



## Angular cuts applied to the long-term transmission expansion planning problem

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### ABSTRACT

This paper presents a methodology to solve the long-term transmission expansion planning problem incorporating *angular cuts* based on the constraints imposed by circuits with low capacity-reactance product over other circuits with high capacity-reactance product in parallel with them. These cuts aim to reduce the search space of the problem and the total computing time of this NP-hard problem. To construct these *angular cuts* the method uses information of the power flows obtained with the system when it is solved using the transport model, the hybrid model and the *low effort* transmission model. The problem considers high voltage alternating current (HVAC) investment options in lines and substations. A traditional Southern Brazilian test system is used to show the efficiency of the proposed approach. The tests are performed solving the transmission planning problem using the reduced disjunctive DC model for cases with and without *angular cuts*.

**KEYWORDS.** Transmission network expansion planning, Angular cuts, Optimization, Disjunctive model, Low effort transmission model.

### 1. Introduction

The electricity market is increasingly becoming competitive and demanding both technically and economically. In order to meet the ever-increasing demand from consumers, electric power companies need new planning tools in to economically supply their consumers and technically maintain the power system operating with stability and reliability. The transmission expansion planning (TEP) problem in an electric power system defines *where, how many, and when* new transmission components (lines and substations) must be added to the system so that they meet the predicted power demand and to assure its operation is viable for a pre-defined planning horizon. The high associated costs and high complexity of the electric system, which is constantly growing and has the presence of different decision agents with conflicting objectives (supply and demand), encourage the research of methodologies that contribute to the improvement and use of the system.



The objective in the transmission network expansion planning (TNEP) problem is to determine the least-cost investment in the options to be added to the transmission network in order to guarantee a proper system operation in the future. The static approach performs a single investment at the beginning of the planning horizon and determines where and how many devices should be installed; with such an approach, only one planning horizon is analyzed.

The mathematical model of the TEP problem is a mathematical optimization problem that belongs to the category of mixed integer nonlinear programming (MINLP) problems, solving an investment and operation problem in an integrated way, which is a very complex and computationally demanding problem [Escobar 2004]. This problem presents many locally optimal solutions, and when the system size increases, the number of local solutions grows exponentially. Therefore, researchers usually employ a variety of approaches to obtain high quality solutions for this problem. The most researched planning is called static planning in which the open access is not considered. Static planning determines the minimum cost solution by employing a mathematical optimization problem approach that considers the current network as part of the future solution. The integer feature of the investment variables produces a high number of solution alternatives that make it an *NP-hard* problem for large and complex electrical systems. In the traditional transmission expansion planning [Garver 1970], the operation problem is solved using the DC model [Escobar 2004] [Escobar 2012] which is a relaxed version of the AC model. The DC model is simpler, easy to implement and considered the ideal model to represent the network in long term planning [Escobar 2010]. In this paper, an equivalent of the DC model: a *reduced linear disjunctive model* is used [Rahmani 2013]. This model is an Integer Mixed Linear Problem (IMLP) with continuous variables: power flow and angles, and integer variables: investment in lines and substations.

The high-performance computers design in recent decades and advances in optimization techniques has produced changes in the power system planning from experimental designing to intelligent and cost effective design [Gómez 2017] [Domínguez 2012]. When analyzing bigger Transmission systems is highly important to reduce the solution space, in this paper is proposed the use of *angular cuts* in exact methods based in branch and cut. Using this procedure, the idea is to find a linear inequality that induces constraints close to the facets of the convex hull of the IMLP problem.

## 2. Mathematical model

### 2.1 Nomenclature

Sets:

$\Omega$ , Subset of new or existing corridors for additions.

$B$ , Set of system nodes.

Parameters:

$C_{ij}$ , Cost associated with the construction of a circuit in the corridor  $i - j$ .

$n_{ij}^0$ , Number of existing circuits in the corridor  $i - j$

$\gamma_{ij}$ , Susceptance of one circuit in the corridor  $i - j$ .



- $g_i$ , Generation at the node  $i$ .  
 $d_i$ , Demand at the node  $i$ .  
 $\bar{f}_{ij}$ , Maximum power flow through one circuit in the corridor  $i - j$ .  
 $\bar{n}_{ij}$ , Maximum number of additions allowed in the corridor  $i - j$ .  
 $\bar{g}_i$ , Maximum power generation of node  $i$ .  
 $M$ , Parameter related to the angular aperture.  
 $\bar{\theta}$ , Maximum nodal angle of node.

