ANALYSIS OF PROCESS FLEXIBILITY AND THE CHAINING PRINCIPLE IN LOT SIZING PROBLEMS

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ABSTRACT

We analyse the value of process flexibility in the context of a deterministic lot sizing problem with backlogging, where several types of products can be made on several alternative resources. The lot sizing problem consists in determining the quantity of products to be produced in each period of a finite time horizon, in order to meet a given demand and to minimize total costs. When multiple parallel resources are present, the standard assumption is that each product can be produced on any of the resources, i.e. we have complete resource flexibility. However, it may be very costly in practice to install resources that have complete flexibility, especially if the products are very different. Therefore, it might be interesting to only implement a limited amount of flexibility where each resource can produce only certain types of items. In order to study the value of such process flexibility, we perform several analyses.

KEYWORDS. Lot Sizing. Process Flexibility. Long Chain.

OC - Combinatorial Optimization

1. Introduction

The literature on flexibility covers a wide spectrum of issues ranging from strategic decisions such as capacity planning [Fine and Freund, 1990; Jordan and Graves, 1995] to detailed operational issues such as the number of tool changes [Tang and Denardo, 1988a]. The concept of process flexibility in a supply chain defines the type of products that can be manufactured on various alternative resources such as plants or machines. Making decisions about process flexibility is a very complex task, first because the benefits of flexibility are very difficult to calculate and second because the number of possible product assignment configurations grows exponentially with the number of items and alternative resources. Since total process flexibility can be very expensive, it is very important to study ways to implement a limited amount of flexibility to balance the costs and the benefits.

[Jordan and Graves, 1995] analysed the value of manufacturing process flexibility in a stochastic model with a single period and single stage production environment where multiple products can be produced on different capacitated plants. Each plant can be either dedicated to one specific product or flexible to produce several different products. Demand is random and it is possible that some of the demand will be lost if there is insufficient capacity. More flexibility will allow one to satisfy more of the total demand. The main insight from the paper is that almost all of the benefits of flexibility, can be achieved by implementing only a small amount of flexibility, but in a smart way. To analyse the value of process flexibility, [Jordan and Graves, 1995] introduced the concept of ”chaining”. A ”chain”is a group of items and plants which are all connected, directly or indirectly, by product assignment decisions. Within a chain, a path can be traced from any item or plant to any other item or plant via the product assignment links. The key idea behind chaining is that excess capacity can be shifted - to some extend - along the chain. The benefits of flexibility do not come only from having more items assigned per plant, but also from creating longer chains. The intuition behind this concept is easy to grasp. In the stochastic context presented in [Jordan and Graves, 1995], the longer the chain of items and plants, the greater the opportunities are for shifting capacity for building items with lower than expected demand to those with higher than expected demand.

This chaining principle has, to the best of our knowledge, not yet been explored in a lot sizing context. Therefore, the main objective of this work is to analyse the trade-off between the benefits of process flexibility and its cost in a lot sizing context. More specifically, we analyse the value of process flexibility in the context of the deterministic lot sizing problem with backlogging which consists basically of determining the size of production lots, i.e. the amounts of each item to be produced in each of the periods in the planning horizon, in a way that minimizes total costs, respects the resource availability and meets the known demand of the items.

The work is organized as follows. In Section 2, we provide the mathematical models for the lot sizing problem with plant flexibility. Section 3 presents the analysis of process flexibility on a classical formulation and shows the computational results considering the chaining concepts. In Section 4, we present the development and computational results considering the formulation with process flexibility as a decision variable. Finally in Section 5, we present our conclusions.

2. Mathematical Models for the Lot Sizing Problem with Process Flexibility

In this section, we present two different models for the lot sizing problem with multiple resources and limited flexibility. We first present a formulation that is based on the classical formulation of the lot sizing problem with unrelated parallel machines, i.e. the setup costs and times are different for some product on different machines. This formulation is based on the formulation of [Trigeiro et al., 1989] for the single machine problem, and has been studied in [Toledo and Armentano, 2006], [Fiorotto and de Araujo, 2014] and [Fiorotto et al., 2015]. Subsequently, we present a new optimization model that considers the possibility of investing in flexibility and determines the optimal flexibility configuration for a given budget.

We observe that in the multi-plant models, if there is no production transfer between the plants, then the model represents a parallel machine problem. Each plant corresponds to a machine. As the proposed models are based on parallel machines problems and these models are widely researched in lot sizing literature, from now on, without loss of generality we will use parallel machines instead multi-plant.

There are many studies in the literature on the lot sizing problem with parallel machines considering complete machine flexibility. For the problem with identical machines, [Lasdon and Terjung, 1971] propose a heuristic for a lot sizing and scheduling problem without setup time. [Carreno, 1990] also proposes a heuristic for this problem with setup time and constant demand. [Jans, 2009] proposes new constraints to break the symmetry that is present due to the identical machines and tests his approach using a network reformulation for the problem. For the unrelated parallel machines case, [Toledo and Armentano, 2006] relax the capacity constraints and propose a Lagrangian heuristic to solve the problem. [Fiorotto and de Araujo, 2014] use a network reformulation of the problem and instead of the capacity constraints, they relax the demand constraints using Lagrangian relaxation and also propose a heuristic to find feasible solutions. [Fiorotto et al., 2015] present hybrid methods using Lagrangian relaxation and Dantzig-Wolfe decomposition and find better lower and upper bounds compared to [Toledo and Armentano, 2006] and [Fiorotto and de Araujo, 2014].

