MONITORING BIVARIATE PROCESSES WITH A SYNTHETIC CONTROL CHART BASED ON SAMPLE RANGES

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ABSTRACT

The RMAX chart was proposed to control the covariance matrix of two quality characteristics. The monitoring statistic of the RMAX chart is the maximum of two standardized sample ranges from bivariate observations of two quality characteristics. In this article, we investigate the performance of two synthetic RMAX charts. The first synthetic chart signals when a second point, not far from the first one, falls beyond the warning limit. The second synthetic chart additionally signals when a sample point falls beyond the control limit. The performance of the synthetic RMAX charts are compared with the performance of the standard RMAX chart and the generalized variance $|\mathbf{S}|$ chart. If the aim is to detect moderate or even small changes in the covariance matrix, the synthetic run rules enhances the performance of the standard RMAX chart.

KEYWORDS. RMAX chart. Bivariate processes. Synthetic run rules.

Paper Topics (Statistics, Probabilistic Models)

1. Introduction

Alt (1985) was the first researcher to propose a chart to control the covariance matrix of bivariate processes, the generalized variance $|\mathbf{S}|$ chart. The $|\mathbf{S}|$ chart is not simple to deal with once the monitoring statistic of the chart depends on the determinant of the sample covariance matrix. Moreover, the chart is slow in signaling changes in the covariance matrix. Costa and Machado (2008) proposed a simpler and more efficient statistic to control the covariance matrix of bivariate processes. Their VMAX statistic is the maximum of two variances from the sample observations of two quality characteristics. Costa and Machado (2009), Machado and Costa (2008) and Machado et al. (2008, 2009) also worked with the VMAX statistic. Alternatively, Costa and Machado (2011) proposed the use of the RMAX statistic to control the covariance matrix of bivariate processes. The RMAX statistic is the maximum of two standardized ranges from the sample observations of two quality characteristics.

Wu and Spedding (2000) introduced the synthetic chart that is an integration of the Shewhart $\overline{X}$ chart and the Conforming Run Length (*CRL*) chart. The *CRL* is the number of conforming samples between two consecutive nonconforming samples. According to the synthetic run rule, the control chart signals when a second point, not far from the first one, falls beyond the warning limits. Davis and Woodall (2002) obtained the steady-state properties of the $\overline{X}$ chart with the synthetic run rule. The results of their studies motivated other researchers to consider the synthetic run rule as an alternative to enhance the performance of the control charts.

Khoo et al. (2010) proposed a synthetic double sampling chart, which combines the double sampling $\overline{X}$ chart and the *CRL* chart for monitoring the process mean. Wu et al. (2010) proposed a scheme comprising a synthetic chart and an $\overline{X}$ chart, denoted as the Syn-$\overline{X}$ chart, for monitoring the process mean. In this scheme, a nonconforming sample is the one with an $\overline{X}$ value larger than the upper warning limit $UWL = \mu_0 + w\sigma_{\overline{X}}$ or smaller than the lower warning limit $LWL = \mu_0 - w\sigma_{\overline{X}}$. The Syn-$\overline{X}$ chart signals when a sample point falls beyond the control limits or when $CRL<L$, where L is a specified positive integer.

More recently, Zhang et al. (2011) evaluated the performances of the synthetic chart when the process parameters are estimated. The synthetic $\overline{X}$ chart is the name they used for the synthetic chart proposed by Wu and Spedding (2000). They demonstrated that when the number of samples during Phase I is small, the performances of the synthetic chart with known parameters and with estimated parameters are quite different.

Haridy et al. (2012) proposed a combined scheme comprising a synthetic chart and an *np* chart, which has always a better overall performance than the individual synthetic chart and individual *np* chart. An optimal design of procedure for a synthetic chart able to monitor the mean based on the Median Run Length (MRL) was suggested by Khoo et al. (2012). Costa and Machado (2015) considered the Markov chain approach to obtain the properties of the synthetic and side-sensitive synthetic double sampling $\overline{X}$ chart. Lee and Khoo (2017) proposed a combined synthetic and $|\mathbf{S}|$ chart for monitoring the covariance matrix of multivariate processes. Lee and Khoo (2017a) studied the performance of the synthetic $|\mathbf{S}|$ chart based on Median Run Length.