Variables:

- $f_{ij,k}$ , Power flow in the  $2^{(k-1)}$  elements of the corridor  $i - j$ .  
 $f_{ij}^o$ , Total power flow through the existing circuits.  
 $\theta_i$ , Angle at node  $i$ .  
 $y_{ij,k}$ , Represents the addition of  $2^{(k-1)}$  elements in the corridor  $i - j$

## 2.2 Reduced Linear Disjunctive Model

The disjunctive linear model proposed by several authors is a transformation applied to the DC model to convert the Mixed-Integer Non-Linear Programming (MINLP) type problem to a Mixed-Integer Linear Programming (MILP) typed [Rahmani 2013], with binary variables instead of continuous variables. This transformation is achieved by representing each possible addition of transmission lines and/or substations by binary decision variables (1 to represent that the element is added and 0 to indicate otherwise). The inclusion of these binary variables implies a separation of the quadratic terms present in the DC model (products between  $\theta_i$  and  $n_{ij}$ ), however, it is necessary to incorporate into the model a big  $M$  parameter of very large value, so as to include the Kirchhoff's second law associated with binary variables whose value is 1, or that does not otherwise affect the model. The most interesting aspect of the disjunctive linear model is that is a problem of linear programming with binary variables, shares its optimal global solution with that of the DC model. One of the disadvantages of applying this model arises from the increase in the size of the problem that occurs when introducing the binary variables.

The equation which represents the *angle aperture constraint* or Kirchhoff's second law, for every  $2^{(k-1)}$  elements candidates to be added to the transmission system, becomes strict within the system of equations when the decision variable  $y_{ij,k}$  takes the value of 1. Otherwise, the big  $M$  parameter ensures that particular constraint is irrelevant for the model.

The *angle aperture constraint* represents the product between the maximum power flow of one circuit (capacity in p.u.) and its reactance, (in p.u.), and is denominated *capacity-reactance constraint*, in this work which represent a stronger constraint in the power transmission process. These constraints produce more investment cost in the long-term planning (DC model) than the planning without *capacity-reactance constraints* or transport model.



$$\begin{aligned}
 \text{Min} \quad & \sum_{(i,j) \in \Omega} c_{ij} \sum_{k=1}^K 2^{k-1} y_{ij,k} \\
 \text{S.a} \quad & \sum_{(l,i) \in \Omega} (f_{li}^o + \sum_{k=1}^K f_{li,k}) - \sum_{(i,l) \in \Omega} (f_{il}^o + \sum_{k=1}^K f_{il,k}) + g_i = d_i & \forall_i \in B \\
 & |f_{ij}^o| - n_{ij}^o \bar{f}_{ij} \leq 0 & \forall_{ij} \in \Omega \\
 & |f_{ij,k}| - (2^{k-1} y_{ij,k}) \bar{f}_{ij} \leq 0 & \forall_{ij} \in \Omega, k \in K \\
 & f_{ij}^o - \gamma_{ij} n_{ij}^o (\theta_i - \theta_j) = 0 & \forall_{ij} \in \Omega \\
 & -M(1 - y_{ij,k}) \leq f_{ij,k} - (\theta_i - \theta_j) 2^{k-1} \gamma_{ij} \leq M(1 - y_{ij,k}) & \forall_{ij} \in \Omega, k \in K \\
 & \sum_{k=1}^K 2^{k-1} y_{ij,k} \leq \bar{n}_{ij} & \forall_{ij} \in \Omega \\
 & y_{ij}^k \geq y_{ij}^{k+1} \\
 & |f_{ij}^o| \leq n_{ij}^o \bar{f}_{ij} \\
 & |f_{ij,k}| \leq y_{ij,k} \bar{f}_{ij} \\
 & 0 \leq g_i \leq \bar{g}_i & \forall_i \in B \\
 & -\bar{\theta} \leq \theta_i \leq \bar{\theta} \\
 & y_{ij,k} \in \{0,1\} & \forall_{ij} \in \Omega, k \in K \\
 & f_{ij} \longrightarrow \text{Unrestricted} & \forall_i \in \Omega \\
 & \theta_i \longrightarrow \text{Unrestricted} & \forall_i \in B
 \end{aligned}$$

