MODELING MAINTENANCE SERVICE CONTRACTS USING DISCRETE EVENT SIMULATION

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ABSTRACT

This paper presents a model and a simulation solution approach for the problem of the interaction between the original equipment manufacturer (OEM), which sells a device and offers repair services, and the buyers of the device, who can also choose to hire a product warranty (PW). This model allows for equipment degradation to be considered and assumes minimal repairs; times until failures are conditioned to equipment age. We also present an application example and sensitivity analysis on several of the model's parameters.

KEYWORDS. Maintenance Service Contracts. Discrete Event Simulation. Game Theory.

Paper topics: SIM – Simulation; MP – Probabilistic Models.

1. Introduction

For many companies, success highly depends on operational efficiency and, consequently, adequate maintenance performance. In order to achieve such performance, maintenance outsourcing has been a growing practice [Murthy et al. 2015], [Guedes et al. 2015]. The decision to outsource may be result of several characteristics, as stated by [Jackson and Pascual 2008] and [Murthy and Jack 2014]: reduce costs, improve service, obtain expert skills, improve processes and improve focus on core activities. For many companies, equipment maintenance has become the main profit source [Guajardo et al. 2012].

In many scenarios, the Original Equipment Manufacturer (OEM) provides repair service for their products, frequently offering a Product Warranty (PW) or repair services on demand. This paper presents a simulation approach for the interaction between the OEM and its customers, concerning the purchase of a piece of equipment and possible acquisition of a PW. The model proposed here extends the model presented by [Ashgarizadeh and Murthy 2000], in which the service provider (in this paper considered to be the OEM) and the customers interact in a Stackelberg game. Discrete event simulation allows us to overcome some limitations of the analytical methodology developed in [Ashgarizadeh and Murthy 2000], as well as consider a broader range of scenarios and equipment reliability characteristics. This is also an extension on the model presented in [Santana et al. 2016], in which a similar problem case was approached with use of simulation.

In the problem considered in this paper, the OEM must decide how many devices to sell, and also must set the prices charged for equipment warranty and repairs. Customers analyze the prices charged by the OEM and then decide whether to purchase the equipment, and whether to hire the warranty. The OEM is considered risk-neutral, while customers are risk-averse; we assume complete and perfect information, i.e., OEM and customers are completely aware of the equipment reliability behavior, as well as each other's behaviors. Under these assumptions, this problem can be modeled as a Stackelberg game.

Further description for the model is given in section 2; section 3 presents the model solution; section 4 presents an application with sensitivity analysis; and section 5 has some concluding remarks and suggestions for further extension of this work.

2. Model description

2.1. Notation

The following notation will be used in this paper:

- A_p: customer strategy p;
- A^*: customer optimal strategy;
- C_b: price of equipment;
- C_r: cost of a repair to the OEM;
- C_s: price of single repair service;
- $C_s^*(M)$: optimal C_s given M;
- k: the Weibull distribution shape parameter;
- L: length of product warranty;
- L_{op}: total operational time during L;
- L_{ov}: total overtime during L;
- M: number of units sold and serviced by the manufacturer;
- M_m: maximum allowed M;
- P_w: price to hire product warranty;
- $P_w^*(M)$: optimal P_w given M;
- R: revenue per operational unit time of an equipment;
- t_0: equipment age in the age-conditioned Weibull distribution;

- $U(w)$: customer's utility associated with wealth w;
- y: time between the failure of a unit and its repair;
- α: penalty per unit time when y exceeds τ;
- β: customer's risk aversion parameter;
- μ: repair rate for each equipment;
- π: OEM's expected profit;
- θ: the Weibull distribution scale parameter;
- τ: time limit for the OEM to repair a failed unit since its failure.

2.2. Equipment failures and repairs

The manufacturer (OEM) sells a total of M units and provides repair service for all of them. Each customer decides whether to purchase a device for a price of C_b and decides whether to hire a PW with a duration of L from the OEM by making a payment of a price P_w. If the customer does not hire the PW, the OEM still repairs the equipment for a price of C_s each time it fails.

