PROJECT PORTFOLIO SELECTION WITH RESTRICTIONS USING STOCHASTIC OPTIMIZATION BY SIMULATION

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ABSTRACT

Often companies face the need to select projects, since usually the resources are not enough for executing all. This limitation is reflected in the availability of financial support and also in limiting the other necessary resources (people, equipment, or other). Several selection ways have been presented in literature. One interesting approach is based on mean-Gini evaluation, due to be a less restrictive alternative. However, the decision-maker also faces the challenge of uncertainty over the projects' data, as they are based on estimates. Thus, this work proposes a project portfolio selection approach by mean-Gini evaluation, modeling parameters with triangular probability distribution (as recommended by Project Management Institute), applying stochastic optimization and Monte Carlo simulation. It is exemplified by a ten real projects example.

KEYWORDS. Portfolio Selection. Mean-Gini. Uncertainty.

Main Area: SIM - Simulation

1. Introduction

Usually, when having a list of projects to execute, companies are forced to decide to select a subset of them, due to resource constraints [Abbassi et al., 2014]. This need for decision has been one of the critical problems of project management [Jafarzadeh et al., 2015]. In this way, decision makers need to prioritize projects and define the best subset of projects to be executed [Perez e Gomez, 2014]. The defined subset of projects can be treated as a portfolio.

The application of the portfolio analysis allows managing the risk in a group of assets (which, in the case of the present work are the projects) to determine the combinations that offer the lowest risk for a given level of expected return or higher expected return for a given risk level. These portfolios are called efficient. The same rationale applied to the analysis of investment portfolios can be used in the selection of project portfolios [Eilat et al., 2006].

The work of Markowitz [1952] pioneered the optimal strategy to maximize return and minimize the risk for portfolio selection. In its proposition, a frontier of efficient portfolios (called simply efficient frontier) can be found by comparing the values of the expected returns and the corresponding variances of each portfolio under evaluation (called MV portfolio). However, Feldstein [1969] has demonstrated that MV portfolio presents reliable results only when returns are normally distributed or when the utility function of the decision maker is quadratic. These assumptions are very restrictive for real-world project selection problems [Better e Glover, 2006]. In this way, while the MV portfolio has been extensively explored, as in the works of Levy e Levy [2014]; Lizyayev [2012]; Zopounidis et al. [2014], some alternatives have been proposed.

Shalit e Yitzhaki [1984] presented an interesting proposal for selecting project portfolios using the comparison of mean and Gini coefficient of each portfolios under evaluation (MG portfolio). It is less restrictive than the MV portfolio. Alessandra Cillo [2014] used this approach to select investment portfolios. Already in the works of Ringuest et al. [2004]; Ringuest e Graves [2005]; Gemici-Ozkan et al. [2010], the MG portfolio was employed for project portfolio selection.

Perez e Gomez [2014] state that resource scarcity is a reality in companies. Also Dutra et al. [2014] state that, in general, there are not enough resources to support all project options. These aspects highlight the importance of considering resource availability for project execution, although it is not taken into account in some works.

Once portfolio selection occurs before knowing projects real return, it is done based on estimates. Despite the various tools known for estimation, its result naturally leads to uncertainty [Kitchenham e Linkman, 1997]. In this case, it is important to consider uncertainty in project portfolio selection, as presented in the works of Marcondes et al. [2017] and Marcondes e Leme [2015].

One way recommended by the Project Management Body of Knowledge Guide (PMBOK®), from Project Management Institute, to deal with project uncertainty is through the use of triangular probability distribution. It allows quantitative evaluation of projects, using a range of values (instead of single values for each parameter) and allows to reflect in the analysis the worst-case, best-case and most likely parameters estimates. The same PMBOK® also recommends the use of simulation in this evaluation, which according to Bertrand e Fransoo [2002] and Better e Glover [2006] is usually employed when the mathematical model is very difficult (or impossible) to be treated.

Thus, this work proposes an approach for project portfolio selection through simulation, using mean-Gini and considering resource constraints. In addition, the proposal takes into account the uncertainty project estimates (return and resource demand), which are characterized by the triangular probability distribution and selection done by a stochastic optimization problem.