### 3. Angular cuts in the transmission expansion planning problem.

Due to the combinatorial explosion that present transmission expansion planning problems, has not possible found its optimal solution for large scale systems using exact methods. Usually, the complexity of the problem is directly related to the size of the system to be analyzed, however, other factors produced complexity how the connectivity of the nodes or how well the system is enmeshed [Applegate 2003] [Darvishi 2015]. Hence, the importance of developing methodologies that improve the way in which the search for the solution space is performed [Dobson 2010]. It seeks to provide mechanisms that allow reduce the size of the solution space using technical information of the problem. If the electrical model of the transmission system includes the angle aperture limit, some constraints of maximum angular distance or *capacity-reactance constraints* can be over dimensioned respect to the true limit. This effect is produced in the parallel connection of circuits of low *capacity-reactance* product with circuits of high *capacity-reactance*. When the system is meshed, the parallel connection appears between a set of circuits.

The methodology proposed in this work applied the following ideas:

- If the *capacity-reactance* limits of the circuits are relaxed in the transmission expansion planning problem, the power flows use the shortest and lowest cost paths.
- The *angle aperture* constraints are obligatory limits for the existing transmission paths. In general, the new transmission paths have a low probability of selection for the future



network, in systems with a high number of investment options and low number of selected options. These constraints might be unnecessary for these circuits in a first approximation.

- c) If the cost of the investments is ignored, the problem with angle aperture constraints shows a *low effort* transmission model. The relaxation of the cost shows attractive paths in the DC transmission model.

The known *capacity-reactance* constraints permit to build *angular cuts* close to the facets of the convex hull. The combination of the three relaxed solutions of the problem permit identified sub-spaces of good quality where an exact method can realize exhaustive search [Fang 2011].

The primary aim of our work is to use the cutting-plane based on *angular cuts* for large-scale problems. Instances of this size are well beyond the reach of current (exact) solution techniques. The *angular cut* method can be used to provide strong lower bounds on the optimal investment of the transmission planning since the difference between the DC model (exact model) and the transport model (more relaxed model) are the angular constraints. A separation algorithm for *angular cut* constraints is an algorithm that, given any optimal solution of the relaxed problem (for example, the transport model):  $x^*$ , returns one or some constraints that are violated by  $x^*$  in the exact model (DC model), or an exit message. Separating algorithms that return an exit message only if all constraints are satisfied by  $x^*$  are called exact.

All the mathematical models for the transmission expansion planning problem includes capacity constraints. The capacity-reactance constraints are only necessary if the circuits form loops with other circuits with less capacity-reactance constraint. In electrical systems with high mesh level, the number of loops can increase exponentially. In consequence, can be prohibitive identifying and verifying each capacity-reactance constraint violated in the loops of the system. For large size systems, some potentials capacity-reactance constraints are better identified that can be violated in the exact model or develop a method for identifying moderate angular cut constraints and generated stronger angular cut constraints close to the convex hull of the MILP problem.

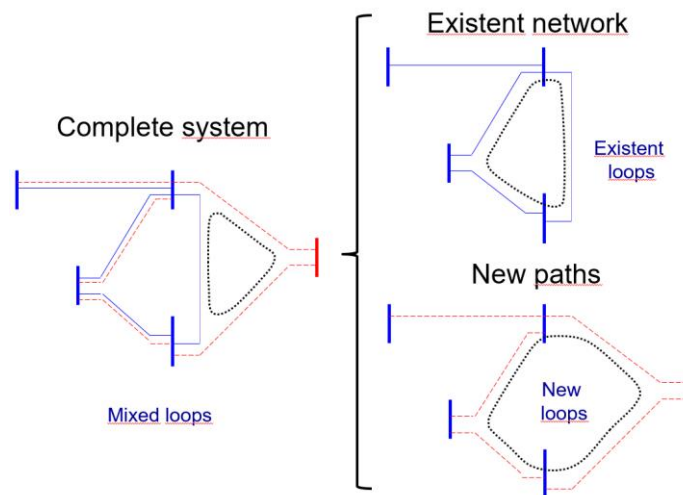


Figure 1. Loops over current paths, new paths and mixed: current and new paths.



