Wavelength Requirements in Optical Backbone Networks Modelled as Twin Graphs

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ABSTRACT

One of the most important problems in optical telecommunication networks is the Routing and Wavelength Assignment (RWA) problem, that consists in selecting routes and wavelengths to establish communication between the node pairs of the network. In the most common version of the RWA problem, the goal is to minimize the number of wavelengths required by the network. Optical networks can be modelled as a graph, with nodes representing the optical routers, and optical fiber links interconnecting some pairs of nodes. In this work, we investigate the use of twin graphs as an alternative to model optical backbone networks, as recently proposed in the literature. The work naturally extends to flexgrid optical networks. Twin graphs are suitable for resilient and cost-effective optical networks, because of the following property: any single node failure causes no impact on the pairwise hopcounts in the remaining network; and no other graphs with fewer links satisfy this property.

KEYWORDS. Optical Networks. Routing and Wavelength Assignment Problem. Twin Graphs.

OR in Telecommunications and Information Systems. Theory and Algorithms in Graphs. Statistics.

1. Introduction

A telecommunications network is called an optical network when the physical medium used for the transmission of information between network nodes consists in fiber optic cables. In these networks, to establish a communication between a pair of nodes, it is necessary a continuous wavelength available along a particular route connecting them.

One of the most important problems in optical networks is the Routing and Wavelength Assignment (RWA) problem, that consists in selecting routes and wavelengths to establish communication between the node pairs of the network. Since the number of wavelengths is related to the cost of the network, in the most common version of the RWA problem, the goal is to minimize the number of wavelengths required by the network to meet all communication demands [Murthy and Gurusamy, 2002].

Optical networks can be modelled as graphs, with nodes representing the optical routers, and optical fiber links interconnecting some node pairs. From this modelling, it is possible to analyse their physical topologies by using metrics from graph theory. Such analyses can contribute to the design of new optical networks, and to put forward changes in existing ones, in order to improve their performance and cost-effectiveness.

For instance, [Baroni and Bayvel, 1997] analysed the number of wavelengths required in real optical networks, which were modelled as 2-connected random graphs such that $0.1 < \alpha < 0.4$, where α refers to the link density. The traffic demand was considered uniform, each demand between a pair of nodes uses a single wavelength and flows through a shortest path, i.e., a geodesic path connecting the node pair. The number of wavelengths was estimated by a heuristic algorithm. It was observed that the number of wavelengths is almost independent of the network order, but strongly depends on the link density.

In [Pavan et al., 2010], a set of 29 real-world optical backbone network topologies have been investigated, and some graph properties of them have been identified, in order to generate graphs that mimic these properties. Among these properties, we highlight that the real-world optical backbone topologies in this set have average node degree ranging from 2 to 4, and 19 of these 29 topologies are 2-connected graphs, i.e., they present at least 2 node-disjoint paths between any pair of nodes.

Recently, a new way to model optical backbone topologies has been proposed [Paiva et al., 2013], based on a special family of 2-connected graphs called twin graphs [Farley and Proskurowski, 1997]. Twin graphs are suitable for resilient and cost-effective optical networks, because: i) each of them provides at least two equal length paths, in number of links, for all non-adjacent node pairs, which means that all distances on the graph remain unchanged after any single node failure; and ii) no other graphs with fewer links satisfy this property [Farley and Proskurowski, 1997]. This family of graphs have shown interesting properties with respect to fault tolerance, resilience, cost (in number of links) and scalability [Paiva et al., 2013].

In this paper we compare topological characteristics of existing networks with topological characteristics of networks modelled as twin graphs, in order to verify the advantages and disadvantages of this new model compared to existing networks. In particular, the wavelength requirements are used for comparing the cost of these networks. The topological characteristics, i.e., the invariants considered in this paper are: number of nodes, maximum degree, minimum degree, average degree, link density, diameter, average distance, link connectivity, node connectivity, algebraic connectivity, average link betweenness, maximum link betweenness, and minimum link betweenness.

The paper is organized as follows. Section 2 presents concepts from graph theory used throughout the paper. Section 3 describes the methodology used to obtain our results, which are presented and discussed in Section 4. Section 5 brings conclusions and future works.

2. Graphs and invariants

A graph is a mathematical structure $G = G(V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is a non-empty set of vertices or nodes and $E = \{e_1, e_2, \ldots, e_m\}$ is a set of links interconnecting them.

Each link $e_k = \{v_i, v_j\}$ corresponds to an interconnection between nodes v_i and v_j in G. The number of nodes is $n = |V(G)|$, and the number of links is $m = |E(G)|$.

A graph that has no link orientation, no self-loops, i.e., links $e_k = \{v_i, v_i\}$, and no more than one link between any pair of nodes is called a simple graph.

Two nodes v and u are said to be adjacent if there is a link between them. The degree of a node v is the number of links adjacent to v in G. The minimum degree among all nodes in G is denoted as $\delta = \delta(G)$. Analogously, the maximum degree among all nodes in G is denoted as $\Delta = \Delta(G)$. The average degree of G, denoted as $\langle d \rangle$ is given by $2m/n$.

