



## **INCORPORATING UNCERTAINTY INTO TACTICAL CAPACITY PLANNING: A ROBUST OPTIMIZATION MODEL FOR AN ENGINEERING-TO-ORDER INDUSTRIAL SETTING**

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### **ABSTRACT**

Production planning literature is rich of approaches and models proposed to cope with different forms of uncertainty. In traditional optimization models, specific probability distributions for random variables are assumed known. In many real-world situations, however, decision-makers do not have precise information about future demand. This is particularly critical in engineering-to-order (ETO) production systems which are associated with high complexity and uncertain situations. Within this context, the objective of this paper is to present a robust optimization reformulation of a tactical capacity planning model to incorporate information relative to process uncertainty. This study refers to an action research that was realized in a real-world ETO industrial setting. The model aims at solution robustness (or stability) and intends to enhance and support the decision-making process of the studied setting.

**KEYWORDS.** Production planning, Mathematical modelling, Uncertainty.

**Paper topics:** AD&GP - OR in Administration & Production Management, IND - OR in Industry



## 1 Introduction

In the engineering-to-order (ETO) context, the production flow is entirely driven by actual customer orders with the decoupling point located at the design stage (Gosling and Naim, 2009; Grabenstetter and Usher, 2014; Powell et al., 2014). ETO processes are highly knowledge intensive and are often built on tacit knowledge, as product structures are subject to constant changes in terms of design and full automation of production processes is often not feasible due to the customer specific requirements (Willner et al., 2014). The ETO context is associated with chaotic production in high-complexity and high-uncertainty situations (Gosling and Naim, 2009; Yang, 2013), where the ability to address instability in demand and to respond to demand modifications over time is crucial (Hans et al., 2007; Zorzini et al., 2008). Some of the most common problems associated with ETO planning processes are difficulties in estimating lead-times and delivery dates and conflicts between projects and manufacture schedules (Pandit and Zhu, 2007). Management must rely on a rough estimation of the impact of an incoming order on resource utilization and eventually adjust capacity, as micro process planning has not been performed yet (Grabenstetter and Usher, 2014). In general, detailed information becomes available only gradually leading to a lack of information in the tactical planning stage. In other words, there is a significant level of uncertainty in this context.

Proactive planning approaches try to incorporate information about uncertainty in the baseline plans so that it can be protected, as well as possible, against future disruptions, i.e., aiming at stability (Herroelen and Leus, 2004; Chtourou and Haouari, 2008; Deblaere et al., 2011; Alfieri et al., 2012). In traditional decision support models that consider uncertainty, precise values for parameters and specific probability distributions for random variables are assumed, as in stochastic models (Wullink et al. 2004; Ebben et al., 2005; Bertsimas and Thiele, 2006; Lusa and Pastor, 2011; Zhen, 2012). Such models have been successfully used in particular situations where an accurate probabilistic description of the randomness is available. In practice, however, the decision-maker often does not have such precise information (Bredstrom et al., 2013), especially when there is a lack of historical data and when the demand refers to new items subject to a challenge in estimating probabilities.

In these cases, robust optimization may be applied as it is well-suited to the limited information available in real-life problems (Bredstrom et al., 2013, Gorissen et al., 2015). According to this approach, the random variables are modeled as uncertain parameters belonging to a convex uncertainty set and the decision-maker protects the system against the worst case within that set. Roughly defined, a robust production plan is the one that remains valid (i.e., feasible or inexpensive to turn into feasible) regardless of the variability resulting from the uncertainty inherent to the production process (Aghezzaf et al., 2010).

Within this context, the objective of this paper is to present a robust optimization reformulation, based on Bertsimas and Sim (2004)'s framework, of a tactical capacity planning model to include information relative to the variability of critical parameters in the planning process. This study, which is detailed in Carvalho et al. (2016), refers to an action research that was realized in a real-world ETO industrial setting. The reformulation proposed incorporates uncertainty relative to the production activities processing times and permits the adjustment of the decision-maker's attitude towards uncertainty. Real data is used to evaluate the behaviour of the proposed robust model for different levels of protection against the degree of conservatism of the plans generated. This robust model aims at solution robustness (or stability) and intends to enhance and support the decision-making process of the studied setting.

The remainder of the paper is organized as follows. Section 2 briefly presents the theoretical background on robust optimization. Section 3 describes the real-world problem studied. In Section 4 a deterministic planning model is presented and Section 5 offers the robust optimization reformulation of this model. Section 6 considers its application in a real-world case and Section 7 presents the conclusions of this study.