Figure 1 shows the three types of loops in an electrical system. Each one produces one type of *strong* angular cut constraints. The more simplest case is when two existing circuits with different *capacity-reactance* product are connected in parallel, figure 2. In this case, the *capacity-reactance* constraints for the circuits are:

$$\text{Limit for circuit a: } f_{ij a} \cdot x_{ij a} \leq (\theta_i - \theta_j)_{max}$$

$$\text{Limit for circuit b: } f_{ij b} \cdot x_{ij b} \leq (\theta_i - \theta_j)_{max}$$

Where:

$$(\theta_i - \theta_j)_{max} = \min \{ \bar{f}_{ij a} \cdot x_{ij a}, \bar{f}_{ij b} \cdot x_{ij b} \}$$

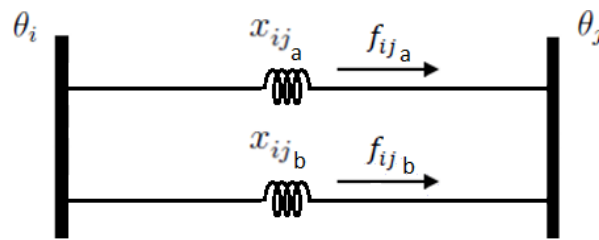


Figure 2. Circuits with different *capacity-reactance* limits.

In this case, if the circuit *b* has the low *capacity-reactance* product, the *moderate angular cut* is:

$$f_{ij a} \cdot x_{ij a} \leq \bar{f}_{ij a} \cdot x_{ij a}$$

And the *strong angular cut* is:

$$f_{ij a} \cdot x_{ij a} \leq \bar{f}_{ij b} \cdot x_{ij b}$$

In the parallel connection, circuit *b* limits the operation of circuit *a*.

In this work, when three or more existent circuits form a loop, the strong *capacity-reactance* is generated if the three relaxed models: transport model, hybrid model and low effort model, show thus the same direction of flow. As shown in the following example:

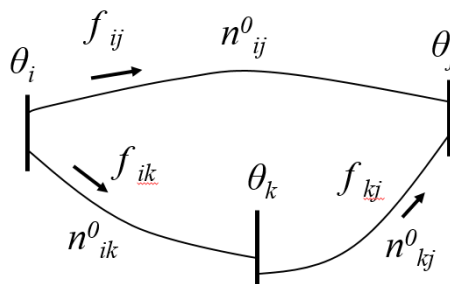


Figure 3. Loop between three existing parallel circuits.



In figure 3, the capacity-reactance limit of the circuits  $ik$  and  $kj$  are added and the result is compared with the capacity-reactance limit of the circuits  $ij$ . If the sum of capacity-reactance limits of the circuits  $ik$  and  $kj$  are greater than the capacity-reactance limit of the circuit  $ij$ , the *stronger capacity-reactance* constraint added to the problem is the constraint:

$$(f_{ik} \cdot x_{ik} / n_{ik}^0) + (f_{kj} \cdot x_{kj} / n_{kj}^0) \leq \bar{f}_{ij} \cdot x_{ij}$$

If the sum of capacity-reactance limits of the circuits  $ik$  and  $kj$  are smaller than the capacity-reactance limit of the circuit  $ij$ , the *stronger capacity-reactance* constraint added to the problem is the constraint:

$$f_{ij} \cdot x_{ij} / n_{ij}^0 \leq (\bar{f}_{ik} \cdot x_{ik}) + (\bar{f}_{kj} \cdot x_{kj})$$

In this constraints, the flows  $f_{ij}$ ,  $f_{ik}$  and  $f_{kj}$  are the total active flow in the corridors and it is necessary to divide it over the number of existing circuits in the path. If the circuits  $ij$ ,  $ik$  or  $kj$  are investment options, the formulation is different. Figure 4 shows a loop where the  $ij$  corridor is an existing path and the  $ik$  and  $kj$  corridors are new paths. The network is represented by the *reduce disjunctive model* and, in figure 4, is considered 0,1,2 or 3 additions in each new path.  $y_{ik,1}$ ,  $y_{ik,2}$ ,  $y_{kj,1}$  and  $y_{kj,2}$  represent binary variables. The flows  $f_{ik,1}$ ,  $f_{ik,2}$ ,  $f_{kj,1}$  and  $f_{kj,2}$  represent continuous variables. In general, if the binary variable  $y_{pq,m} = 1$  then adds  $2^{(m-1)}$  circuits between the buses  $p$  and  $q$ , each one with reactance  $x_{pq}$ , and the power flow  $f_{pq,m}$  represents the total flow in the  $2^{(m-1)}$  circuits.