The link density $\alpha(G)$ of a simple graph G with n nodes and m links is the ratio between m and the number of links of a complete graph with n nodes. Thus, $\alpha(G) = 2m/(n(n-1))$.

A path can be defined as a sequence of vertices and links, without repetition. The distance between two nodes u and v, denoted as $dist(u, v)$, is the number of links in a shortest path between u and v. A shortest path between u and v is called a geodesic. The diameter of a graph G, denoted as $diam = diam(G)$, is the number of links of the largest geodesic in G. The average distance of G, denoted as $\langle dist \rangle$, is given by the sum of the distances between every pair of nodes, over the number of node-pairs.

A graph G is said to be connected if there is at least one path between each pair of nodes in G. Otherwise, G it is called a disconnected graph.

The node connectivity of a graph G corresponds to the smallest number of nodes which need to be removed from G for obtaining a disconnected (or trivial) graph. Analogously, the link connectivity of G corresponds to the smallest number of links that need to be removed from G for obtaining a disconnected graph.

One of the invariants considered in this work is the average link betweenness. The betweenness centrality is proportional to the fraction of shortest paths between pairs of nodes passing through a given vertex (or link). This metric was first proposed to vertices in [Freeman, 1977], and then extended to links in [Girvan and Newman, 2002].

More precise definitions of vertex and link betweenness are given as follows [Comellas and Gago, 2006]. Let $G = G(V, E)$ be a graph, and let u, v and $w \in V(G)$. Denote $\sigma_{uv}(w)$ as the number of shortest paths from u to v that go through w, and denote σ_{uv} as the total number of shortest paths from u to v in G. Then, the betweenness of a vertex w, denoted as B_w, is given by:

$$B_w = \sum_{u,v \neq w} b_w(u, v), \tag{1}$$

where:

$$b_w(u, v) = \frac{\sigma_{uv}(w)}{\sigma_{uv}}. \tag{2}$$

Analogously, the betweenness of a link $e = uv$ in G, denoted as B^e, is given by:

$$B^e = \sum_{u \neq v} b^e(u, v), \tag{3}$$

where:

$$b^e(u, v) = \frac{\sigma_{uv}(e)}{\sigma_{uv}}, \tag{4}$$

where $\sigma_{uv}(e)$ denotes the number of shortest paths from u to v that go through link e, and σ_{uv} denotes the total number of shortest paths from u to v.

The average link betweenness is given by:

$$\overline{B}^E = \frac{\sum_{e \in E} B^e}{m}. \tag{5}$$

Another invariant considered is the algebraic connectivity [Fiedler, 1973], that is a metric obtained from the Laplacian matrix of a graph. Let $G = G(V, E)$ be a graph with n nodes, the Laplacian matrix of G is given by $L(G) = D(G) - A(G)$, where $D(G)$ is the diagonal matrix of vertex degrees of G, and $A(G)$ is the adjacency matrix of G.

The adjacency matrix of a graph G, denoted as $A(G)$, is a matrix where entries a_{ij} are 1 if there is a link between the nodes v_i and v_j, and 0 otherwise.

Then, the algebraic connectivity of a graph G is defined as the second smallest eigenvalue of the Laplacian matrix of G. The algebraic connectivity of G is denoted as $\mu_2 = \mu_e(G)$. According to [William et al., 2011], this measure represents the network potential immunity to failure and thus it is important in analyses of network survivability.

2.1. Twin Graphs

Let $G = G(V, E)$ be a simple graph. The neighborhood $\Gamma(v)$ of a node $v \in V(G)$ is the set of nodes which are adjacent to v in G. If two nodes $u, v \in V(G)$ have the same neighborhood, then (u, v) is a twin pair in G.

A graph G is 2-connected if and only if there are at least 2 node-disjoint paths between each pair of nodes $u, v \in G$. Since node-disjoint paths are also link-disjoint, a physical topology modelled as a 2-connected graph survives to any single link/node failure, because in this case each pair of nodes is still connected by a path avoiding the failure. However, there are no restrictions on the number of links of this path, with relation to the number of links of a geodesic in the original graph. In this sense, the class of 2-geodetically-connected graphs (2-GC for short) is defined as a particular subset of the 2-connected graphs. A graph is 2-GC if and only if there are at least 2 node-disjoint geodesics between each pair of non-adjacent nodes $u, v \in G$. Thus, in the case of any single link/node failure, each pair of non-adjacent nodes is still connected by an alternative geodesic which avoids the failure.

The class of twin graphs consists of 2-GC graphs minimizing the number of links, that is, minimal 2-GC graphs. Moreover, all minimal 2-GC graphs are twin graphs, except the cube graph Q_3 and the cycle graph C_3. Each twin graph has $n \geq 4$ nodes and $m = 2n - 4$ links [Farley and Proskurowski, 1997].