Although the lot sizing problem on parallel machines with a limited amount of flexibility is a natural and more realistic extension of the standard assumption, there has only very limited research been done on this topic. In their application in the tire industry, [Jans and Degraeve, 2004] discuss a problem where not all types of tires can be produced on all types of heaters. [Xiao et al., 2015] propose a hybrid Lagrangian and simulated annealing based heuristic for the capacitated parallel machine lot sizing and scheduling problem where not all machines are eligible to produce all items. Moreover, they consider that for each item there is a machine preference increasing the quality of the items and the machine reliability.

2.1. Lot Sizing on Parallel Machines with Limited Flexibility

The problem will first be modeled as a single stage lot sizing model with multiple parallel machines and multiple products, where a specific product can be made on some machine(s), but possibly not on some others. The formulation that has been proposed for the parallel machine problem with total flexibility (see e.g., [Toledo and Armentano, 2006]) can easily be adapted to include limited flexibility.

For the mathematical formulation of the problem, we consider the following sets and input parameters:
$I = \{1, ..., n\}$: set of items;
$J = \{1, ..., r\}$: set of machines;
$T = \{1, ..., m\}$: set of periods;
I_j: set of items *i* that can be produced on machine *j*;
J_i: set of machines *j* that can produce item *i*;
d_{it}: demand of item *i* in period *t*;
$sd_{it\tau}$: the sum of the demand for item *i*, from period *t* until period τ ($\tau \geq t$);
hc_{it}: unit inventory cost of item *i* in period *t*;
bc_{it}: unit backlog cost of item *i* in period *t*;
sc_{ijt}: setup cost for item *i* on machine *j* in period *t*;
vc_{ijt}: production cost of item *i* on machine *j* in period *t*;
st_{ijt}: setup time for item *i* on machine *j* in period *t*;
vt_{ijt}: production time of item *i* on machine *j* in period *t*;
Cap_{jt}: capacity (in units of time) of machine *j* in period *t*.

The decision variables are then defined as follows:
x_{ijt}: number of units produced of item *i* on machine *j* in period *t*;
y_{ijt}: binary setup variable, indicating the production or not of item *i* on machine *j* in period *t*;
s_{it}: quantity of inventory of item *i* at the end of period *t*;
b_{it}: quantity of backlog of item *i* at the end of period *t*.

The mathematical formulation of the problem is then as follows:

$$Min \sum_{j\in J}\sum_{i\in I_j}\sum_{t\in T}(sc_{ijt}y_{ijt} + vc_{ijt}x_{ijt}) + \sum_{i\in I}\sum_{t\in T}(hc_{it}s_{it} + bc_{it}b_{it}) \tag{1}$$

Subject to:

$$s_{i,t-1} + b_{it} + \sum_{j\in J_i} x_{ijt} = d_{it} + s_{it} + b_{i,t-1} \qquad \forall i \in I, t \in T \tag{2}$$

$$x_{ijt} \leq min\{(Cap_{jt} - st_{ijt})/vt_{ijt}, sd_{i1m}\}y_{ijt} \qquad \forall j \in J, i \in I_j, t \in T \tag{3}$$

$$\sum_{i\in I_j}(st_{ijt}y_{ijt} + vt_{ijt}x_{ijt}) \leq Cap_{jt} \qquad \forall j \in J, t \in T \tag{4}$$

$$y_{ijt} \in \{0,1\},\ \ x_{ijt} \geq 0 \qquad \forall j \in J, i \in I_j, t \in T \tag{5}$$

$$s_{it} \geq 0,\ \ s_{i0} = 0,\ \ s_{im} = 0,\ \ b_{it} \geq 0,\ \ b_{i0} = 0 \qquad \forall i \in I, t \in T \tag{6}$$

The objective function (1) minimizes the total setup, production, inventory and backlog costs. The constraints (2) guarantee the inventory balance in each period. Demand that cannot be satisfied on time can be backlogged. Next are the machine setup constraints (3) and the capacity limits (4). In order to make the formulation stronger, we limit the production for each item in constraints (3) by both the sum of the demand and the maximum possible production with the available capacity. Finally, constraints (5) and (6) define the variables domains.

Observe that if $J_i = J,\ \forall i \in I$, then we have the total flexibility case. If $|I_j| = 1,\ \forall j \in J$ then this indicate that all machines are dedicated to one product, and the problem can be separated per item.

2.2. Lot Sizing with Process Flexibility as a Decision Variable

A related question that came up was if we can do better than the long chain with the same number of additional flexibility links. In order to check this we developed another new model that finds the best flexibility configuration given a limited number of additional links or a limited budget.

Therefore, we consider the possibility of investing in flexibility. The investment of upgrading a machine for a specific product becomes a binary decision variable and there is a global budget on the investment decisions. Such a model will enable us to check the structure of the optimal flexibility configuration for various levels of the global budget.

For the mathematical formulation of the problem, we consider the following additional parameters:
fc_{ij}: flexibility investment cost for producing item *i* on machine *j*;
$Fmax$: budget to invest in flexibility.

The additional decision variables are then defined as follows:
z_{ij}: binary variable, indicating that machine *j* can produce item *i* or not.