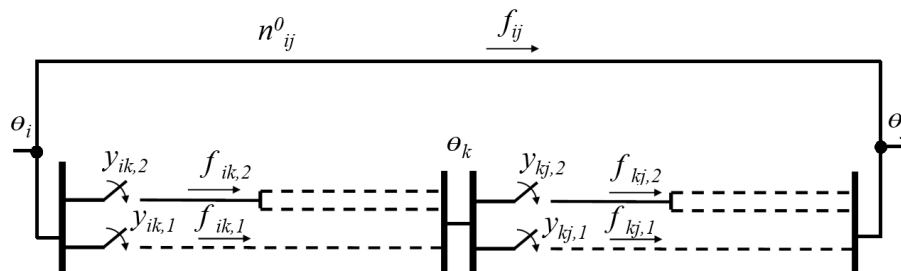


Figure 4. Loop between one existing path and two path options.

If the sum of *capacity-reactance* limits of the circuits  $ik$  and  $kj$  are greater than the *capacity-reactance* limit of the circuit  $i-j$ , the *stronger capacity-reactance* constraint added to the problem is the constraint:

$$\left\{ f_{ik,1} \cdot y_{ik,1} + (f_{ik,2} / 2) [y_{ik,2} * (1 - y_{ik,1})] \right\} x_{ik} + \left\{ f_{kj,1} \cdot y_{kj,1} + (f_{kj,2} / 2) [y_{kj,2} * (1 - y_{kj,1})] \right\} x_{kj} \leq \bar{f}_{ij} \cdot x_{ij}$$

In general, if the option between the buses  $p$  and  $q$  has  $m > 1$  binary variables, for each path is necessary to build the next expression:



$$\{f_{pq,1} y_{pq,1} + (f_{pq,2}/2) [y_{pq,2} (1-y_{pq,1})] + \dots + (f_{pq,m}/2^{(m-1)}) [y_{pq,m} (1-y_{pq,1})(1-y_{pq,2})^* \dots (1-y_{pq,(m-1)})]\} x_{ik}$$

These mixed loops are identified for the three relaxing models in a previous step.

#### 4. Test and results

The transmission expansion planning problem considering *angular cuts* in the reduced linear disjunctive model is implemented using AMPL and solved using CPLEX [Escobar 2012]. The methodology is probed on the Southern Brazilian test system shown in figure 5 to validate the methodology. The system is solved with and without *strong capacity-reactance* constraints. Table 1 shows the results when the next constraints are added to the model.