When operational, each equipment unit generates revenue of R per unit time and is subject to failures. When there is one or more units at failed state, they are repaired by the OEM, one at a time, at constant average rate μ. Repairs are considered to be minimal, so they do not restore degradation of equipment, i.e., when a equipment is repaired, it returns to operational state with the same age as before the failure; the next time until failure is conditioned to this age. Times until failures follow a Weibull distribution conditioned to the age of the equipment, as shown in Eq. (1). x is the time until the failure; t_0 is the age of the device; θ is the Weibull scale parameter; k is the Weibull shape parameter.

$$f(x|t_0;\theta,k) = \frac{k}{\theta}\left(\frac{x+t_0}{\theta}\right)^{k-1} e^{\left(\frac{t_0}{\theta}\right)^k - \left(\frac{x+t_0}{\theta}\right)^k} \tag{1}$$

When covered by the PW, the OEM has a time limit of τ to return the equipment to operational state; if the time y between the failure of a unit under PW until the completion of its respective repair is longer than τ, the customer receives compensation from the OEM of a value $\alpha(y\text{-}\tau)$, where α is the penalty rate parameter.

A G/M/1 queue with finite population of size M (total number of equipment/customers served) is adequate to describe the behavior of failures and repairs as described above [Gross et al. 2008], [Kleinrock 1975].

2.3. Customer's decision problem

Customers must decide whether to purchase the device. If they choose to purchase, it is also necessary to decide if a PW will be hired. These decision options are listed below:

- A_0: customers do not purchase the equipment;
- A_1: customers purchase the equipment and hire a PW;
- A_2: customers purchase the equipment but do not hire a PW.

Customers are risk-averse and choose the option that maximizes their expected utility [Varian 1992]. All customers are considered homogeneous with respect to their risk behavior. Their utility associated with a wealth w is given by Eq. (2), where β is the risk aversion parameter.

$$U(w) = \frac{1 - e^{-\beta w}}{\beta} \tag{2}$$

In this model, we assume perfect information for the players, i.e., the OEM and the customers can estimate their expected returns. Considering the customers, they can estimate their expected utility from the choice for each of the strategies when given the prices of the PW (P_w) and repairs on demand (C_s), as well as the number of equipment units to be sold (M). If customers choose A_0 and do not purchase the equipment, their expected utility equals zero, since no gain or loss of wealth is observed.

When the decision is for option A_1, each customer purchases an equipment with price C_b and hires PW with price P_w. No further payment from the customer is needed when failures occur. The equipment does not generate revenue when in failed state. If the unit is not returned to operational state within τ time units, the customer receives compensation of α for each time unit after τ; the time between τ and the completion of repair will be referred here as overtime. PW has length L. The total operational time during L is denoted by L_{op}, note that $L_{op} \leq L$. The sum of overtimes during the PW is denoted by L_{ov}. The wealth associated with A_1 is given by Eq. (3).

$$w(A_1) = RL_{op} + \alpha L_{ov} - C_b - P_w \tag{3}$$

For decision option A_2, customers purchase the equipment by paying C_b but do not hire a PW. Each failure during L requires the customer to pay a repair price of C_s. The equipment does not generate revenue in failed state and no compensation is received when time until repair takes too long. The wealth associated with A_2 is given in Eq. (4), where N is the number of failures during L.

$$w(A_2) = RL_{op} - C_b - C_s N \tag{4}$$

To choose among the strategies, customers take into account the expected utility associated with each respective wealth given by each option. Due to this and their homogeneity with respect to risk behavior, all M customers choose the same strategy.

2.4. Manufacturer's decision problem

The OEM must define P_w, C_s and M in order to maximize their expected profit, given that customers choose among A_0, A_1 and A_2 depending on these parameters. When customers choose A_0, the OEM has expected profit equal to zero. When customers choose A_1, the OEM's expected profit is expressed by Eq. (5), where C_r is the cost of each repair for a unit. N_j is the number of failures and $L_{ov,j}$ is the overtime for customer j, respectively.

$$\pi(P_w, C_s, M; A_1) = E\left[\sum_{j=1}^{M}\left(P_w - C_r N_j - \alpha L_{ov,j}\right)\right] \tag{5}$$

If customers decide for option A_2, the OEM's expected profit is as in Eq. (6).