The remainder of this article is organized as follows: Section 2 introduces portfolio selection through mean-Gini. Section 3 presents a way of considering uncertainty by using the triangular distribution. Section 4 presents the project portfolio selection proposal, and the numerical results are presented in Section 5 by means of an example. Finally, Section 6 presents the conclusions of the paper.

2. Mean-Gini Portfolio

The expected portfolio return depends on the expected return on each included project. The expected return calculation of a project is done by means of the expected value of the random variable that describes it. Thus, the portfolio's expected return (R_P) (or the mean portfolio return) can be calculated by the equation:

$$R_P = \sum_{j=1}^{N} w_j r_j \tag{1}$$

where:

- N is the total of projects under evaluation;
- r_j is the expected return of project j;
- $w_j \in \{0, 1\}$ represents the decision of exclude or include project j in portfolio, respectively.

The Gini coefficient is a measure of statistical dispersion. This coefficient is a well-known measure of income inequality that is widely used [Rogerson, 2013]. Despite its original purpose, it has been applied in various forms of dispersion measure between values. Thus, Shalit e Yitzhaki (1984, 1989, 2005) presented the application of Gini coefficient values for portfolio risk analysis, where such an approach appears as an alternative to the traditional use of variance. It is a simple understanding measure and easy to be presented to the decision maker [Ringuest et al., 2004].

The calculation of the Gini coefficient, as proposed by Shalit e Yitzhaki [1984], can be done by:

$$\Gamma = 2\text{cov}\left[R, F\left(R\right)\right] \tag{2}$$

where:

- R is the stochastic variable for portfolio return;
- $F\left(R\right)$ is its cumulative probability distribution.

Based on this values, an optimization problem can be built for selecting a portfolio, as following:

$$\begin{aligned} \text{minimize} \quad & \Gamma \\ \text{subject to} \quad & \\ & R_P \geqslant R_0 \end{aligned} \tag{3}$$

where R_0 is the minimum return expected by the decision maker.

Someone can build an efficient frontier if this optimization problem is repeated for $\underline{R_P} \leqslant R_0 \leqslant \overline{R_P}$ ($\underline{R_P}$ and $\overline{R_P}$ are the lowest and the highest possible mean portfolio returns, respectively). However, it does not consider any resource restriction in the selection.

Given that the available resources are generally not enough for the execution of all projects, as stated by Dutra et al. [2014], and that this has been one of the most critical problems in management of projects, as stated by Jafarzadeh et al. [2015], the present work considers it. In this way, the selection policy incorporates this restriction in its application.

Thus, for including this restriction in the problem, Equation 3 can be rewritten as:

$$\begin{aligned} \text{minimize} \quad & \Gamma \\ \text{subject to} \quad & \\ & R_P \geqslant R_0 \\ & \sum_{i=1}^{N} w_j z_j \leqslant z_0 \end{aligned} \tag{4}$$

where:

- z_j is the demand for resource of project j;
- z_0 is the resource limit available for the chosen portfolio execution.

3. Uncertainty

As stated by Kitchenham e Linkman [1997], uncertainty is inherent in the estimation process. It reflects the lack of knowledge about a future event [Gregory et al., 2011]. That is, any estimation carries with it the uncertainty about the estimated value. Thus, in the selection of project portfolios, mechanisms must be used to deal with uncertainty.

According to PMI [2013], one of the ways to treat uncertainty in projects is through the use of triangular probability distribution. It presents a more direct way of considering the results obtained by three point estimate, which evaluates the resources according to the worst-case, best-case and most likely views. It is a continuous probability distribution with a lower limit a, an upper limit b and a mode c (most likely value or value that repeats the most), so that $a < b$ and $a \leqslant c \leqslant b$. Figure 1 illustrates the triangular probability density function.

Because it is easier to define its parameters (it allows the decision maker to more directly convert its estimates into parameters of the distribution [1]), the triangular distribution is often more

[1]Mainly, when the three-point estimate is used, which is one of estimation tools recommended by PMBOK®.