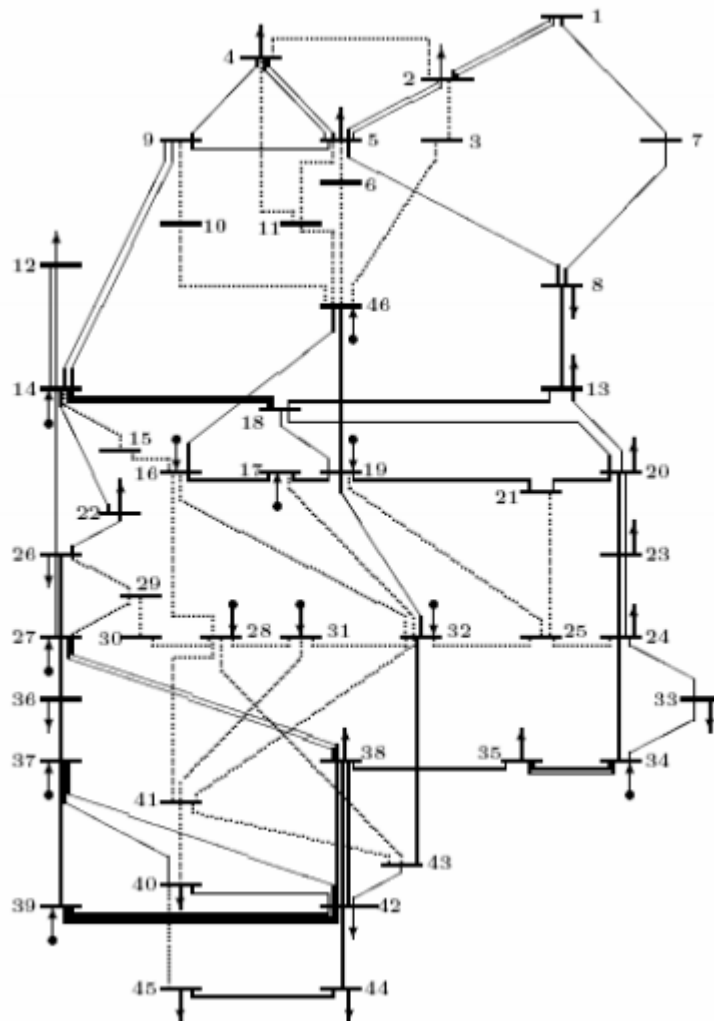


Figure 5. Southern Brazilian test system.





The following constraints were constructed following the methodology explained above and are added to the reduced linear disjunctive model and tested in the Southern Brazilian test system.

$$\begin{aligned} \text{Constraint 1: } & (\theta_2 - \theta_8) \leq \bar{f}_{12} x_{12} + \bar{f}_{17} f_{17} x_{17} + \bar{f}_{78} x_{78} \\ \text{Constraint 2: } & \frac{f_{33-34}^0}{n_{33-34}^0} x_{33-34} + \frac{f_{24-33}^0}{n_{24-33}^0} x_{24-33} \leq \bar{f}_{24-34} x_{24-34} \\ \text{Constraint 3: } & (\theta_{32} - \theta_{42}) \leq \bar{f}_{32-43} x_{32-43} + \bar{f}_{42-43} x_{42-43} \\ \text{Constraint 4: } & (\theta_{19} - \theta_{14}) \leq \bar{f}_{18-19} x_{18-19} + \bar{f}_{14-18} x_{14-18} \end{aligned}$$

As a result, the base case is used to compare the traditional solution with the addition of the new constraints to the model, and then analyzed to verify if the results show a better behavior, as shown in the table below. *Ticks* it's a measure of how much algorithmic work is required to obtain a provably optimum (MAWOPO) independently of the computer on which it is run on as shown in the first part of table 1 followed by the number of iterations and nodes created in the Branch and Cut process which are the key point in to verify the methodology.

|                                 | MAWOPO<br>(units) | MAWOPO<br>(%) | MIPs simplex<br>iterations | Branch&Bound<br>nodes |
|---------------------------------|-------------------|---------------|----------------------------|-----------------------|
| <b>Base Case (BC)</b>           | 731.73            | 100           | 31389                      | 1432                  |
| <b>BC + Constraint 2</b>        | 629.57            | 86.1          | 26210                      | 926                   |
| <b>BC + Constraints 1 and 2</b> | 491.74            | 67.2          | 19536                      | 973                   |
| <b>BC + Constraint 3</b>        | 437.57            | 59.8          | 12912                      | 571                   |
| <b>BC + Constraint 4</b>        | 544.47            | 74.4          | 18777                      | 926                   |

Table 1. Southern Brazilian test system with and without *strong capacity-reactance* constraints

The constraint 1 limits the maximum *angular distance* between the buses 2 and 8 in the all possible parallel connections with the series 1-2, 1-7 and 7-8; the constraint 2 limits the capacity-reactance sum of the circuits 33-34 and 24-33; the constraint 3 fix a lower bound for the angular distance of the buses 32 and 42; and the constraint 4 fix a lower bound for the angular distance of the buses 19 and 14. The table 1 shows the effect of these constraints over the MAWOPO, over the number of iterations of the Mixed Integer Programming problem and over the number of Branch and Bound nodes. The base case (BC) resolved the MILP problem without *strong capacity-reactance* constraints. In the other cases, one or two *strong constraints* are added. In the cases with *strong capacity-reactance* constraints the MAWOPO are less than the base case. The more effective strong capacity-reactance constraint was constraint 3 which reduces the algorithmic work to 59,8%.

## 5. Conclusions

The reduced disjunctive model used in transmission expansion planning problems is an NP-hard problem in large size systems. The transmission model is simpler. The difference between this model is the *capacity-reactance* constraints of the circuits. Identified *moderated capacity-reactance* constraints permit to build the *angular cuts* close to the facets of the convex hull.

The combination of the three relaxed solutions of the problem permit identifying *strong capacity-reactance* constraints for this problem which is highly important at the start of the analysis to improve the performance.



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