$$\pi(P_w, C_s, M; A_2) = E\left[\sum_{j=1}^{M}\left[(C_s - C_r)N_j\right]\right] \tag{6}$$

3. Model solution

3.1. Customer's optimal strategy

Given a set of P_w, C_s and M, the customer must estimate their expected utility associated with Eqs. (3) and (4) and make their decision [Ashgarizadeh and Murthy 2000].
A_0 results in expected utility equal to zero, as shown in Eq. (7).

$$E[U(A_0; P_w, C_s, M)] = E\big[U\big(w(A_0)\big)\big] = 0 \tag{7}$$

A_1 results in expected utility given by Eq. (8).

$$E[U(A_1; P_w, C_s, M)] = E\big[U\big(w(A_1)\big)\big] = \frac{1}{\beta}\Big(1 - e^{\beta(C_b+P_w)}E\left[e^{-\beta(RL_{op}+\alpha L_{ov})}\right]\Big) \quad (8)$$

A_2 results in expected utility as in Eq. (9).

$$E[U(A_2; P_w, C_s, M)] = E\big[U\big(w(A_2)\big)\big] = \frac{1}{\beta}\Big(1 - e^{\beta C_b}E\left[e^{-\beta(RL_{op}-C_s N)}\right]\Big) \quad (9)$$

3.2. Manufacturer's optimal strategy

Due to the assumption of perfect information, the OEM is aware that customers decide based on their expected utilities, being able to predict which strategy customers will choose given a set of P_w, C_s and M. In order to obtain maximum profit, the OEM estimates optimal P_w and C_s according to different values of M, until an optimal M is found, yielding the greatest possible profit to the OEM.

Since $E[U(w(A_0))] = 0$, the OEM wants to find $P_w^*(M)$ and $C_s^*(M)$, optimal values for P_w and C_s given M, respectively, that also make $E[U(w(A_1))] = 0$ and $E[U(w(A_2))] = 0$, meaning that $E[U(w(A_0))] = E[U(w(A_1))] = E[U(w(A_2))] = 0$. In other words, the expected utilities yielded by each of the strategies are equal. In this scenario, customers do not have preference for any of the strategies over the others. This allows the OEM to set the service prices so that customers choose the option that results in the greatest OEM's expected profit. The OEM sets $P_w = P_w^*(M)$ and $C_s > C_s^*(M)$ when $\pi(P_w^*(M),C_s^*(M),M;A_1) > \pi(P_w^*(M),C_s^*(M),M;A_2)$ (strategy A_1 is more lucrative than strategy A_2), inducing customers to choose option A_1. If $\pi(P_w^*(M),C_s^*(M),M;A_2) > \pi(P_w^*(M),C_s^*(M),M;A_1)$ (strategy A_2 is more lucrative than strategy A_1), the OEM sets $P_w > P_w^*(M)$ and $C_s = C_s^*(M)$, causing customers to choose option A_2. The OEM also sets $M = M^*$ by choosing M^* that maximizes $\pi(P_w^*(M),C_s^*(M),M;A_1)$ or $\pi(P_w^*(M),C_s^*(M),M;A_2)$, which means that M^* is the number of customers that result in the greatest possible profit.

In order to find $P_w^*(M)$, as mentioned before, the OEM sets $E[U(w(A_1))] = 0$, which results in Eq. (10) when solving for $P_w^*(M)$.

$$P_w^* = -C_b - \frac{1}{\beta}\Big(\ln E\left[e^{-\beta(RL_{op}+\alpha L_{ov})}\right]\Big) \quad (10)$$

In the case of $C_s^*(M)$, the OEM sets $E[U(w(A_2))] = 0$, finding Eq. (11). Notice, however, that C_s must be found by using a numerical method, since Eq. (11) does not present a closed form for C_s.

$$\beta C_b + \ln E\left[e^{-\beta(RL_{op}-NC_s^*)}\right] = 0 \quad (11)$$

3.3. Simulation approach

In order to find the solution for the model presented in this paper, we must find, for each customer, the number of failures (N), the total downtime (L_{op}), and the total overtime (L_{ov}), present in Eqs. (8), (9), (10) and (11). Therefore, the model presented in section 2 was replicated via discrete event simulation (DES) [Ross 2013]. This allowed us to extend the model developed in [Santana et al. 2016] and [Ashgarizadeh and Murthy 2000], in which times until failures follow an exponential probability distribution. In the case of [Ashgarizadeh and Murthy 2000], in order to solve the problem analytically, it was necessary to consider the approximation $L_{op} \approx L$. In the model presented in this paper, due to using a DES approach, we were able to simulate failures following the conditioned Weibull distribution given in Eq. (1) and use simulated values for L_{op}, thus not using the aforementioned approximation.