Two methods for building twin graphs were proposed in [Chang et al., 1996]. Both methods are explained as follows.

2.1.1. Generating twin graphs

Let G_1 be a twin graph with n nodes. Starting from G_1 we can obtain a new twin graph G_2 with $n+1$ nodes by identifying a twin par $(u_1, v_1) \in G_1$ and adding a new node w_2 to that pair. For instance, we can take G_1 as the cycle C_4, which is a twin graph, and generate recursively new twin graphs by means of the previously proposed method. This process is illustrated in Figure 1.

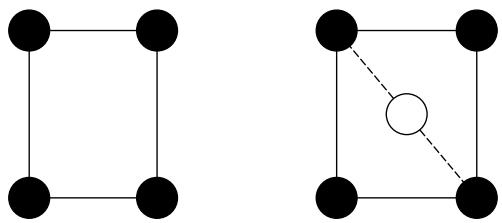


Figure 1: Generating a twin graph by adding a new node.

Other way to build a twin graph is by merging two twin graphs G_1 with $n_1 = |V(G_1)|$ and G_2 with $n_2 = |V(G_2)|$. For this we identify twin pairs $(u_1, v_1) \in G_1$ and $(u_2, v_2) \in G_2$ and connect them by using 4 new links $e_1 = \{u_1, u_2\}$, $e_2 = \{u_1, v_2\}$, $e_3 = \{v_1, u_2\}$ and $e_4 = \{v_1, v_2\}$. Figure 2 illustrates this process and the twin graph G obtained by merging both twin graphs shown in Figure 1. Notice that G is a twin graph with $n = 4 + 5 = 9$ nodes.

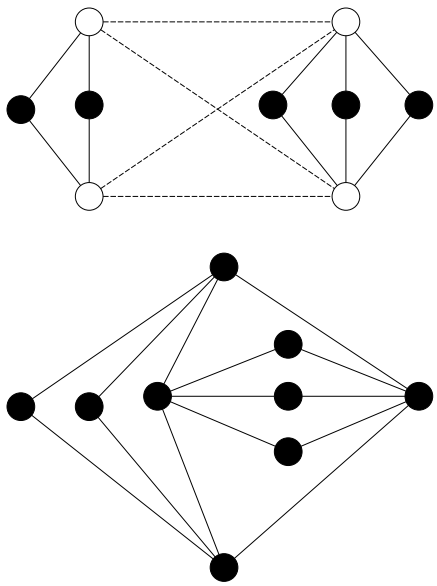


Figure 2: Generating a twin graph by merging two twin graphs.

3. Methodology

This work analyses the modelling of optical backbone networks as twin graphs in comparison with real networks. This analysis is carried out considering, in particular, the cost of the network, represented here by the wavelength requirements.

We have considered two sets of graphs: a set T of twin graphs, and a set R of real networks. The set T consists of all twin graphs from 9 to 17 nodes, that totalizes 742 graphs. The set R of real networks was taken from the set of 29 networks investigated in [Pavan et al., 2010]. It consists of the 12 real networks for which the number of nodes also ranges from 9 and 17. Table 3 lists the networks in R, their number of nodes, average degree, and connectivity.

	Number of nodes	Average degree	Connectivity
Arnes	17	2.35	1
Austria	15	2.93	1
Bren	10	2.20	2
Cesnet	12	3.17	2
Germany	17	3.06	2
Italy	14	4.14	2
Mzima	15	2.53	2
NSFnet	14	3.00	2
RNP	10	2.40	2
Spain	17	3.29	2
VBNS	12	2.83	2
Vianet	9	2.67	1

Table 1: Description of the real-world networks considered in this paper.

For each graph of both sets T and R, we computed the number of wavelengths, using the methodology described in [Cousineau et al., 2015], and several other invariants, such as number of nodes, maximum degree, minimum degree, average degree, link density, diameter, average distance, link connectivity, node connectivity, algebraic connectivity, average link betweenness, maximum link betweenness, and minimum link betweenness.

All computations of this work where performed using the IGRAPH package available for the software R.

4. Results and discussion

For comparing the efficiency in the use of wavelengths by optical backbone networks modelled as twin graphs (T), as opposed to real networks (R), we will carry out analyses by fixing the network order (n) and investigate what can be expected from their other topological characteristics.

The distribution of the number of wavelengths by the order of the networks were analysed for both groups T and R, as shown in Figure 3. For these network sets, we observe that the number of wavelengths required by a network modelled as a twin graph is less than or at most equal to the one required by a real network of the same order.

Whereas the real networks in R need between 9 and 38 wavelengths, twin graphs of the same orders presented a reduced requirement, using between 5 and 18 wavelengths. This result shows that, when modelling an optical backbone network as a twin graph, one could build an order n network such that, in addition to having all properties of this class of graphs, it requires less wavelengths than any order n real network in R.