Mathematical formulation:

$$Min \sum_{j\in J}\sum_{i\in I}\sum_{t\in T}(sc_{ijt}y_{ijt} + vc_{ijt}x_{ijt}) + \sum_{i\in I}\sum_{t\in T}(hc_{it}s_{it} + bc_{it}b_{it}) \tag{7}$$

Subject to:

$$s_{i,t-1} - b_{i,t-1} + \sum_{j \in J} x_{ijt} = d_{it} + s_{it} - b_{it} \qquad \forall i \in I, t \in T \qquad (8)$$

$$x_{ijt} \leq min\{(Cap_{jt} - st_{ijt})/vt_{ijt}, sd_{itm}\}y_{ijt} \qquad \forall j \in J, i \in I, t \in T \qquad (9)$$

$$\sum_{i \in I}(st_{ijt}y_{ijt} + vt_{ijt}x_{ijt}) \leq Cap_{jt} \qquad \forall j \in J, t \in T \qquad (10)$$

$$y_{ijt} \leq z_{ij} \qquad \forall j \in J, i \in I, t \in T \qquad (11)$$

$$\sum_{i \in I}\sum_{j \in J} fc_{ij}z_{ij} \leq Fmax \qquad (12)$$

$$y_{ijt} \in \{0,1\}, \;\; z_{ij} \in \{0,1\}, \;\; x_{ijt} \geq 0 \qquad \forall j \in J, i \in I, t \in T \qquad (13)$$

$$s_{it} \geq 0, \;\; s_{i0} = 0, \;\; s_{im} = 0, \;\; b_{it} \geq 0, \;\; b_{i0} = 0, \qquad \forall i \in I, t \in T \qquad (14)$$

The objective function (7) and constraints (8)-(10) are similar to the previous formulation. The constraints (11) guarantee that a machine can be set up to produce a specific item in a specific period only if this machine has the flexibility to produce this item. Constraint (12) limits the budget for investing in flexibility. We model the flexibility investment as part of a budget constraint, instead of putting it in the objective function. This will allow us to trace the trade-off between the level of flexibility and the operational costs. Due to constraints (11) and (12) the binary condition on the variables z_{ij} can be relaxed.

Note that we have not imposed in the models that the backlog of the last period must be equal to zero ($b_{im} = 0$). In other words, we consider the possibility of not satisfying all demand. We are not imposing the backlog to be zero in the final period because we will test several instances with different levels of capacity. Therefore, if we required that $b_{im} = 0$, we would have many infeasible instances.

3. Analysis of Process Flexibility Configurations

We will analyse the concept of process flexibility in a lot sizing context. The base case for the comparison is the case in which each machine is dedicated to exactly one specific product. In the deterministic lot sizing case, the value of flexibility will be apparent if for this base case (i.e., with only dedicated machines), not all of the demand can be satisfied on time leading to costly backorders. In such a case, adding flexibility (i.e. some machines can produce several types of products instead of just one) can decrease the amount of backlog and hence the total cost. A first objective is to analyse the effect of long chains, such as proposed in [Jordan and Graves, 1995], and compare this against several other cases such as the base case with only dedicated machines, the case with random flexibility where there is no specific pattern in the augmented flexibility, and the case of total flexibility where each machine can make every product. A second objective is to analyse how different parameters such as the number of machines and items, the backlog costs, the setup costs and setup times have an impact on the value of flexibility. The value of this manufacturing flexibility will next be compared to the value of increasing capacity on dedicated machines.

3.1. Setup of the Computational Tests

Using the ideas proposed by [Jans, 2009], we adapted a standard data set proposed by [Trigeiro et al., 1989] which was originally proposed to test the single-machine capacitated lot-sizing problem with setup times. This standard set is used in many computational experiments as a benchmark test set [Belvaux and Wolsey, 2000; Van Vyve and Wolsey, 2006; Degraeve and Jans, 2007; de Araujo et al., 2015]. We used the problem sets $F1 - F20$ and $G51 - G60$. The $F1 - F20$ set contains 20 instances with 6 items and 15 periods. The $G51 - G60$ set consists of 5 instances with 12 items and 15 periods and 5 instances with 24 items and 15 periods. For $F1 - F20$, $G51 - G55$ and $G56 - G60$, the original capacity level was set at 728, 1456 and 2912, respectively. For each of the 30 problem instances, we created identical parallel machine problems, i.e. the capacities are the same for each machine, and for a given item, the setup time, unit production time and setup cost are the same on each machine. Note that in these [Trigeiro et al., 1989] instances, the unit production costs are not considered. Furthermore, the demand is different in different time periods, but the average demand per period for each item is equal to 100. Our base case is the case with dedicated machines, i.e. each machine can make exactly one product, and hence we have as many machines as we have products. We set the backlog costs for each item equal to 100 times the inventory holding cost. The choice of the capacity levels was based on preliminary tests to have a broad range of problems so that the solutions have different levels of backlog. As such, by changing the capacity level each original single-machine test problem resulted in 12 different parallel machine test problems. As a result, 360 different test problems were created.

For each instance, we tested the effect of using different flexibility configurations. Figure 1 shows an example with 6 items, 6 machines and 4 of the 5 flexibility configurations that we will analyse in this section. The first case (case (a)) is the dedicated case. In cases (b) and (c), we have added additional links to increase the flexibility. The number of additional links (on top of the base case) is equal to the number of items. However, the flexibility was added in different ways. In case (b) we have 3 clusters of 2 machines, whereas in case (c) we have a long chain. The goal is to show the impact of the same level of flexibility when it is being added in different ways. In the final case (d), all the flexibility links are present. In the case of 12 and 24 machines this figure is extended in a straightforward way. For the clustered configuration, we have 6 clusters of 2 machines for the 12 machine case, and 12 clusters of 2 machines for the 24 machine case.