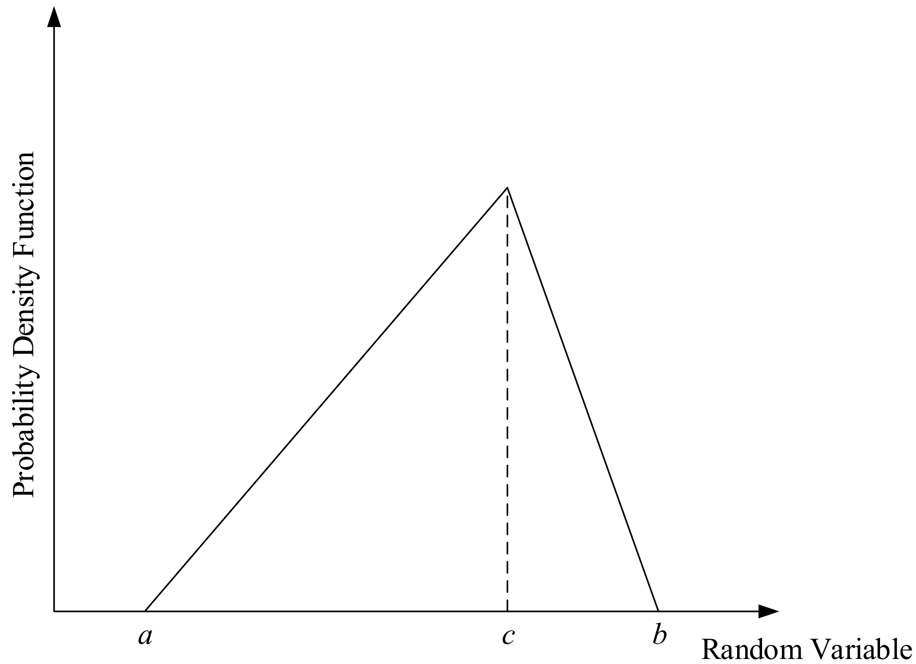

Figure 1: Triangular Probability Density Function Example.

used than the beta distribution in project management [van Dorp e Kotz, 2002; Johnson, 1997; Stein e Keblis, 2009; Yang, 2005].

Mathews [2009] used the triangular distribution for cost evaluation and project return, as it is a convenient approach for practical purposes. This application can be done by assigning to parameter a the lowest expected value, to parameter b the highest expected value and to parameter c the most probable expected value [van Dorp e Kotz, 2002; Mathews, 2009; Yang, 2005].

4. Stochastic Optimization for Project Portfolio Selection

Based on what was presented in Sections 2 and 3, this section presents the proposed approach of this paper for portfolio selection. It considers the use of mean-Gini portfolio, resource constraint, and treatment of uncertainty by employing the triangular probability distribution and simulation.

Thus, considering the optimization problem presented in Equation 4 and the uncertainty over resource estimation parameter, it can be rewritten as:

$$\begin{aligned} &\text{minimize} && \Gamma \\ &\text{subject to} && \\ & && R_P \geqslant R_0 \\ & && \text{Prob}\left[\sum_{i=1}^{N} w_j z_j \leqslant z_0\right] \geqslant P_0 \end{aligned} \tag{5}$$

where P_0 is the lowest acceptable probability for meeting resource restriction. As described in Section 2, also in this case an efficient frontier can be built varying R_0 value.

Figure 2 presents the simulation sequence used in the proposed project portfolio selection and detailed below.

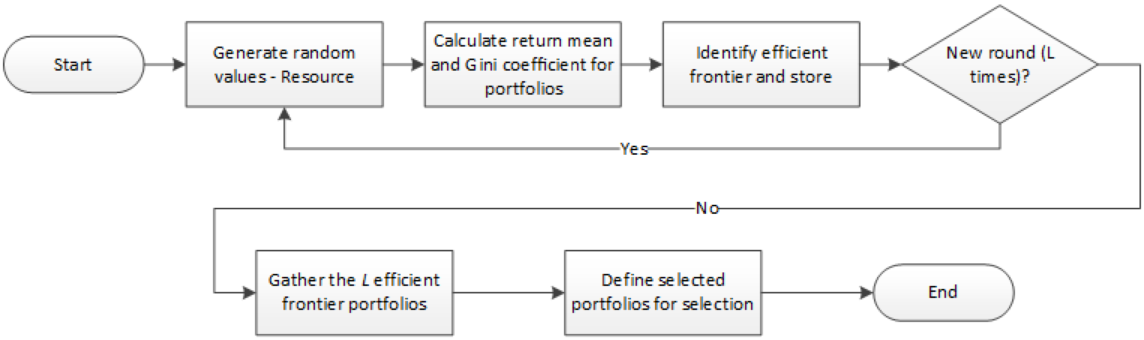

Figure 2: Project Portfolio Selection Simulation Sequence.