The Histogram 4(a) and the Boxplot 4(b) show the unidimensional distribution of the number of wavelengths for the sets T and R. Both groups show asymmetric distributions for the number of wavelengths, with the average of T (14.8) and R (18.6) being affected by extreme values. The median values of T and R are 16 and 16.5, respectively, showing that the central point of data is quite near. The graphs in T show few values above the median, whereas the networks in R show quite distant median values, resulting in a positive asymmetric distribution of the data. Also, the variability is greater in R than in T, and the absolute average deviation results in 7.41 for the networks in R, and in 2.97 for the graphs in T.

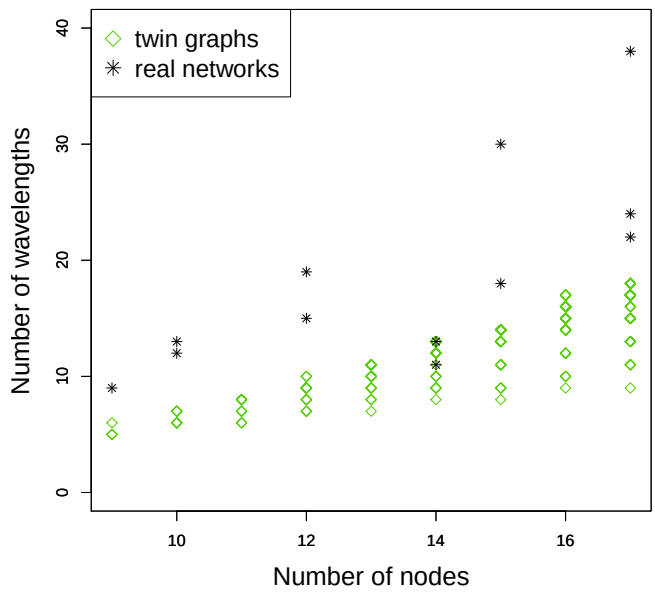


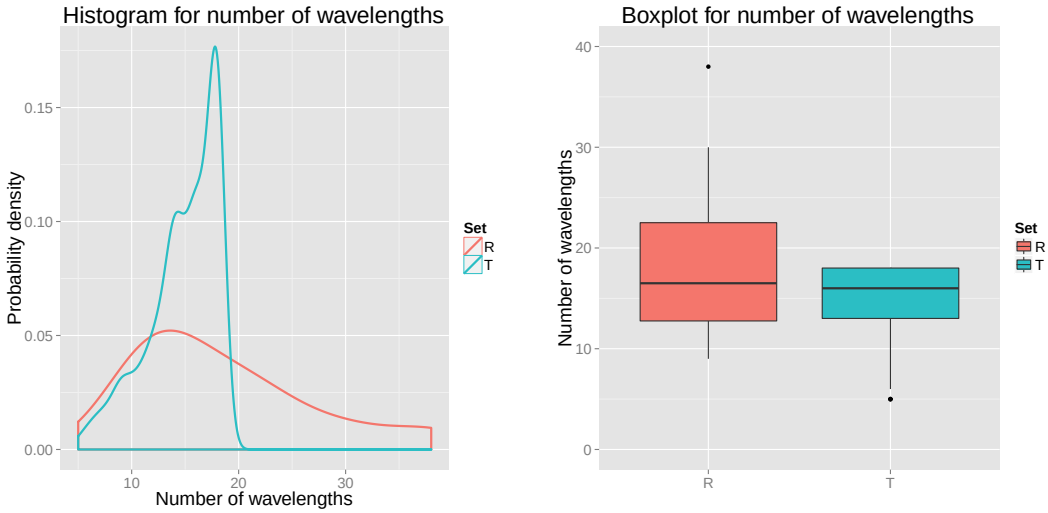
Figure 3: Distribution of the number of wavelengths by the number of nodes, for both sets T and R.

As shown in [Paiva et al., 2013], twin graphs have $2 \leq \langle d \rangle < 4$. That range is close to the average degree of networks in R, $2.2 \leq \langle d \rangle \leq 4.14$, as shown in Table 3.

For a fixed n, we observe in Figure 5(a) that twin graphs generally show average degree greater than that presented by real networks. This invariant can also be seen as a means to assess the cost of a network [Pavan et al., 2010]. Thus, according to that criterion, the twin graphs are less attractive as topology model, since the decrease in the use of wavelengths for this class is

associated with an increase in the average degree. Then, it is important to evaluate the cost-benefit of prioritizing one of those parameters with respect to the other.

Figure 5(b) shows the behavior of the variable link density α for both groups T and R. For a fixed n, we see that most existing networks in R are less dense than twin graphs in T. When comparing Figure 5(b) with Figure 3, we find that the reduction on the use of wavelengths for twin graphs can be due to their higher link density. Therefore, in the case of designing an order n network where α is a critical variable, modelling the network as a twin graph would be beneficial if the cost of adding new links is less than the cost of using more wavelengths.



(a) (b)

Figure 4: Histogram 4(a) and Boxplot 4(b) of the number of wavelengths.