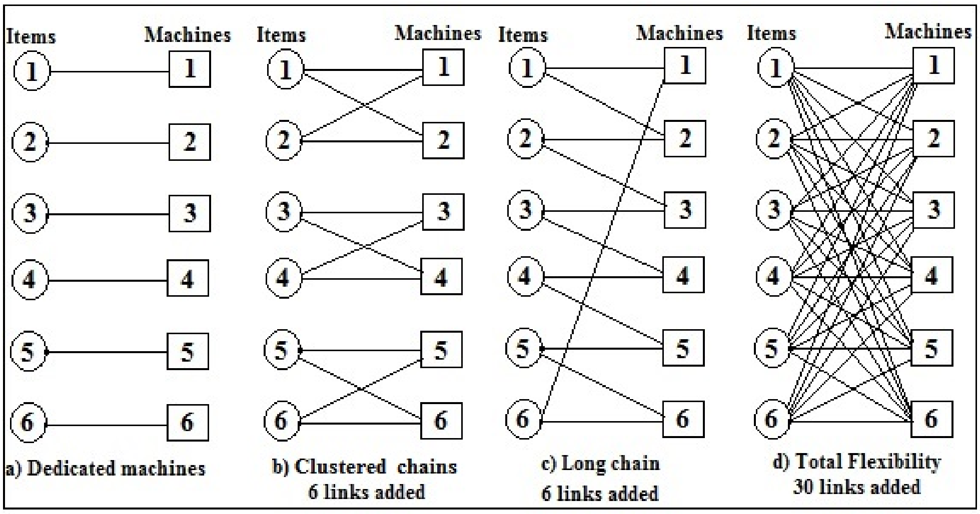

Figura 1: Flexibility configuration for 6 items.

The random flexibility configuration is not presented in Figure 1. In this random flexibility configuration, we add the same amount of links as in the clustered and the long chain, i.e. 6 additional links for the 6 machine case, 12 additional links for the 12 machine case and 24 additional links for the 24 machine case. The additional links are added randomly. Since there are many ways in which links can be added randomly to the dedicated case, we generated 10 different random flexibility configurations. We also note that there are many different long chains and clustered configurations possible, by changing the sequence of the items. Therefore, we generated 10 different long chains and clustered configurations, by changing the order of the items randomly and keeping the same structure of links between machines and items.

Formulation (1)-(6) (with the appropriate configuration of the links) is used to analyse the following cases: dedicated, clustered, random, long chain and total flexibility. The formulations were modeled in C++ using the concert technology and CPLEX 12.6 as solver. The tests were done on a computer with 2 Intel(R) Xeon(R) X5675 processors, 3.07GHz with 96GB of RAM and the Linux operating system. Moreover, when solving the formulation (1)-(6), we have limited the computational time in all instances to 600 seconds.

3.2. Discussion of the Computational Results

In Table 1 we consider different levels of capacity (Cap) and give the average upper bounds found for all flexibility configurations. We set the upper bounds found by the formulation with dedicated machines to 100% and calculate the other values relative to this. For each of the Clustered, Random and Long chain, we generated 10 different configurations. The upper bound (UB) reported are averages over these ten configurations. We also report the minimum upper bound (min) and maximum upper bound (max) obtained over these 10 configurations.

Just as in [Jordan and Graves, 1995], we observe that the value of flexibility depends on the capacity level. For all forms of flexibility (clustered, random, long chain and total flexibility), we observe that the benefits of flexibility are the highest for medium levels of capacity, but these benefits follow an inverted U-shape and are lower for high and low levels of capacity. Upon further analysis, our results clearly show that:

1) The benefits of the long chain are very close to the ones obtained by total flexibility for high levels of capacity. However, for low capacity levels (i.e. lower than 100), the long chain still has a substantial performance difference compared to the total flexibility. When the capacity levels becomes smaller, the performance difference between these two configurations typically increases, and reach up to approximately 13% for the lowest capacity level and 24 items. The reason is that in the optimal solution with full flexibility and low capacity levels, some items might not be produced at all because the backlog cost of that item is lower than for other items. In such a case it is better to focus on first satisfying the demand of the products with a high backlog cost (all other thing being equal). The long chain provides less flexibility and leads possibly to results where focusing solely on satisfying the products with high backlog costs is not possible. This happens for example if there are many products with a low backlog cost in the system, and some machines will have to be linked to two products which both have a low backlog cost (and thereby forcing some production of a product with low backlog cost).

2) By comparing the performance of the random and long chain configuration, we see that the difference between the benefits is on average 3.11% in favor of the long chain. This difference is minimal for low or very high capacity levels, but higher for the capacity levels in between. The random configuration is substantially worse than the average chain typically for the capacity levels 100 to 130. We observe a link with the variability within the random configuration. At capacity levels of 100 to 130, we observe indeed the highest difference between the best and worst random configuration, whereas this difference becomes smaller when the capacity is lower.

3) The clustered approach is overall much worse than the other approaches (including the average random one). The performance difference between the clustered and average long configuration is 12.2% overall. Only at the lowest capacity levels, the difference between the clustered approach and the random approach or average chain is very small (less than 1%).