The complete solution of the model is done through the following steps: (i) set the initial population size M = 1 (number of devices/customers) (ii) simulate the G/M/1 queue considering the description given in subsection 2.2, returning the values for N, L_{op} and L_{ov}; (iii) estimate P_w and C_s by using Eqs. (10) and (11), respectively; (iv) estimate the OEM's profit for each customer strategy, using Eqs. (5) and (6); (v) increment M by 1 ($M = M + 1$) and repeat steps (ii)-(iv), until the M that maximizes the OEM's expected profit is found (this can be detected when the OEM's expected profit decreases for both A_1 and A_2 when M is incremented).

4. Application and analysis

4.1. Application example

A numerical example to illustrate the model will be presented in this section, allowing us to analyze its behavior. The steps given in subsection 3.3 were followed, implemented using C++ programming language. The simulation was replicated 1,000,000 times. The parameters used in this application are as follows: θ = 4,000 h; k = 2; μ = 0.05 / h; α = 3 (10^3) \$ / h; β = 0.2; τ = 24 h; C_b = 900 (10^3) \$, C_r = 4 (10^3) \$; R = 0.18 (10^3) \$ / h; L = 8,760 h.

Results are shown in Table 1. The OEM decides to sell the equipment to $M = M^* = 34$ customers; the price of the PW is set as $P_w = P_w^*(M^*)$ = 682,006; the price of each repair on demand is set as $C_s = C_s^*(M^*)$ = 37,707; customers choose strategy A_1, buying the equipment and purchasing a PW; the OEM's expected profit is $\pi(P_w^*,C_s^*,M^*;A^*)$ = 13,067,343.

Table 1 – Main results of the numerical examples

Variable	Value
M^*	34
P_w (\$)	682,006
C_s (\$)	37,707
A^*	A_1
$\pi(P_w,C_s,M^*;A^*)$ (\$)	13,067,343

Table 2 shows further information about the queue performance, as well as the expected penalties received by each customer and expected total paid by the OEM. Each equipment unit is expected to fail 4.61 times during L; each unit is expected to remain in failed state (downtime) for 174.46 h (2% of L); the expected overtime for each device is 93.11 h; each customer receives a expected value in penalties of \$ 279,330; in total, the OEM expects to pay \$ 9,497,220 in penalties. Note, however, that the OEM's expected profit of \$ 13,067,343 already takes into account the payment of these expected penalties, i.e., the penalties are not deduced from its given profit. Also notice that customers are willing to pay a high value for P_w, which is equal to \$ 682,006, the equivalent of 75.8% of the device purchase price; one of the reasons this price is relatively high, is due to the fact that customers expect beforehand to receive \$ 279,330 in penalties, which is also a considerably high value, but results in the customers' willingness to pay for the PW to be high as well.

Table 2 – Queue performance measures

Variable	Value
Expected server idle time (h)	5,661.96
Expected number of failures $E[N]$	4.61
Expected downtime $E[T - L_{op}]$ (h)	174.46
Expected overtime $E[L_{ov}]$	93.11
Expected penalties per customer (\$)	279,330
Expected total penalties paid by the OEM (\$)	9,497,220

4.2. Sensitivity analysis

In this section, we present a sensitivity analysis for several of the model's parameters, showing how each parameter affects the behavior of the OEM, the customers, and the interaction between them.

Table 3 shows how the model's results behave when the Weibull distribution scale parameter θ varies. When θ is smaller, the device fails more often, and thus its availability is reduced, i.e., the expected downtime increases. As a result, the OEM choose lower values of M (sells the equipment to fewer customers) when θ is lower, and higher values of M when θ is higher. The OEM's expected profit is proportional to θ.