Figures 5(c)- 5(e) show the distribution of invariants related to link betwenneess for fixed values of n. On average, the link betwenneess is higher for networks in R than in T. Despite the minimum value for the link betwenneess is lower in real networks, the average link betweenness $\overline{B}^E$ is affected by the maximum values that were higher than those found in twin graphs. The increase in link betwenneess is related to increased congestion in the links, which influences the greater wavelength requirements. With this result, we see that the best use of wavelengths by graphs in T is result of lower average congestion presented by their links. In the case where congestion is a critical variable in the design of the network, using a twin graph topology is shown as an alternative to decrease the wavelength requirements.

Figure 5(f) shows the distribution of maximum degree Δ. For the graphs in T, $4 \leq \Delta \leq 15$, whereas for networks in R, $3 \leq \Delta \leq 10$. By fixing n, we can obtain networks modelled by twin graphs having a widely varying values of Δ, it is possible to increase Δ at most to $n-2$ while still using less wavelengths than the required in R. Thus, the twin graphs modelling is interesting for networks having central nodes (nodes with a high value of maximum degree), providing lower cost due to the reduced use of wavelengths.

For the group T, it was obtained $1.61 \leq \langle dist \rangle \leq 3.12$. This result is consistent with that obtained for the networks in R, where $1.87 \leq \langle dist \rangle \leq 3.02$. Figure 6(a) shows that, for a given n, $\langle dist \rangle$ shows no considerable difference of behavior between the two groups. Thus, in relation to $\langle dist \rangle$, twin graphs are comparable with real networks, and also reduce considerably the requirement of wavelengths, as shown in Figure 3. The two groups neither show considerable difference of behavior in the diameter, according to Figure 6(b). For graphs in T, $2 \leq$diam≤ 8. Such a result is close to that obtained in R, where $3 \leq$diam≤ 6.

The real networks in R showed lower or at most equal algebraic connectivity μ_2 than