This analysis shows that choosing well where adding flexibility is as important as the amount of flexibility (i.e. the number of links). Furthermore, the results show the importance of studying the process flexibility for lot sizing problems especially for problems where the capacity is tight and is not enough to meet demand without backlog.

In Table 2 we show the averages of the gaps (columns Gap) and computational times in seconds (columns T(s)) of the results obtained. We see that the problem with only dedicated machines is much faster to solve than any other flexible configuration. Among the flexible configurations, the clustered configuration is the easiest to solve, the random flexibility and long chain have on average a similar time performance and the total flexibility presents the worst performance on average. With respect to the optimality gaps, we observe that it is zero or very small for the dedicated and

		Dedicated	Clustered			Random			Long chain			Total
items	Cap	UB	max	min	av.	max	min	av.	max	min	av.	UB
6	150	100	98.69	97.76	98.34	98.07	97.48	97.69	97.48	97.48	97.48	97.48
	140	100	77.50	67.83	71.26	69.69	62.73	65.12	62.83	62.59	62.67	62.57
	130	100	62.38	46.15	53.05	46.26	29.99	36.83	30.08	29.93	30.00	29.66
	120	100	69.47	58.41	62.00	51.73	31.40	42.90	31.87	31.45	31.57	30.76
	110	100	79.54	70.36	73.12	64.91	46.90	57.80	47.29	46.92	47.05	46.37
	100	100	77.73	72.51	74.29	64.47	53.74	60.56	54.75	53.74	54.33	52.41
	90	100	75.29	72.23	73.56	65.72	61.11	63.84	62.66	60.97	61.91	57.62
	80	100	76.08	72.33	73.93	70.23	68.74	69.43	70.06	67.52	68.73	62.70
	70	100	78.83	75.00	76.85	76.91	74.87	75.62	76.58	74.11	75.37	67.94
	60	100	83.33	80.41	81.94	83.14	81.18	81.89	82.36	80.79	81.79	72.82
	50	100	87.28	84.96	86.14	86.97	85.28	86.00	86.53	85.19	86.03	76.54
	40	100	93.45	91.79	92.35	92.63	92.05	92.25	92.82	91.91	92.37	82.94
12	150	100	87.85	80.59	83.56	84.19	79.47	80.64	79.59	79.46	79.48	79.45
	140	100	58.40	45.67	53.20	51.77	40.39	43.23	40.59	40.26	40.41	40.19
	130	100	55.18	32.01	42.37	40.59	12.86	21.83	13.15	12.60	12.85	12.51
	120	100	68.16	46.21	55.73	47.15	22.04	32.90	22.81	21.92	22.24	21.51
	110	100	74.12	61.51	68.67	60.08	40.88	50.11	44.01	38.41	41.57	37.36
	100	100	78.46	67.21	73.45	70.62	52.38	61.20	59.39	50.67	54.99	45.96
	90	100	79.46	68.95	75.07	77.10	61.28	68.63	68.72	61.04	64.62	54.42
	80	100	81.43	72.72	77.48	79.98	70.64	74.71	76.21	69.45	72.49	62.27
	70	100	83.88	76.74	80.40	82.27	77.62	79.58	80.80	76.65	78.49	68.40
	60	100	85.87	80.47	83.35	85.10	80.74	83.44	84.27	81.44	83.04	72.50
	50	100	88.43	84.00	86.63	88.12	84.69	86.75	87.59	85.22	86.61	75.66
	40	100	91.63	89.06	90.62	92.17	89.39	90.86	91.44	89.53	90.52	80.05
24	150	100	86.16	80.07	82.00	82.23	78.28	79.20	78.42	78.25	78.31	78.28
	140	100	60.18	39.21	51.66	37.52	27.83	30.86	28.08	27.75	27.91	27.86
	130	100	62.50	42.74	54.25	29.79	11.49	18.00	11.39	10.49	10.86	16.09
	120	100	69.59	55.02	63.44	41.42	29.15	32.44	28.46	26.60	27.33	26.96
	110	100	76.42	66.74	71.82	53.75	44.02	48.44	47.16	43.01	45.45	39.65
	100	100	79.07	69.56	73.48	61.91	53.74	58.19	59.26	52.24	55.97	45.41
	90	100	80.59	70.75	74.29	69.79	62.83	66.01	68.11	61.48	64.86	50.94
	80	100	81.82	72.74	76.53	75.19	70.37	72.90	74.44	69.94	72.27	59.33
	70	100	83.22	75.82	79.48	80.29	76.38	78.56	79.15	75.77	77.93	66.10
	60	100	85.65	80.11	83.27	85.58	81.97	83.41	84.30	80.99	82.81	72.19
	50	100	88.87	84.78	87.46	89.59	86.54	87.57	88.63	85.67	87.19	74.71
	40	100	92.36	90.43	91.42	92.88	90.69	91.58	91.75	90.62	91.21	77.42

Tabela 1: Comparison of upper bounds for different configurations

clustered configuration. On the other hand, for the total flexibility case, the gaps are very significant for some capacity levels, especially for the instances with 12 and 24 items. Note that the gaps resulting from optimizing a given long chain are significantly smaller for most instances than those resulting from optimizing the total flexibility. This explains why in Table 1 for some instances with 24 items, the long chain solution is better than the total flexibility solution.