Table 3 – Effect of variations on θ

θ (h)	M	P_w ($)	C_s ($)	A	$\pi(P_w,C_s,M^*;A^*)$ ($)
3500	24	682,275	32,403	A_1	8,375,125
3750	28	681,890	35,077	A_1	10,586,947
4000	34	682,006	37,707	A_1	13,067,343
4250	39	681,665	39,967	A_1	15,822,131
4500	45	681,463	42,449	A_1	18,869,456

Table 4 presents the effects of variations on the Weibull distribution shape parameter k. When k is increased, the frequency of failures increases over time, that is, k dictates how fast the equipment degrades. Similar to variations on θ, when failures occur more often (when k is higher) the OEM chooses a lower value of M, and vice versa. Beyond varying the number of customers, the OEM's expected profit is lower when k is higher.

Table 4 – Effect of variations on k

k	M	P_w ($)	C_s ($)	A	$\pi(P_w,C_s,M^*;A^*)$ ($)
1.8	44	681,729	39,734	A_1	17,847,003
1.9	38	681,707	38,870	A_1	15,272,940
2.0	34	682,006	37,707	A_1	13,067,343
2.1	29	681,830	36,239	A_1	11,167,206
2.2	26	682,106	35,293	A_1	9,547,597

The effect on the model due to variations on μ can be found in Table 5. When μ is lower, the OEM is not able to repair as much equipment units over time as when μ is higher; repairs take longer for lower values of μ. Therefore, the number of customers M is proportional to μ. Also, when $\mu = 0.03$, the OEM induces customers to choose not to purchase a PW, since the OEM would have to pay too much in penalties if customers purchased a PW. When μ is increased, the OEM's expected profit also increases, due to its capacity to serve more customers, therefore receiving higher revenue.

Table 5 – Effect of variations on μ

μ (h^{-1})	M	P_w ($)	C_s ($)	A	$\pi(P_w,C_s,M^*;A^*)$ ($)
0.03	32	691,642	29,075	A_2	3,495,336
0.04	24	683,302	37,535	A_1	8,142,875
0.05	34	682,006	37,707	A_1	13,067,343
0.06	44	680,914	37,401	A_1	18,379,703
0.07	54	679,968	37,209	A_1	23,979,146

Variations on the customers' risk aversion parameter β are displayed in Table 6. For all values of β tested, the number of customers M remained the same. However, changes can be noticed. For lower values of β (which means customers are less risk-averse), customers accept to pay more for either the PW or repairs on demand. Consequently, it is possible to observe that the OEM's expected profit is inversely proportional to β.

Table 6 – Effect of variations on β

β	M	P_w (\$)	C_s (\$)	A	$\pi(P_w, C_s, M^*; A^*)$ (\$)
0.10	34	693,298	41,317	A_1	13,444,848
0.15	34	686,050	38,701	A_1	13,191,644
0.20	34	682,006	37,707	A_1	13,067,343
0.25	34	679,297	37,010	A_1	12,972,090
0.30	34	677,235	35,857	A_1	12,903,125

Table 7 shows the effects of variations on R, the customers' revenue per operational hour of the device. When R is higher, customers accept to pay more for the services and, consequently, the OEM's expected profit is higher. The OEM sells a slightly higher number of equipment when R is raised.

Table 7 – Effect of variations on R

R (\$ 10^3)	M	P_w (\$)	C_s (\$)	A	$\pi(P_w, C_s, M^*; A^*)$ (\$)
0.170	32	594,333	32,797	A_1	10,224,717
0.175	33	638,164	35,042	A_1	11,622,338
0.180	34	682,006	37,707	A_1	13,067,343
0.185	34	725,556	39,983	A_1	14,548,292
0.190	35	769,401	42,069	A_1	16,065,545

Table 8 shows how the model behaves when α, the penalty per hour of overtime, varies. When α is low, the OEM can serve more customers without paying as much in penalties, therefore raising its expected profit. For high values of α, the OEM pays too much in penalties, making it induce customers not to acquire the PW. For instance, when α = 5.0, customers do not hire the PW.