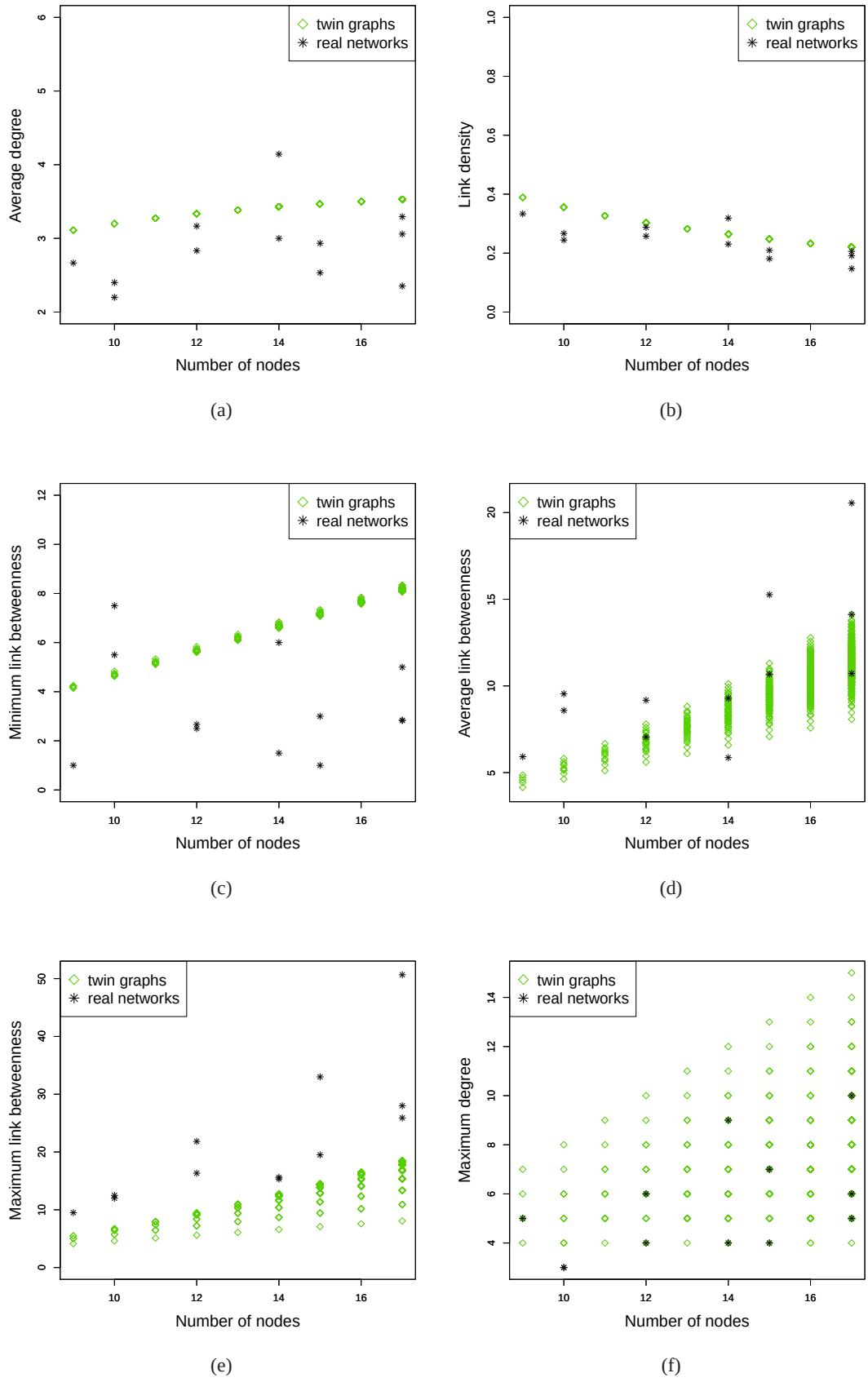


Figure 5: Distribution of the average degree 5(a), the link density 5(b), the minimum link betweenness 5(c), the average link betweenness 5(d), the maximum link betweenness 5(e), and the maximum degree 5(f), by number of nodes.

graphs in T, as Figure 6(c) shows. For R, $0.19 \leq \mu_2 \leq 0.81$, whereas for T, $0.27 \leq \mu_2 \leq 2$. According to [William et al., 2011], μ_2 is directly related to the number of independent paths in the graph. Therefore, the increase of μ_2 tends to be inversely proportional to the requirement of wavelengths and proportionally associated with the increasing of robustness of the network. For fixed values of n, it is possible to find twin graphs showing better results for μ_2 than the values obtained by real network in R. Based on this criterion, the choice of a twin graph as network topology conciliates the use of wavelengths with protection to network failures in a better way than real networks in R. The twin graphs that maximized the algebraic connectivity ($\mu_2 = 2$) are known as complete bipartite graphs.