		Dedicated		Clustered		Random		Long chain		Total	
itens	Cap	Gap	T(s)	Gap	T(s)	Gap	T(s)	Gap	T(s)	Gap	T(s)
6	150	0	0.03	0	3.73	0	75.16	0	59.21	0	20.80
	140	0	0.02	0	3.98	0.2	73.13	0	27.88	0.1	68.28
	130	0	0.01	0	6.18	0.9	291.63	0.8	282.35	2.5	381.16
	120	0	0.01	0	23.35	2.6	476.30	4.3	514.66	7.6	583.49
	110	0	0.01	0	13.85	0.2	215.89	0.2	130.58	1.8	515.08
	100	0	0.01	0	7.64	0.1	75.44	0	4.84	0.5	384.96
	90	0	0.01	0	3.05	0.1	18.23	0	1.60	0.2	230.25
	80	0	0.01	0	1.11	0	0.36	0	0.34	0.1	146.86
	70	0	0.01	0	0.14	0	0.03	0	0.04	0.1	105.16
	60	0	0.01	0	0.02	0	0.02	0	0.02	0	1.90
	50	0	0.01	0	0.05	0	0.04	0	0.04	0	4.82
	40	0	0.01	0	0.03	0	0.04	0	0.04	0	0.03
12	150	0	0.08	0	47.25	0.2	325.52	0.3	346.62	0.2	237.31
	140	0	0.06	0	27.28	0.3	409.58	0.3	476.11	0.4	473.34
	130	0	0.04	0.2	82.33	3.7	496.90	4.5	544.91	7.9	600
	120	0	0.03	0.3	139.63	4.6	579.47	8.8	600	14.8	600
	110	0	0.03	0.1	79.59	0.5	365.72	0.6	279.87	2.6	600
	100	0	0.03	0	44.89	0.1	146.33	0.1	66.79	0.9	600
	90	0	0.02	0	14.86	0	10.68	0	5.93	0.5	600
	80	0	0.02	0	3.00	0	1.10	0	0.65	0.2	509.58
	70	0	0.03	0	0.94	0	0.37	0	0.28	0.1	333.75
	60	0	0.03	0	0.10	0	0.07	0	0.07	0.1	157.01
	50	0	0.03	0	0.07	0	0.06	0	0.07	0	0.96
	40	0	0.03	0	0.06	0	0.06	0	0.06	0	0.37
24	150	0	0.19	0.1	139.74	0.3	573.86	0.3	550.73	0.4	600
	140	0	0.18	0.4	250.87	0.9	600	0.6	600	1.4	600
	130	0	0.14	0.3	329.87	8.1	600	9.7	600	27.8	600
	120	0	0.14	0.4	395.73	5.3	600	6.5	600	20.3	600
	110	0	0.11	0.2	340.07	0.7	542.75	0.8	545.05	4.3	600
	100	0	0.12	0.1	248.45	0.1	205.73	0.1	151.97	2.4	600
	90	0	0.12	0	39.33	0	9.28	0	7.05	0.8	600
	80	0	0.12	0	4.77	0	1.52	0	1.79	0.3	600
	70	0	0.13	0	1.14	0	0.47	0	0.47	0.2	487.52
	60	0	0.13	0	0.20	0	0.19	0	0.19	0	88.25
	50	0	0.12	0	0.19	0	0.18	0	0.20	0	12.78
	40	0	0.11	0	0.13	0	0.12	0	0.14	0	4.05

Tabela 2: Average gaps and CPU time for different configurations

Aiming to further analyse the effect of the chaining concept, Table 3 shows the structure of the solutions for all instances considering the flexibility configurations addressed. We present the total backlog calculated as $\sum_{i=1}^{n}\sum_{t=1}^{m} b_{it}$ (columns back.), the average percentage of capacity utilization (columns CU) in the solutions and the number of setups (columns setup).

Regarding the total number of backlog we see that the total flexibility and long chain have very similar levels for most instances. On the other hand, the clustered and the dedicated configurations have much higher levels of backlog for some instances. Note that a higher number of backlog does not necessarily mean a worse total production cost. For example, when the capacity is equal to 70 or 80, the long chain configuration presents on average less backlog than the total flexibility. However, the total production cost of the long chain is significantly higher than the total flexibility (see Table 1). This happens because of the difference in backlog cost between items. Table 3 also shows that the overall percentage of capacity utilization and number of setups is very similar for the long chain and total flexibility when the capacity level is higher than 50. It is also in line with [Jordan and Graves, 1995]. However, when the capacity level is equal to 50 or 40 we may have links in the chain configuration that cannot be used because of the length of the setup times. The reason is that in the data set used, the setup times can be up to 50 for some items. If this happens, then the item cannot be produced if the available capacity is 50 or less. Note that for the long chain there is some available capacity and the number of setups is less than 90, 180 and 360 for the instances with 6, 12 and 24 items (machines), respectively, which correspond to one setup on each machine per period. On the other hand, we see that there are considerable differences in the capacity utilization and the number of setups considering the results obtained with dedicated machines and clustered configurations. Note that the capacity is better utilized for the long chain and total flexibility configurations and the difference compared to the dedicated machine reaches 45.05% for some instances with 24 machines ($Cap = 40$). Moreover, we do see that the number of setups increases significantly for some instances in the long chain and total flexibility. It makes sense that the more we use the flexibility compared to the dedicated case, the more setups will be performed.