In this article, we investigate the performance of two synthetic RMAX charts. The first synthetic chart signals when a second point, not far from the first one, falls beyond the warning limit. The second synthetic chart additionally signals when a sample point falls beyond the control limit.
The paper is organized as follows. In section 2, we describe the RMAX and the synthetic RMAX charts. In Section 3, we investigate the performance of the proposed charts and compare it with the

performance of the RMAX chart proposed by Costa and Machado (2011) and the generalized variance $|\mathbf{S}|$ chart. Conclusions are in Section 4.

2. The RMAX chart

In this section, we introduce the RMAX chart proposed by Costa and Machado (2011) to detect changes in the covariance matrix $\mathbf{\Sigma}$ of bivariate processes. The sample points plotted on the RMAX chart are the larger value of two standardized sample ranges, $W_i = R_i / \sigma_i,\ i = 1, 2$, where R_1 and R_2 are, respectively, the standardized sample ranges from the observations of the first and the second quality characteristics. The process is considered to start with the covariance matrix on target $(\mathbf{\Sigma} = \mathbf{\Sigma}_0)$, where

$$\mathbf{\Sigma}_0 = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}. \tag{1}$$

The occurrence of the assignable cause changes the covariance matrix from $\mathbf{\Sigma}_0$ to $\mathbf{\Sigma}_1$,

$$\mathbf{\Sigma}_1 = \begin{bmatrix} a_1 \cdot a_1 \cdot \sigma_1^2 & a_1 \cdot a_2 \cdot \sigma_{12} \\ a_1 \cdot a_2 \cdot \sigma_{12} & a_2 \cdot a_2 \cdot \sigma_2^2 \end{bmatrix}. \tag{2}$$

After the occurrence of the assignable cause it is assumed that at least one a_i becomes larger than one, $i = 1, 2$. With the standard RMAX chart in use, an out-of-control signal is triggered by a sample point falling beyond the control limit. Costa and Machado (2011) obtained the properties of the RMAX chart, that is, a closed theoretical expression to obtain the false alarm risk α and the power of detection $P=1-\beta$, being α and β, respectively, the well-known Type I and Type II errors. They observed that the coefficient of correlation, $\rho = \sigma_{12} / (\sigma_1 \sigma_2)$, has minor influence on the properties of the RMAX chart. Based on that, we fixed ρ=0.5.

2.1. The synthetic RMAX chart

The standard RMAX chart combined with the synthetic rule, shortly the synthetic RMAX chart, signals when a sample point falls beyond the control limit or when a second point, not far from the first one, falls beyond the warning limit. In order to measure the distance between the two points beyond the warning limits, we attributively classify the samples as conforming and nonconforming - being conforming when their sample points fall below the warning limit and nonconforming when their sample points fall beyond the warning limit. The distance is measured by the *CRL*, the number of conforming samples between two consecutive nonconforming samples plus the ending nonconforming one; in other words, the first of the two consecutive nonconforming samples is the reference to compute the *CRL*. A *CRL* lower than or equal to a specified positive integer *L* ($CRL \leq L$) triggers a signal.

When the control limit goes to infinite, the synthetic RMAX chart only signals when a second point, not far from the first one, falls beyond the warning limit. In order to distinguish the two synthetic charts, the one with a control limit will be the synthetic RMAX chart and the other one without a control limit will be the pure synthetic RMAX chart.

Figure 1 shows the pure synthetic RMAX chart. The sample is classified as nonconforming when the value of the monitoring statistic RMAX falls beyond the control limit *W*. Samples 9 and 13 are nonconforming (Figure 1). In this case, *CRL* = 4 (13th sample – 9th sample = 4). As *CRL* < *L* (=5), the pure synthetic RMAX chart signals an out-of-control condition.

Figure 2 shows the synthetic RMAX chart. The sample is also classified as nonconforming when the value of the monitoring statistic RMAX falls beyond the warning limit *W*. Samples 3 and 9 are nonconforming (Figure 2). In this case, *CRL* = 6 (9th sample – 3th sample = 6). As *CRL* > *L* (=5), the synthetic RMAX chart does not signal at sample 9. However, the synthetic RMAX chart signals at sample 13, once the value of the monitoring statistic RMAX is beyond the control limit *C*.