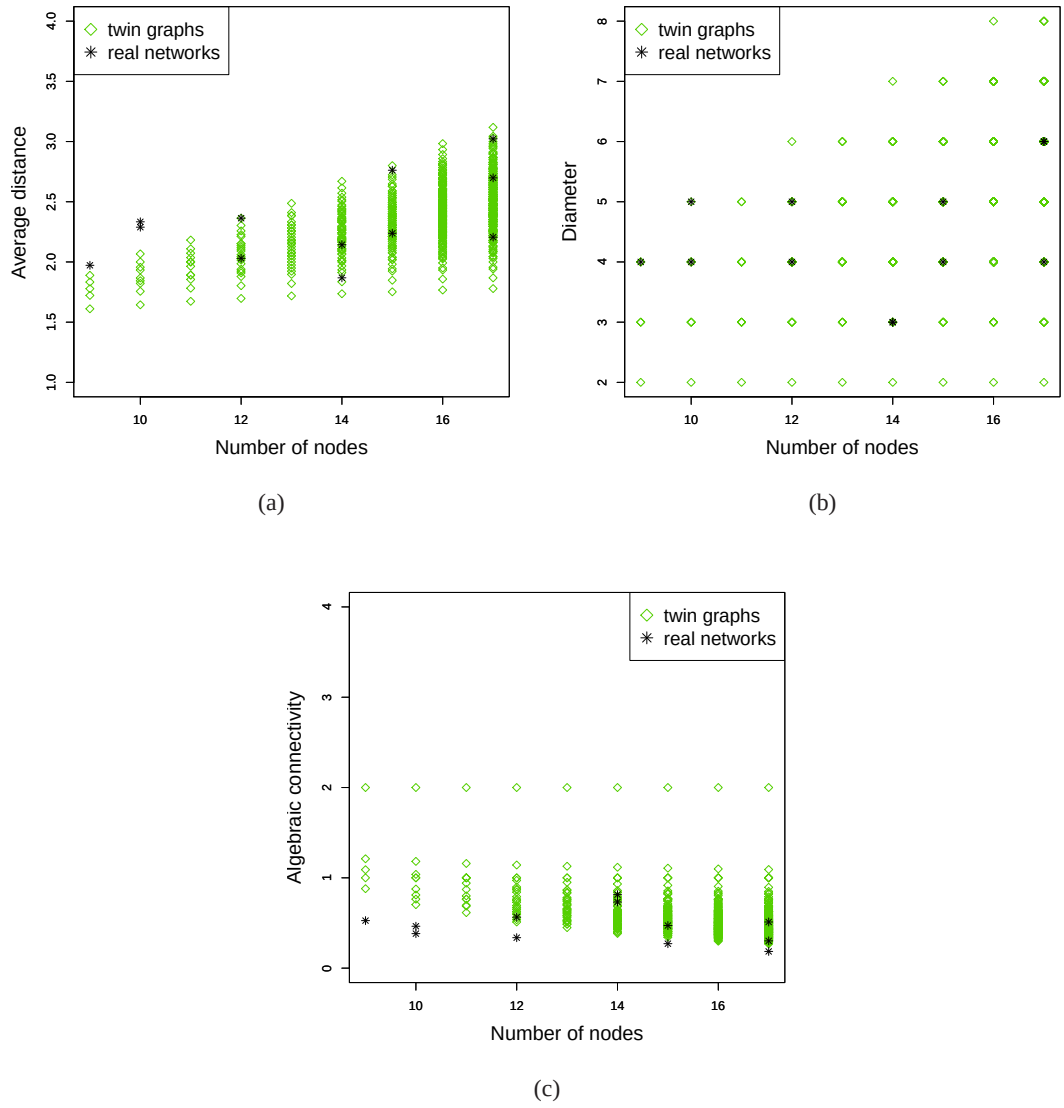


Figure 6: Distribution of the average distance 6(a), the diameter 6(b) and the algebraic connectivity 6(c) by number of nodes.

For the other invariants, the twin graph class shows constant values, with δ, node connectivity and link connectivity equal to 2. The real networks in R also show δ and link connectivity equal to 2, for all networks. The results for node connectivity are presented in Table 3.

We close this section by presenting the results obtained when the number of wavelengths

is jointly distributed with the invariants in study. For this, we consider the invariants that the above analyses have indicated in association with the wavelength requirements. These invariants are the average degree $\langle d \rangle$, the link density α, and the maximum link betweenness B^{maxE}, shown in Figures 7 and 8.

When considering the entire sample, the invariants $\langle d \rangle$ (Fig 7(a)) and α (Fig 7(c)) show good correlation with the number of wavelengths. However, by fixing the number of nodes n (Figs 7(b) and 7(d)), both of these invariants are constant. Thus, they do not explain the variation in the wavelength requirements. Given that, when designing an optical network, n is an input parameter of the problem, not a variable, it is necessary to find invariants that show good correlation with the number of wavelengths for each fixed n.

The maximum link betweenness B^{maxE} presented a very strong positive linear correlation $\rho = 0.995$ with the wavelength requirements, see Fig 8(a). A simple linear regression was performed to predict the number of wavelengths based on B^{maxE}. The regression equation obtained is $0.385 + 0.976 * B^{maxE}$, with R^2 of 0.99 and residual stardard error of 0.335. The $t - test$ presented strong evidence that both these model coefficients are significantly different from zero ($p - value < 0.001$). These results show a significant correlation between the variables maximum link betweenness and wavelength requirements.

For n fixed, B^{maxE} also showed a very strong positive linear correlation with the number of wavelengths. Given $n = 17$ (Fig 8(b)), the result for linear correlation was $\rho = 0.983$, pointing out that the number of wavelengths is proportionally related to the maximum link betweenness. Notice that this result agrees with the one found for the entire sample.

Moreover, these result shows how a single link could affect the wavelength requirements. For a twin graph of order $n = 17$, the wavelength requirements range from 9 to 17, whereas the maximum link betweenness ranges from 8.07 to 18.5. Therefore, B^{maxE} appears as an important invariant to be considered in solving the RWA problem, since the control on this invariant showed to be related to the reduction of the wavelength requirements.

5. Conclusion

We have presented the results obtained from comparative analyses between modelling networks as twin graphs and real networks. Several invariants of graph theory were compared in order to identify the main differences between the topological characteristics of these groups. The results help us to decide when twin graphs are indeed advantageous for the design of optical networks. Notice that this work naturally extends to the study of Elastic Optical Networks.

Our analyses showed that twin graph topologies lead to networks requiring less wavelengths, when compared to a set of real networks of the same orders. This reduction is usually achieved by the higher density and the lower congestion observed in twin graphs. Thus, an optical backbone network modelled as a twin graph could benefit from both fault tolerance and cost savings caused by the greater efficiency in using wavelengths.

In addition, we have found an invariant which presents a very strong correlation with the number of wavelengths. The maximum link betweenness showed to be directly related to the wavelength requeriments. Then, this invariant can be then used for solving the RWA problem, in order to minimize the network cost.