4. Analysis of Process Flexibility as a Decision Variable

The investment in flexibility is usually part of a wider investment program to be implemented over a certain period of time and include important decisions in order to increase the benefits obtained. In this section we analyse and compare the benefits of investing in flexibility with the costs to implement it.

To decide where to add flexibility, we compared various configurations, including the chain concept discussed in Section 3. Based on the computational results in Section 3 we observed that a small amount of flexibility, if added in the right way, can reach almost all the benefits obtained by total flexibility for many instances. However, we also observed that the chain principle does not perform very well for the instances with very tight capacity. The objective of this section is to see if we can do better than the long chain, given the same (or lower) number of links. We use formulation (7)-(14) to determine the optimal configuration for a given number of links. As such, we trace the trade-off between the benefits of flexibility and the level of flexibility.

4.1. Computational Results

The formulation $(7) - (14)$ was also modeled in C++ using the concert technology and CPLEX 12.6 as solver. We have considered the same data sets which were proposed in Section 3.1 (sets $F1 - F20$, $G51 - G55$ and $G56 - 60$) and the same capacity levels. Note that in constraints (12) we have considered $fc_{ij} = 1$ and $Fmax$ equal to the number of links we are allowed to add. It is also important to note that in these data sets, we are considering identical machines so to avoid symmetrical solutions we have fixed the dedicated configuration before solving all instances. In other words, we add the the constraints $z_{ii} = 1 \; \forall i$ in the formulation. Moreover, due to the

		Dedicated			Clustered			Random			Long chain			Total		
items	Cap	back.	CU	setup	back.	CU	setup	back.	CU	setup	back.	CU	setup	back.	CU	setup
6	150	4	80.1	79	2	80.2	80	1	80.2	80	0	80.2	80	0	80.2	80
	140	268	86.3	83	55	86.9	84	17	87.1	85	1	87.2	85	0	87.2	85
	130	1278	92.5	87	620	93.8	89	257	95.0	90	83	95.3	91	87	95.4	91
	120	3818	97.3	89	3022	98.4	91	2653	99.5	92	2465	99.7	93	2473	99.8	94
	110	8217	99.5	90	8093	99.7	91	8324	99.8	92	8544	99.9	92	8557	99.9	92
	100	14371	99.9	90	14608	99.8	91	14891	99.8	91	14825	99.9	91	14953	99.9	91
	90	21166	100	90	21279	99.9	90	21168	99.9	91	21142	99.9	90	21100	99.9	91
	80	28182	100	90	27749	99.8	90	27539	99.9	90	27296	99.9	90	27500	99.9	90
	70	35292	100	90	34408	99.9	90	34100	99.9	90	34001	99.9	90	34133	99.9	90
	60	42434	100	90	40295	99.9	90	40065	99.9	90	40720	99.9	90	39531	99.9	90
	50	48344	90.7	82	45754	98.3	88	45522	98.1	88	46144	98.4	89	44107	99.9	90
	40	56113	81.4	73	53674	91.7	82	53431	94.5	85	54004	94.2	85	51748	99.9	90
12	150	122	79.7	157	21	80.0	158	6	80.0	158	0	80.1	159	00	80.1	159
	140	881	85.9	166	263	86.7	168	38	87.2	170	6	87.3	170	10	87.2	170
	130	3490	91.3	172	2073	93.9	178	617	94.8	181	79	95.5	183	9964	95.8	186
	120	9067	95.5	177	7445	98.3	181	5946	99.0	185	5172	99.7	187	5210	99.8	192
	110	17394	98.5	179	17194	99.5	182	17003	99.8	184	17308	99.8	184	17422	99.9	187
	100	28701	99.7	180	29991	99.8	182	29877	99.9	182	31297	99.9	182	32279	99.9	186
	90	41946	99.9	180	43644	99.9	183	42680	99.9	182	45135	99.9	181	46047	99.9	184
	80	55874	100	180	56598	99.8	181	54815	99.9	181	57255	99.9	181	57846	99.9	183
	70	70070	100	180	69037	99.8	181	67332	99.9	180	69173	99.9	180	67730	99.9	182
	60	84369	99.9	180	79973	99.9	180	78627	99.9	180	80017	99.9	180	78424	99.9	181
	50	98712	85.0	153	90803	100	180	90244	98.2	177	89672	96.7	174	87564	99.8	180
	40	110923	68.3	123	102379	93.3	168	102749	89.8	162	102110	91.5	165	97044	99.8	180
24	150	257	80.0	316	36	80.2	318	8	80.2	318	0	80.2	318	0	80.2	318
	140	2221	86.2	332	946	87.0	336	97	87.4	340	5	87.5	340	1	87.5	342
	130	8979	91.6	343	5885	93.5	352	1510	95.5	364	395	96.1	367	1236	96.0	368
	120	21414	95.5	350	17834	98.1	363	14335	99.5	371	13420	99.7	371	13874	99.4	373
	110	39508	98.3	357	38440	99.7	365	38868	99.8	367	38321	99.8	368	40799	99.8	373
	100	63107	99.6	359	64366	99.8	363	64934	99.9	364	63794	99.9	364	66372	99.7	369
	90	89610	99.9	360	91640	99.7	363	90597	99.9	362	89853	99.9	362	93305	99.9	366
	80	117670	100	360	117558	99.7	362	115373	99.9	361	115688	99.9	361	118503	99.9	365
	70	146175	100	360	141827	99.9	362	139553	99.9	360	139135	99.9	360	140871	99.9	364
	60	174795	100	360	164319	99.8	360	162232	99.9	360	160703	99.9	360	162217	99.9	362
	50	203482	84.1	303	187775	100	360	187069	97.1	350	185398	97.2	350	175477	99.8	360
	40	227641	54.9	198	210429	86.7	312	211018	79.7	287	208870	80.7	291	195728	99.8	360

Tabela 3: General structure of the solutions

complexity of the model, we consider again the computational time limit equal to 36000 seconds (10 hours). We observe that even with this increase in the processing time, the solver does not find the optimal solution for many of the instances analysed, which shows the difficulty of solving this problem.