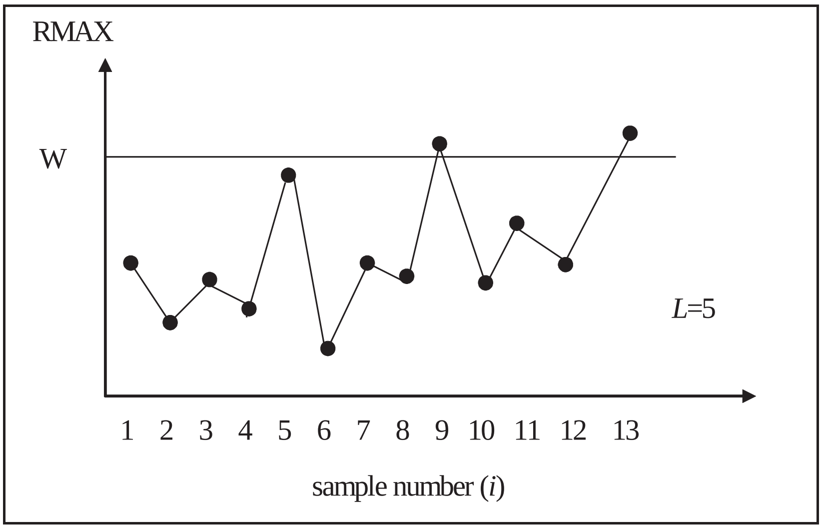

Figure 1: The pure synthetic RMAX chart

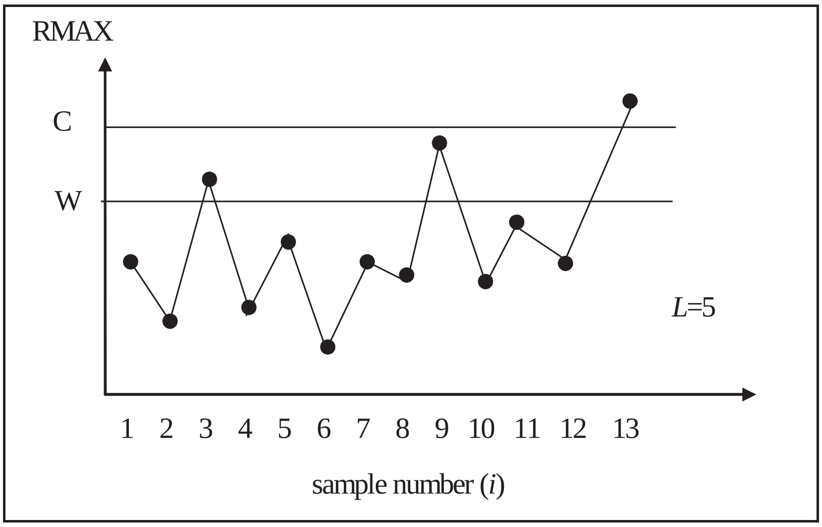

Figure 2: The synthetic RMAX chart

3. The performance of the synthetic RMAX charts

In this section, we compare the speed with which the generalized variance $|\mathbf{S}|$ chart, the standard RMAX chart (*Std* chart), the pure synthetic RMAX chart (*PSyn* chart) and the synthetic RMAX (*Syn* chart) chart signal changes in the covariance matrix. According to Davis and Woodall (2002), the proper parameter to measure the performance of a synthetic chart is the steady-state average run length (*SSARL*), that is, the *ARL* value obtained when the process remains in-control for a long time before the occurrence of the assignable cause. When the process is in-control, the *SSARL* measures the rate of false alarms. A chart with a larger in-control *SSARL* ($SSARL_0$) has a lower false alarm rate than other charts. A chart with a smaller out-of-control *SSARL* has a better ability to detect process changes than other charts. The in-control *SSARL* is an input parameter, and the warning limit *W* is the adjusting parameter to obtain to the desired in-control SSARL. Following the work of Machado and Costa (2014), we also considered the Markov chain approach to obtain the *SSARL*s of the synthetic RMAX charts. The transition matrix of the Markov chain is used to obtain the steady-state ARLs of the synthetic *RMAX c*hart.

