.2585	0.0950	0.4215	0.1434	0.1158
X_9	0.7213	0.3844	1.0000	0.1177	1.0000	1.0000	0.8427	0.5908
X_{10}	0.0909	0.0909	0.1620	0.0909	0.0909	0.0909	0.0909	0.0909

Table 9 – Levels of Satisfying Infrastructure/Logistics Criteria

ID	C_9	C_{10}
X_1	0.2501	0.7000
X_2	0.1958	1.0000
X_3	0.0909	1.0000
X_4	0.1036	1.0000
X_5	0.1937	0.7000
X_6	0.3126	0.7000
X_7	1.0000	1.0000
X_8	0.1822	1.0000
X_9	0.3495	0.7000
X_{10}	0.1547	0.7000

Table 10 – Levels of Satisfying Environmental Criteria – Part 1

ID	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
X_1	0.3000	0.7000	0.3000	1.0000	0.9140
X_2	0.3000	0.7000	0.7000	1.0000	0.2705
X_3	0.3000	1.0000	0.3000	0.2500	0.9768
X_4	0.3000	1.0000	0.3000	0.5000	1.0000
X_5	0.3000	0.3000	0.7000	0.7500	0.9193
X_6	0.7000	0.3000	1.0000	0.7500	0.0909
X_7	0.3000	0.7000	0.2500	0.2500	0.4031
X_8	0.7000	1.0000	1.0000	0.7500	0.5408
X_9	0.3000	0.3000	0.3000	1.0000	0.8303
X_{10}	0.3000	0.3000	0.7000	0.7500	0.6995

Table 11 – Levels of Satisfying Environmental Criteria – Part 2

District	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
X_1	0.3587	0.7562	0.7066	1.0000	0.3000
X_2	1.0000	1.0000	0.2897	1.0000	0.7000
X_3	0.6426	0.7876	0.9467	1.0000	0.7000
X_4	0.8811	0.8384	0.8184	0.3000	0.3000
X_5	0.8389	0.8408	0.7645	1.0000	0.3000
X_6	0.7719	0.7610	0.0909	1.0000	0.3000
X_7	0.0909	0.0909	1.0000	0.3000	0.3000
X_8	0.7327	0.7199	0.4866	0.3000	0.3000
X_9	0.2479	0.6400	0.6140	1.0000	0.3000
X_{10}	0.5480	0.9855	0.5213	1.0000	0.3000

The results of ordering of the IDs are defined by the levels of satisfying all considered criteria. In the present work, the IDs have been analyzed to prioritize those of them, which do not have many poorly evaluated criteria. It is important to note that for each ID, the weight of poorly evaluated criteria is greater than the weight of well evaluated criteria, according to the correlations (4) and (5). Considering this, the application of (6) allows one to obtain the aggregated evaluation of each ID. It allows one to order the IDs, as shown on Table 12.

Table 12 – Aggregated Evaluations of the IDs

ID	Aggregated Value
X_5	0.3776
X_9	0.3504
X_4	0.3048
X_7	0.2717
X_2	0.2688
X_1	0.2293
X_8	0.2002
X_6	0.1755
X_3	0.1715
X_{10}	0.1253

5. Conclusion

This paper describes the methodology used for evaluating, comparing, choosing, and/or prioritizing IDs, based on the multicriteria analysis with considering the criteria of socio-economic, infrastructure/logistics, and environmental character. Taking into account that the measurement of the considered criteria is realized in different formats, the use of the Bellman-Zadeh approach to decision making in a fuzzy environment is combined with the application of the OWA operator as well as techniques of fuzzy preference modeling. The application of the results associated with fuzzy preference modeling permits one to transform the qualitative estimates in the quantitative ones, providing the homogeneity of information used for decision making.

The practical results are of multifunctional character. In particular, they can be used for allocating available recourses for successive executing of the Program of Revitalization and Modernization of Industrial Districts of Minas Gerais.

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