In Table 4 we present a detailed comparison between the results obtained by the average long chain and the model considering the flexibility as a decision variable. An overall analysis of the results shows that only a very small amount of flexibility is necessary to get almost the same benefits as the total flexibility for all instances. The results obtained by the flexibility as a decision variable are very similar to the results obtained by the total flexibility for all instances. Therefore, although the long chain configuration does not obtain almost all benefits, in terms of production cost for instances with very tight capacity, the number of links considered is enough to find these benefits.

In Table 4, for the model considering the flexibility as a decision variable we present the results with half (Flex.($l = m/2$)) and the same additional number of links (Flex.($l = m$)) required to build a long chain. Table 4 confirms that the long chain is not a good flexibility configuration to use for the instances with very low capacity. For the capacity levels between 90 and 40 the model considering the flexibility as a decision variable finds substantially better results than the best long chain by adding only half of the links. When allowing the same number of links as in the long chain ($Flex.(l = m)$), the difference with the best long chain becomes even bigger at the low capacity levels, and reaches 12.8% for the instance with 24 items and a capacity level equal to 40. This significant difference is due the fixed structure considered by the long chain configuration (2-flexibility). The structure of the solutions of the model considering the flexibility as a decision variable shows that, specially for the instances with capacity levels equal to 100 or lower, some items with high backlog cost and low setup time should be linked to more than 2 machines and it reaches 7 links for some instances. On the other hand, some items with low backlog cost and high setup times are only linked to the machine fixed to build the dedicated case and, given the limited budget, it seems not beneficial to add a second link.

		Long chain	B. chain	Flex.($l = m/2$)	Flex.($l = m$)	Total
items	Cap	av.	UB	UB	UB	UB
	150	97.48	97.48	97.48	97.48	97.48
	140	62.67	62.59	62.62	62.57	62.57
	130	30.00	29.93	31.26	29.66	29.66
	120	31.57	31.45	36.55	30.70	30.76
	110	47.05	46.92	51.44	46.35	46.37
6	100	54.33	53.74	56.14	52.42	52.41
	90	61.91	60.97	60.55	57.64	57.62
	80	68.73	67.52	64.48	62.75	62.70
	70	75.37	74.11	69.80	67.97	67.94
	60	81.79	80.79	74.86	72.85	72.82
	50	86.03	85.19	78.98	76.62	76.54
	40	92.37	91.91	86.07	82.95	82.94
	150	79.48	79.46	79.46	79.45	79.45
	140	40.41	40.26	40.32	40.21	40.19
	130	12.85	12.60	14.08	12.67	12.51
	120	22.24	21.92	24.53	21.70	21.51
	110	41.57	38.41	40.49	37.72	37.36
12	100	54.99	50.67	49.17	46.16	45.96
	90	64.62	61.04	58.06	54.54	54.42
	80	72.49	69.45	65.49	62.31	62.27
	70	78.49	76.65	71.30	68.48	68.40
	60	83.04	81.44	75.00	72.56	72.50
	50	86.61	85.22	78.93	75.71	75.66
	40	90.52	89.83	82.48	80.13	80.05
	150	78.31	78.25	78.36	78.53	78.28
	140	27.91	27.75	29.34	28.07	27.86
	130	10.86	10.49	22.21	13.74	16.09
	120	27.33	26.60	35.60	29.74	26.96
	110	45.45	43.01	47.03	41.35	39.65
24	100	55.97	52.24	51.11	46.91	45.41
	90	64.86	61.48	55.51	51.67	50.94
	80	72.27	69.94	62.08	59.82	59.33
	70	77.93	75.77	68.15	66.30	66.10
	60	82.81	80.99	74.32	72.24	72.19
	50	87.19	85.67	77.54	74.78	74.71
	40	91.21	90.62	80.40	77.85	77.42

Tabela 4: Comparison between long chain and flexibility as decision variable

5. Conclusion

In this work, the chain principle and process flexibility were studied in the context of deterministic lot sizing problem. In this deterministic environment the value of flexibility appears in case not all the demand can be satisfied on time and the company has to resort to backlogging. In order to better analyse the benefits of the long chain and process flexibility we propose a new optimization models where we consider the possibility of investing in flexibility. Our computational experiments show that, in terms of total production costs, the long chain configuration obtains almost all benefits of the total flexibility for the instances where the capacity level is not tight. However, if the capacity level is tight, the long chain configuration does not present good results. Although the long chain does not perform very well for some instances, analysing the model proposed with the possibility of investing in flexibility, we see that only a small amount of flexibility is typically enough to find almost all benefits of the total flexibility. Moreover, for some instances with tight capacity levels, adding only half of the links required to build a long chain is enough to find better results than the solution found by the best long chain. Finally, we also see that the benefits of process flexibility show decreasing marginal utilities when we add additional links.

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