## 2 Theoretical background



Production planning related literature is very rich of approaches and models that were proposed to cope with different forms of uncertainty (Aghezzaf et al., 2010). A tactical production planning model which does not integrate the variability of critical parameters in the planning process often results in worthless plans or at the best in plans that must be revised frequently. Moreover, a plan based on incorrect data might be infeasible or achieve poor performance when implemented (Bertsimas and Thiele, 2006, Aghezzaf et al., 2010). Many authors (Herroelen and Leus, 2004; Tolio and Urgo, 2007; Chtourou and Haouari, 2008; Van de Vonder et al., 2008; Deblaere et al., 2011; Alfieri et al., 2012; Artigues et al., 2013; Radke et al., 2013) support that the baseline schedule must be robust and therefore should incorporate a certain degree of variability anticipation. According to these authors, the robustness concept may refer to the solution robustness or stability (i.e., the insensitivity of planned activity start times to schedule disruptions) or to the solution quality robustness (i.e., the insensitivity in terms of the objective function). Lagemann and Meier (2014) highlight that robustness includes both stability (i.e., resilience) and flexibility (i.e., ability to adapt to unforeseen events) and therefore a robust plan should contain a stable basic plan and one or several back up plans that need to work together. Policella et al. (2004) defines that a plan is robust when it can absorb disruptions (external events) without loss of consistency while keeping the pace of execution, whereas Khakdaman et al. (2015) interpret that a robust plan is one that remains valid for a longer time and is insensitive to the effects of uncertainties. Within the field of optimization under uncertainty, stochastic models refer to approaches that require full knowledge of the probabilistic information of the uncertain data (Mulvey et al., 1995; Bredstrom et al., 2013). On the other hand, as full knowledge of probabilistic information is rarely available in practice, robust optimization approaches have received a lot of attention as the uncertainty of the parameters is modeled as lower and upper bounds with no need for exact distributions (Bredstrom et al., 2013, Gorissen et al., 2015). Soyster (1973) gave the first steps in robust optimization by proposing a linear optimization model to construct excessively conservative solutions that were feasible for all data in a given uncertainty set without specifying these distributions. To address Soyster's overconservatism and also to retain the advantages of his linear programming framework, Bertsimas and Sim (2004) propose a robust optimization approach to address data uncertainty that allows the degree of conservatism of the solution to be controlled (i.e., protection is provided for the case where only a pre-specified number of the input coefficients changes from its base value). According to this approach, a parameter  $\Gamma$ , also known as the budget of uncertainty, reflects the decision-maker's attitude towards uncertainty.

Gabrel et al. (2014) offer an overview of recent developments in robust optimization and state that this approach has come to encompass several others to protecting decision-maker against stochastic uncertainty. The robust optimization modeling technique has been successfully applied to different applications concerning production, scheduling, inventory, portfolio management, vehicle routing, among others (Alem and Morabito, 2012). Moreover, Gorissen et al. (2015) provide a guide for practitioners to understand and apply this approach.

### **3 Problem description**

The real-world industrial setting considered in this study is a medium-sized company that produces a wide range of customised and complex equipment, such as high-pressure boilers and sophisticated reactors, based on the ETO policy. These equipment are composed of several part components which must be designed by the company's engineers to be produced. This process may take several months (from 3 to 18 months). In this context, the company's managers generate a midterm production plan by assessing simultaneously demand and the available capacity to accommodate demand overloads through meticulous internal capacity adjustments (i.e., by authorizing overtime or hiring more operators) or subcontracting components. They must decide whether to accept or not a given set of projects considering the workload of already committed orders. The team must also define due dates and calculate the overall production costs.



For planning purposes, the company's demands must include the committed workload (i.e., ongoing projects that have been detailed and started to be produced), new projects (i.e., projects which have deadlines but that have not been started in production neither have been detailed by the design engineers) and the incoming orders (i.e., projects, in the order accept/reject phase, that have not yet specified with exactness a release date or a deadline). A major concern highlighted by the company's managers when generating a plan refers to the variability encountered within the production process. For instance, the new projects and the incoming orders are sensitive to uncertainty referring to the estimated processing times (i.e. since design phase is not concluded when the plan is generated). In one particular situation, a significant delay between the original production plan and the executed production of a specific boiler occurred due to the underestimation of processing times. This occurrence resulted in higher costs and additional time to complete the job.