	00..00	0..001	0..010	0..100	...	010..0	100..0	*Signa*
00...00	*A*	*B*	0	0	...	0	0	0
0..001	0	0	*A*	0	...	0	0	*B*
0..010	0	0	0	*A*	...	0	0	*B*
...	..	...	...	...	...	...	...	...
001..0	0	0	0	0	...	*A*	0	*B*
010..0	0	0	0	0	...	0	*A*	*B*
100..0	*A*	0	0	0	...	0	0	*B*
Signal	0	0	0	0	...	0	0	1

The transient states describe the position of the last *L* sample points; "1" means the sample point fell beyond the control limits, and "0" means the sample point fell in the central region. For instance, the transient state (010..0) is reached when the second of the last *L* points falls in the action region and all others points fall in the central region. The events "0" and "1" occur with probabilities A and B=1-A, respectively.

The steady-state ARL is given by $\mathbf{S}^{'}\mathbf{ARL}$, where **S** is the vector with the stationary probabilities of being in each nonabsorbing state and **ARL** is the vector of ARLs taking each nonabsorbing state as the initial state. $\mathbf{ARL} = (\mathbf{I} - \mathbf{R})^{-1}\mathbf{1}$, where **I** is an (*L*+1) by (*L*+1) identity matrix, **R** is the transition matrix given in (2) with the last row and column removed, and **1** is an (*L*+1) by one vector of ones. The vector $\mathbf{S}^{'} = (1/C, B/C, \ldots, B/C)$, with *C*=1+*LB*, was obtained by solving the system of linear equations $\mathbf{S}^{'}\mathbf{R}_{\mathbf{adj}} = \mathbf{S}$, constrained to $\mathbf{S}^{'}\mathbf{1} = 1$. The matrix $\mathbf{R}_{\mathbf{adj}}$ is an adjusted version of **R**, where A and B in the first row are retained and the remaining As are switched by 1 s. The matrix $\mathbf{R}_{\mathbf{adj}}$ is as follows:

$$\begin{pmatrix} A & B & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & 0 & 0 & \ldots & 0 & 0 \end{pmatrix}$$

Tables 1 and 2 give the *SSARL* of the generalized variance $|\mathbf{S}|$ chart, the standard RMAX chart (*Std* chart), the pure synthetic RMAX chart (*PSyn* chart) and the synthetic RMAX (*Syn* chart). In these Tables, ρ=0.5 and $SSARL_0 \cong 370.4$. Considering the case *n*=5, the pure synthetic (the generalized variance $|\mathbf{S}|$) is faster in signaling small (large) disturbances; however, in terms of overall performance, the synthetic chart is a better option. Considering the case *n*=3, the synthetic chart is always the best option, except for large disturbances. In these cases, the generalized variance $|\mathbf{S}|$ chart performs better.

Tables 3 and 4 give the *SSARL* of the generalized variance $|\mathbf{S}|$ chart, the standard RMAX chart (*Std* chart), the pure synthetic RMAX chart (*PSyn* chart) and the synthetic RMAX (*Syn* chart). In these Tables, ρ=0.5 and $SSARL_0 \cong 700.0$. The synthetic chart is always the best option, except for *n*=5 and large disturbances. In these cases, the generalized variance $|\mathbf{S}|$ chart performs better.

The choice of *L* depends on the magnitude of the disturbance the practitioner is interested to detect. With small samples (n=3) and taking into account all possible disturbances, the synthetic charts reach their best performance with the input parameter *L* = 5 or 7.

4. Illustrative example

In this section, we provide an example to illustrate the ability of the pure synthetic RMAX chart (*PSyn* chart) and the synthetic RMAX (*Syn* chart) chart in detecting shifts in the covariance matrix. To this end, we considered a bivariate process whose quality characteristics of interest. X_1 and X_2, are normally distributed. When the process is in-control, the mean vector and the covariance matrix are given by $\boldsymbol{\mu}_0^{'} = (0.0)$ and $\boldsymbol{\Sigma}_0 = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$, respectively.