Table 8 – Effect of variations on α

α (\$ 10^3)	M	P_w (\$)	C_s (\$)	A	$\pi(P_w, C_s, M^*; A^*)$ (\$)
1.0	50	684,796	33,430	A_1	23,463,279
2.0	39	682,876	36,512	A_1	16,837,274
3.0	34	682,006	37,707	A_1	13,067,343
4.0	29	680,945	38,470	A_1	10,464,772
5.0	55	689,272	31,567	A_2	6,744,977

Table 9 shows variations on τ, the maximum time for the OEM to repair the equipment without paying penalties. As well as α, τ changes how much penalty is paid; while α dictates how much penalty is incurred per hour of overtime, τ influences how much overtime occurs, i.e., for lower values of τ, overtime occurs earlier (and thus more often), while for higher values of τ, overtime occurs later (and thus less often). Therefore, for lower values of τ, the OEM is able to serve less customers, while its expected profit is lower; for higher values of τ, more customers are served and the OEM's profit is higher. Customers accept to pay higher prices for hiring a PW when τ is lower.

Table 9 – Effect of variations on τ

τ (h)	M	P_w (\$)	C_s (\$)	A	$\pi(P_w, C_s, M^*; A^*)$ (\$)
12	30	690,534	37,939	A_1	10,150,958
18	32	686,252	37,601	A_1	11,692,397
24	34	682,006	37,707	A_1	13,067,343
30	35	677,399	37,191	A_1	14,279,721
36	35	672,297	37,032	A_1	15,326,099

4.3. Comparison with the analytical approach

For validation and comparison, we replicated the application example given by [Ashgarizadeh and Murthy 2000]. The parameters used were: θ = 1,250 h; k = 1; μ = 0.02 / h; α = 0.06 (10^3) \$ / h; β = 0.1; τ = 70 h; C_b = 300 (10^3) \$, C_r = 5 (10^3) \$; R = 0.015 (10^3) \$ / h; L = 40,000 h.

In our approach, the optimal number of customers is M = 17, the same result obtained by the analytical solution. The prices charged for the warranty and repairs, as well as the OEM's expected prices were different in the two approaches, as found in Table 10. Notice that our approach results in smaller values for prices and OEM's profit. This is due to the approximation used in the analytical method, $L_{op} \approx L$, which considers the equipment to generate revenue for the entire duration of the analyzed time, not considering equipment downtime to stop revenue generation, consequently resulting in higher values in the analytical approach.

Table 10 – Comparison of results from the analytical and simulation methods

Method	*M*	P_w (\$)	C_s (\$)	*A*	$\pi(P_w,C_s,M^*;A^*)$ (\$)
Analytical	17	369,948	6,614	A_1	1,650,205
Simulation	17	334,238	5,803	A_1	1,429,824

5. Conclusions

This paper presented a simulation approach to the problem of the interaction between OEM and customers regarding the purchase of equipment and repair services. This model extends the simulation approach presented in [Santana et al. 2016] by considering equipment degradation with the use of Weibull distributed times until failures, along with minimal repairs. This extension allows the present model to cover a broader range of situations, while also being able to model problems covered by the models of [Santana et al. 2016] and [Ashgarizadeh and Murthy 2000]. We also extend the analytical approach in [Ashgarizadeh and Murthy 2000] by not considering the approximation $L_{op} \approx L$, which was needed in their approach. Regarding computational cost, our approach depends on the efficiency of the simulation methods used; the application example in subsection 4.1 takes about 15 minutes to run in a consumer-grade personal computer. Although the analytical approach given by [Ashgarizadeh and Murthy 2000] requires the evaluation of much fewer equations, the model's metrics from the queueing system must be obtained analytically, and that approach also requires the evaluation of integrals. It can also be noted that the present simulation approach allows for easier adoption of different modeling systems for equipment failures and repairs.

Further improvements on this model are concern of our ongoing research, and are listed as follows: (i) incorporation of imperfect repair effectiveness, characterizing a generalized renewal process (ii) inclusion of different maintenance policies, such as preventive and predictive maintenance interventions; (iii) adoption of a two-dimensional warranty, using, for example, equipment usage rate into account; (iv) consideration of incomplete information, causing the customers and/or the OEM to not be able to know exactly the equipment performance and availability.

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