- **Generate random values - Resource**, based on resource demand three-point estimates, for each project, and applying the triangular probability distribution;
- **Calculate return mean and Gini coefficient for portfolios**, based on the values of projects return three point estimate and applying the triangular probability distribution;
- **Identify efficient frontier and store**, applying MG portfolio and considering resource availability, as in Equation (5);
- **New round?**, repeat L times the previous steps;
- **Gather the L efficient frontier portfolios**, to carry out the final evaluation;
- **Define selected portfolios for selection**, considering the frequency of occurrence of each portfolio in the L efficient frontiers.

For each L simulation rounds, a random resource need value of each project is generated, based on the established triangular distribution. This approach allows one to consider the uncertainty about resource estimation in portfolio evaluation.

In the final selection step, the decision maker can define the degree of risk to be taken in the choice. That is, if she/he does not want to take any risk on resources availability in the execution of her/his projects, she/he can only opt for a portfolio that have been indicated in 100% of the simulation rounds (in this case, $P_0 = 100\%$). If she/he wants to take some risk, the indicated list can include portfolios with lower probability of occurrence (for instance, $P_0 = 90\%$). With this list of indicated portfolios, applying some criteria defined by her/him or her/his organization (greater return, lower risk, less resource consumption, among others), the decision maker can decide which project portfolio will be executed.

Following the classification presented by Eilat et al. [2006]; Urli e Terrien [2010], this work considers the static selection. For it, the evaluation does not include the active projects, which are those that are already in progress, but only the candidate projects, which are those that have not started yet.

5. Example with Real Projects

To exemplify the selection approach proposed in Section 4, this paper draws on a ten real projects example. For them, the three-point estimated return values of each project are provided. The example considers additionally the evaluation of people availability for projects execution (the limited resource considered in this example). Each project presents its demand for people involved, also using three-point estimate.

The projects' values are presented in Table 1, in which the return estimates are listed from the second to the fourth columns and the estimated persons involved listed from the fifth to the seventh columns.

Table 1: Projects Information.

	Return			People		
Project	Best-Case	Most Likely	Worst-Case	Best-Case	Most Likely	Worst-Case
A	98,848	46,080	-20,236	10	12	14
B	19,008	6,360	-6,384	3	4	5
C	36,384	11,620	-15,667	5	6	7
D	37,968	15,777	-15,129	6	7	8
E	40,032	16,152	-14,304	6	7	8
F	16,992	6,744	-4,531	4	5	6
G	57,520	26,647	-22,406	6	8	9
H	80,793	36,412	-20,928	9	10	11
I	17,932	6,837	-5,424	4	5	6
J	35,323	13,615	-15,475	5	6	8

With these values, triangular distributions for return and resources (people) can be established in order to deal with the uncertainty caused by the estimation in the selection process. Thus, for return characterization of each project, parameter a receives the worst-case estimation value, parameter b the best-case estimation, and parameter c the most likely value. In the case of number of people, the parameter designation is inverted: parameter a receives the best-case estimation value, parameter b the worst-case estimation value, and parameter c the most likely value.

Once these values were known, a Monte Carlo simulation was performed. In this simulation, the L value, which characterizes its number of repetitions, was $2,000$. Its execution also considered a limit of 30 people available for projects execution (project development team size in the company) and $P_0 = 90\%$ (defined by the company as a limit to be prepared for a risk contingency, as commented further).

The simulation result led to 27 feasible and efficient portfolios (out of a total of $1,023$ possible portfolios), which were included in the indicated portfolio list with, at least, 90% probability of meeting people availability limit. The indicated portfolios are presented in Table 2 (the sequence of letters indicates the projects included in the portfolio).