We initially generate 5 samples of size $n = 5$ with the process in control. The remaining samples were simulated considering that the assignable cause changed the variability of X_1, that is, $a_1 = 1.50$. Table 5 presents the data of X_1 and X_2, the sample ranges (R_1 and R_2) and the statistic RMAX.

Table 1: The *SSARLs* of the |**S**|, *Std*, *Syn* and *PSyn* charts, *n*=5

	Chart	|**S**|	*Std*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*
	L			2	2	3	3	5	5	7	7
	C			5.50		5.50		5.50		5.50	
	W	4.72	5.37	4.40	4.14	4.54	4.30	4.66	4.44	4.74	4.52
a_1	a_2										
1.25	1.00	74.13	47.12	41.74	49.24	40.29	43.23	39.23	39.20	38.87	38.12
1.50	1.00	26.70	11.85	10.35	13.12	10.00	11.35	9.83	10.43	9.85	10.52
1.75	1.00	13.32	5.22	4.70	6.06	4.60	5.42	4.60	5.18	4.64	5.48
2.00	1.00	8.11	3.15	2.93	3.78	2.90	3.50	2.92	3.44	2.95	3.82
2.50	1.00	4.23	1.82	1.78	2.24	1.78	2.17	1.79	2.17	1.80	2.68
1.25	1.25	21.90	26.15	21.18	22.54	20.37	19.93	19.94	18.52	19.90	18.34
1.50	1.50	5.61	6.64	5.49	6.36	5.35	5.75	5.37	5.57	5.44	5.77
1.75	1.75	2.74	3.10	2.76	3.38	2.74	3.18	2.78	3.18	2.82	3.45
2.00	2.00	1.85	2.00	1.89	2.38	1.89	2.31	1.91	2.32	1.92	2.65
2.50	2.50	1.28	1.31	1.30	1.63	1.31	1.62	1.31	1.62	1.31	2.11

Table 2: The *SSARLs* of the |**S**|, *Std*. *Syn* and *PSyn* charts. *n*=3

	Chart	|**S**|	*Std*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*
	L			2	2	3	3	5	5	7	7
	C			5.50		5.50		5.50		5.50	
	W	6.56	4.94	3.68	3.62	3.84	3.78	4.00	3.94	4.08	4.03
a_1	a_2										
1.25	1.00	113.48	64.71	63.15	75.94	58.34	66.60	54.70	59.88	53.18	57.08
1.50	1.00	51.57	19.00	18.53	25.31	16.99	21.03	15.98	18.47	15.68	17.62
1.75	1.00	29.36	8.81	8.67	12.65	8.09	10.46	7.78	9.33	7.73	9.04
2.00	1.00	19.25	5.32	5.31	8.05	5.05	6.74	4.94	6.15	4.96	6.05
2.50	1.00	10.65	2.92	2.99	4.75	2.92	4.12	2.91	3.90	2.93	3.89
1.25	1.25	44.05	36.72	31.53	36.39	28.91	31.70	27.25	28.81	26.74	27.84
1.50	1.50	13.86	10.57	9.19	11.49	8.53	9.89	8.25	9.14	8.26	9.02
1.75	1.75	6.90	5.05	4.63	6.11	4.41	5.36	4.38	5.11	4.43	5.12
2.00	2.00	4.39	3.17	3.05	4.21	2.97	3.78	2.98	3.68	3.02	3.72
2.50	2.50	2.58	1.88	1.92	2.86	1.91	2.66	1.92	2.65	1.93	2.67