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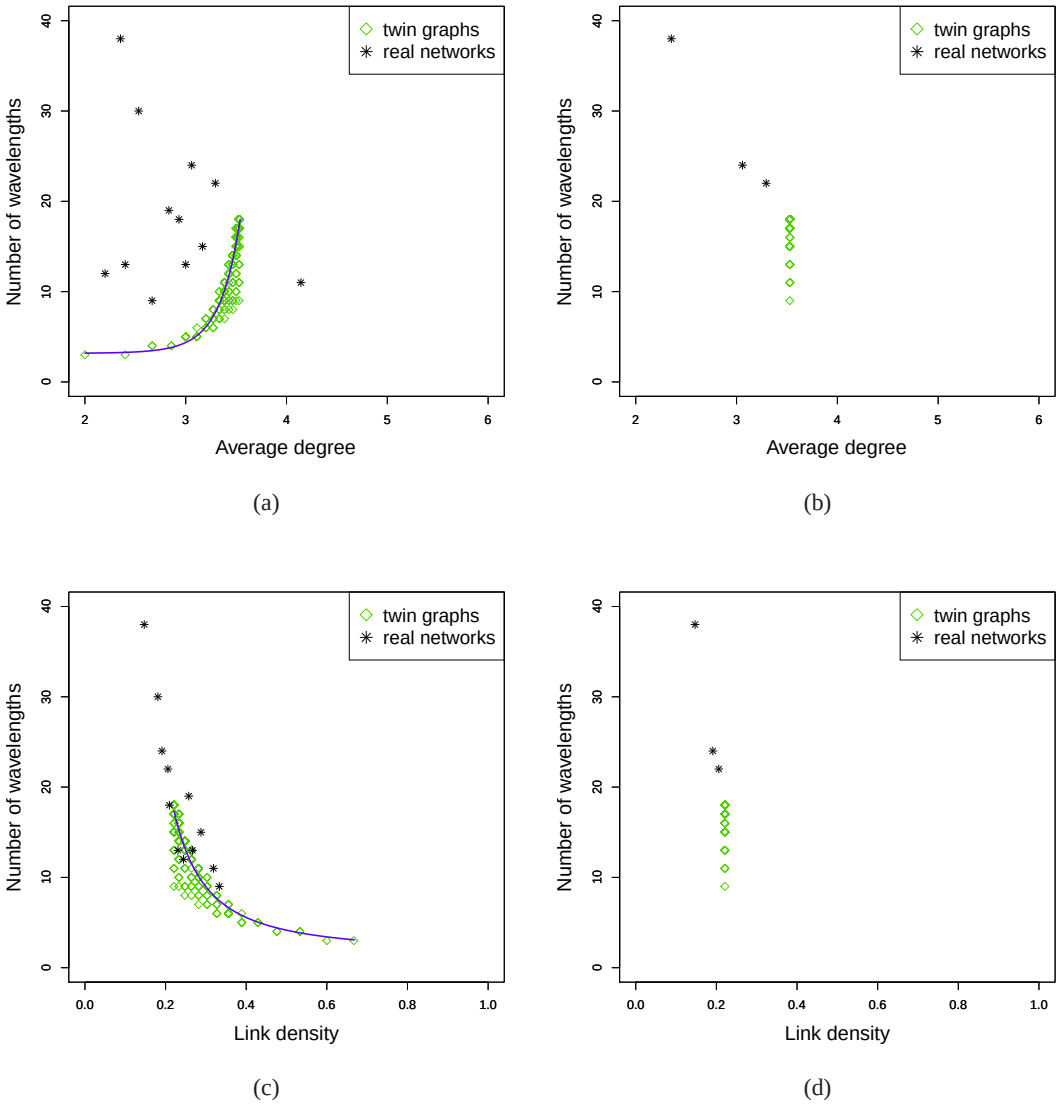
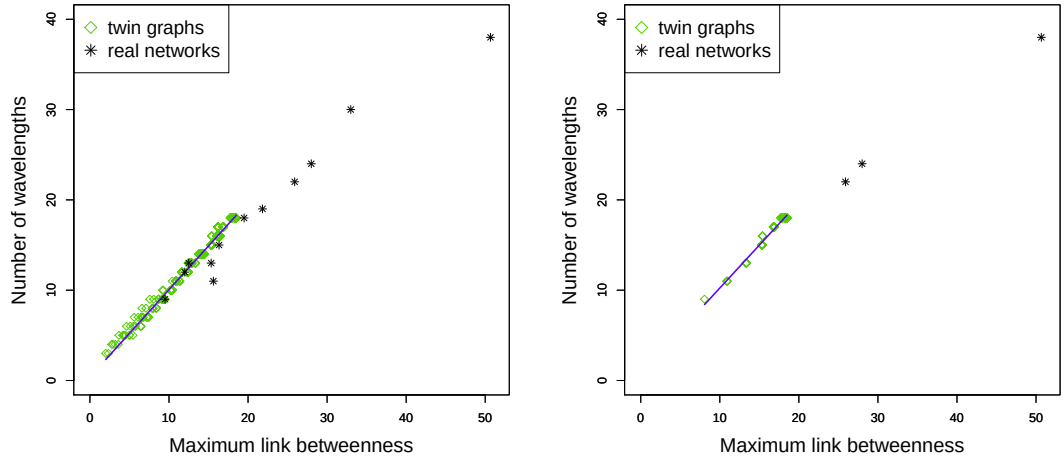


Figure 7: Number of wavelengths with respect to: 7(a) the average degree for all graphs in T and in R, 7(b), the average degree for graphs in T and in R with 17 nodes, 7(c) the link density for all graphs in T and in R, and 7(d) the link density for graphs in T and in R with 17 nodes.



(a) (b)

Figure 8: Number of wavelengths with respect to the maximum link betweenness, 8(a) for all graphs in T and in R, and 8(b) for graphs in T and in R with 17 nodes.

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