These are the portfolios that met the limit of 30 people involvement in 90% of the simulation rounds and that are considered efficient as MG criteria. In this case, if the decision-maker chooses some portfolio with occurrence probability lower than 100% (portfolio ABIJ, for instance), she/he is assuming some risk: project development team could not be sufficient for the execution of all projects, and some overtime work should be needed. However, if the decision-maker knew this information previously, some contingency plan could be prepared for this eventual overtime demand.

For this set of projects, if total number of people to be involved limitation did not exist, the efficient frontier would include 166 portfolios. This information, without limiting the total number of people, can help the decision-maker define whether to seek more resources (people, in

Table 2: Indicated Portfolios.

Portfolio	Return Mean	Gini Coefficient	Probability
ABIJ	55,588	17,681	95%
AFIJ	55,521	17,634	90%
ABFJ	55,440	17,625	95%
BEFGI	44,762	13,575	90%
BCEIJ	40,881	12,338	95%
BCEFJ	40,734	12,286	95%
BCDIJ	40,255	12,233	95%
CDFIJ	40,188	12,057	90%
BDEFI	39,801	11,459	100%
BEFIJ	38,615	10,568	95%
BDFIJ	37,989	10,416	95%
BEIJ	32,720	10,283	100%
EFIJ	32,653	10,229	100%
BEFJ	32,573	10,204	100%
BDIJ	32,094	10,178	100%
DFIJ	32,027	9,999	100%
BEFI	29,164	8,733	100%
BDFI	28,537	8,563	100%
BFIJ	27,351	7,915	100%
BIJ	21,456	7,545	100%
FIJ	21,389	7,458	100%
BFJ	21,308	7,383	100%
BFI	17,899	4,845	100%
BI	12,005	4,138	100%
FI	11,938	3,846	100%
I	6,043	2,865	100%
F	5,895	2,596	100%

the case of this example) to get involved in the projects or to keep the limit set. Still for the same set of projects, if the number of people were taken by the most likely value and not considered uncertainty, the indicated portfolios would be 60, maintaining the same limit of people available for the activities, confirming that the uncertainty causes a great impact on selection (33 probably unfeasible portfolios would be included in the efficient frontier).

Based on the portfolios in the efficient frontier, the decision maker can choose the portfolio to be effectively executed. For this, she/he can use other techniques. A selection based on expected minimum return or at maximum tolerated risk may be employed. As applied by Ringuest et al. [2004], she/he can use the stochastic dominance criterion on efficient frontier portfolios. Or, she/he can employ a project prioritization based on data envelopment analysis and occurrence frequency, as done by Urli e Terrien [2010]. Regardless of the criterion used in defining the chosen portfolio, since it is at the efficient frontier, it is known that there is no other portfolio that is more efficient and feasible, according to the criteria established in this study, with the same level of return and risk.

6. Conclusions

This work presents an approach for static project selection, based on the return and risk of portfolios and considering resource limitation. Portfolio risk is measured by the Gini coefficient, given the advantages it has over using the traditional variance approach. Thus, the evaluation does not depend on the return normal distribution and neither of the decision maker quadratic utility function.

It contributes to the prior proposal of using mean-Gini for project portfolio selection in some ways. First, it proposes a more practical distribution to characterize projects return, using triangular continuous distribution instead of a discrete one. It also introduces the resource limitation in the evaluation, to consider more realistic situation in project management environment. As a new estimation inserted in the selection, resource limitation is also considered as a stochastic variable, leading to an application of stochastic optimization problem. These contributions help to deal better with estimation uncertainty and resource limitation, and they allow to evaluate situations closer to the real ones usually faced by companies.

In both variables, projects returns and needed resources (in the case of the presented example, people involved), the uncertainty is considered through the application of triangular probability distribution. It allows, in a simple and practical way, to reflect in selection simulation the information obtained by the three-point estimates.

The imposition of the resource limitation presented in this paper does not mean that it can not be extended based on the results. It also allows the decision-maker to evaluate whether or not increase this limit, for instance, to include a strategic portfolio in the efficient frontier.

In the example, this approach indicated a set of 27 efficient portfolios that met the constraint (with some risk assumed by the decision-maker), in a set of $1,023$ possible portfolios. It was done based on objective comparison criteria and considering uncertainty. This known risk can help decision-maker to prepare a contingency plan for it.

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