Table 3: The *SSARLs* of the |**S**|, *Std. Syn* and *PSyn* charts. n=5

	Chart	|**S**|	*Std*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*
	L			2	2	3	3	5	5	7	7
	C			6.00		6.00		6.00		6.00	
	W	5.47	5.60	4.37	4.29	4.52	4.45	4.66	4.59	4.74	4.67
a_1	a_2										
1.25	1.00	120.46	71.33	62.76	75.77	57.62	65.16	53.79	57.81	52.25	54.88
1.50	1.00	39.28	15.61	13.52	18.04	12.44	14.98	11.83	13.33	11.70	12.88
1.75	1.00	18.28	6.33	5.71	8.01	5.40	6.79	5.29	6.28	5.33	6.22
2.00	1.00	10.56	3.62	3.41	4.99	3.31	4.35	3.30	4.16	3.34	4.18
2.50	1.00	5.13	1.97	1.97	3.08	1.96	2.82	1.97	2.79	1.98	2.81
1.25	1.25	31.60	38.83	28.99	32.54	26.53	28.17	25.05	25.62	24.67	24.86
1.50	1.50	7.02	8.59	6.70	8.14	6.30	7.13	6.19	6.76	6.27	6.78
1.75	1.75	3.17	3.69	3.21	4.18	3.12	3.79	3.15	3.74	3.21	3.80
2.00	2.00	2.03	2.24	2.14	2.97	2.12	2.78	2.14	2.79	2.17	2.83
2.50	2.50	1.33	1.38	1.42	2.23	1.42	2.17	1.43	2.17	1.43	2.18

Table 4: The *SSARLs* of the |**S**|, *Std. Syn* and *PSyn* charts. n=3

	Chart	|**S**|	*Std*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*	*Syn*	*PSyn*
	L			2	2	3	3	5	5	7	7
	C			5.50		5.50		5.50		5.50	
	W	8.05	5.18	3.91	3.78	4.06	3.95	4.21	4.10	4.30	4.19
a_1	a_2										
1.25	1.00	188.79	99.36	92.17	117.71	86.27	101.87	81.45	90.12	79.29	85.03
1.50	1.00	78.82	25.53	23.23	34.05	21.56	27.74	20.38	23.86	19.98	22.50
1.75	1.00	42.24	10.94	10.05	15.62	9.46	12.67	9.12	11.09	9.06	10.66
2.00	1.00	26.45	6.28	5.88	9.43	5.63	7.76	5.52	6.97	5.53	6.82
2.50	1.00	13.74	3.25	3.16	5.24	3.09	4.48	3.09	4.21	3.11	4.20
1.25	1.25	66.19	55.16	45.10	53.22	41.76	45.69	39.41	40.80	38.57	39.01
1.50	1.50	18.38	13.97	11.37	14.64	10.62	12.37	10.24	11.23	10.22	10.98
1.75	1.75	8.49	6.18	5.29	7.20	5.06	6.21	5.01	5.84	5.06	5.83
2.00	2.00	5.14	3.68	3.33	4.73	3.24	4.19	3.26	4.05	3.30	4.09
2.50	2.50	2.85	2.06	2.00	3.05	1.99	2.82	2.00	2.80	2.02	2.83

Figure 3 shows the pure synthetic RMAX chart with design parameters L=5 and W=4.52. Samples 9 and 12 are nonconforming (RMAX>W). In this case. CRL = 3 (13th sample – 9th sample = 4). As $CRL < L$ (=5), the pure synthetic RMAX chart signals an out-of-control condition at sample 12.

Figure 4 shows the synthetic RMAX chart. Samples 12 and 13 are nonconforming (RMAX>W). In this case, CRL = 1. As $CRL < L$ (=5), the synthetic RMAX chart signals at sample 13. Even though CRL > 5, the synthetic RMAX chart would signal at sample 13, once the value of the monitoring statistic RMAX is beyond the control limit C.

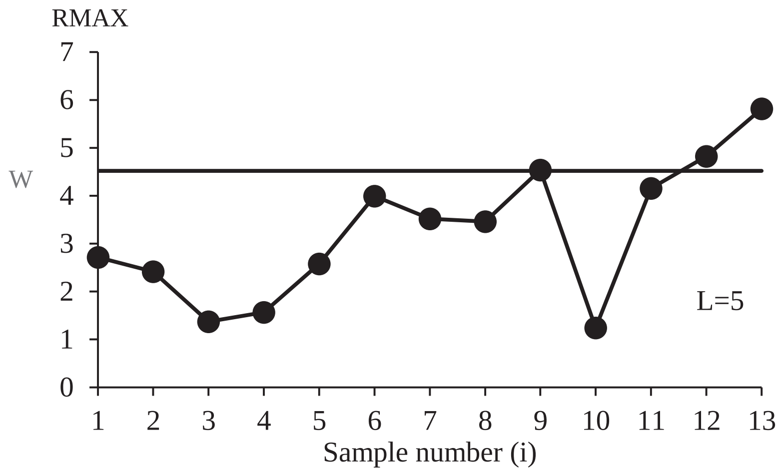

Figure 3: The pure synthetic RMAX chart - example

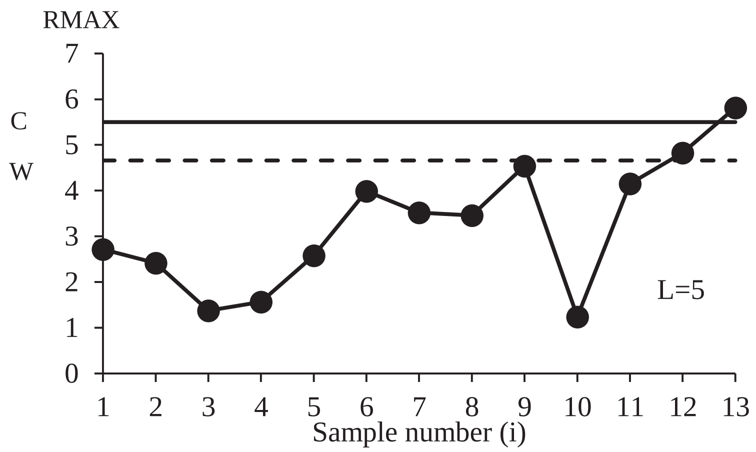

Figure 4: The synthetic RMAX chart - example

Table 5: Values of X_1, X_2. R_1, R_2 and RMAX.

Sample		Observations					RMAX
		1	2	3	4	5	
1	X_1	0.53	-1.83	0.20	0.89	-0.80	2.71
	X_2	-0.27	-1.71	-0.10	-1.30	-1.55	
2		-1.63	-0.86	-0.25	0.78	-1.30	2.42
		-0.54	-0.51	-0.93	0.14	-0.93	
3		-0.27	0.12	-0.10	-0.73	-1.05	1.37
		-0.66	0.71	-0.18	0.22	-0.43	
4		-0.27	-0.68	-0.59	-1.60	-0.23	1.57
		-1.01	-0.31	-0.17	-1.38	0.18	
5		-0.07	0.77	2.02	-0.56	0.15	2.58
		0.48	-0.27	0.07	-1.66	-0.39	
6		2.00	-1.44	0.45	-2.00	-0.04	3.99
		0.70	-1.25	-0.49	0.31	0.26	
7		-0.31	1.96	2.66	-0.87	0.21	3.52
		0.34	0.07	0.96	-0.54	-0.57	
8		-1.22	-1.02	-1.18	0.02	2.00	3.46
		-0.78	0.10	-1.90	1.56	-0.20	
9		-0.96	-0.40	0.71	3.59	0.97	4.54
		-0.17	-2.00	-0.90	0.63	-0.09	
10		0.46	1.52	0.28	0.31	0.51	1.24
		-0.76	0.20	-0.24	0.38	0.33	
11		-0.73	-3.28	0.87	-0.62	-0.97	4.15
		-2.36	-1.56	0.38	-0.98	-0.93	
12		0.84	0.48	2.81	-2.02	-0.23	4.83
		1.07	-0.09	0.04	-0.04	-0.53	
13		-1.40	2.30	4.41	-0.58	3.10	5.82
		-1.90	0.79	0.63	-1.20	-0.15	

5. Conclusions

The general conclusion related to the combined use of the standard RMAX chart with the synthetic run rules is the following: if the aim of the monitoring is to detect large changes in the covariance matrix, the standard RMAX chart doesn't need additional signal rules, such as the synthetic run rules. However, if the aim is to detect moderate or even small changes in the covariance matrix, the synthetic run rules really enhances the performance of the standard RMAX chart.

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