#### 4 The deterministic planning model

To address the studied problem, Carvalho et al. (2015) developed a deterministic Mixed-Integer Linear Programming model to support tactical capacity planning by optimally balancing demand with the available capacity. It is a cost minimization formulation of an ETO production system that exploits capacity flexibility by considering nonregular capacity alternatives (i.e., overtime, subcontracting and hiring personnel). The model also admits the representation of the production flow with multiple processing stages and overlapping activities. The objective function minimizes the overall variable production costs involving production processing and overtime costs, capacity change costs, personnel payroll and subcontracting costs. This model considers several types of constraints summarized in Table 1.

**Table 1: Types of constraints**

Group	Types of constraints	Description
<b>Production flow related</b>	Release date and deadline	To guarantee that activities are processed within their time windows.
	Non-interruption flow	To establish that once started, an activity must be processed in all subsequent periods until it is finished.
	Maximum and minimum intensities	To ensure that the intensity of an activity can never be less than the minimum intensity nor more than the maximum intensity permitted in a single time period.
	Cadenced flow	To guarantee the activities' variable execution progress mode, their overlapping behaviour and the non-fixed precedence relationship among them.
<b>Capacity-related</b>	Regular, overtime and subcontracting	To guarantee the balance within an activity's processing time and its fractions in terms of regular, overtime and subcontracting hours.
	Maximum number of working hours	To establish the upper bound for internal capacity, which is available in terms of regular working hours and overtime working hours
	Availability of employees	To establish the upper bound of available employees, considering is the capacity changes (hiring and firing) along the time periods
	Committed workload	To ensure that the available capacity can never be more than the already committed workload and the incoming orders allocated in a single time period.
	Minimum employment period	To avoid instability by restricting the capacity changes within the time periods



For the sake of simplicity, this paper only presents the parts of the deterministic model which are key to develop the robust optimization formulation. The complete formulation may be found in Carvalho et al. (2015). As the process uncertainty modelled refers to the production activities processing times, only the workload constraint (1) is detailed here as it is directly affected by this type of variability.

$$\sum_a Q_{aw}x_{at} - \sum_{a|Q_{aw}>0} s_{at} \leq (RHE_{wt} + OHE_{wt})NE_{wt} - WH_{wt} \quad \forall w, \forall t | t \leq FC \quad (1)$$

This constraint ensures that, during the fixed capacity periods, the sum of all activities processed in a given workcenter minus what is subcontracted must be equal to or smaller than the internal capacity minus the already committed workload. During the fixed capacity periods, the internal capacity is fixed and the only nonregular capacity options refers to adopting overtime (i.e., changing capacity levels by hiring and firing personnel is not permitted). Table 2 defines the parameters and variables that compose the workload constraint.

**Table 2 - Parameters and variables in the workload constraint**

Parameters	$FC$	fixed capacity periods
	$NE_{wt}$	number of employees allocated at work centre $w$ in period $t$ within de fixed capacity periods
	$OHE_{wt}$	number of overtime hours per employee per period at work centre $w$
	$Q_{aw}$	processing time of activity $a$ at work centre $w$
	$RHE_{wt}$	number of regular working hours per employee per period at work centre $w$
	$WH_{wt}$	number of hours relative to the workload allocated to work centre $w$ in period $t$
Variables	$x_{at}$	processed fraction of activity $a$ in period $t$
	$s_{at}$	number of subcontracted hours processing activity $a$ in period $t$

## 5 The robust optimization approach

This section describes the robust optimization reformulation of the deterministic model previously presented. The techniques introduced by Bertsimas and Sim (2004) are employed to derive robust plans when production activities processing times are independent and bounded random variables. The motivation for adopting this approach includes the practical advantages over other possibilities found in the academic literature. Three main characteristics of the robust formulation were considered when choosing this approach to address the studied problem:

- It does not require knowledge on the probabilistic distributions of the uncertain data (which is difficult to obtain in the studied setting);
- It preserves the linearity of the original deterministic model;
- It allows the decision maker to control the degree of conservatism of the generated plans.

### 5.1 The robust formulation

The processing time of activity  $a$  at work centre  $w$  is given by the parameter  $Q_{aw}$  where  $QD_{aw}$  represents the maximum possible deviation of the activity processing time from its mean value,  $Q_{aw}$ . In the robust model, each entry  $Q_{aw}$  is represented as a symmetric and bounded random variable  $\tilde{Q}_{aw}$  with unknown probability distribution and with values in the interval  $[Q_{aw} - QD_{aw}, Q_{aw} + QD_{aw}]$ . The subset  $K$  represents the set of coefficients  $Q_{aw}$ ,  $a \in K$ , which are subject to uncertainty. Moreover, the random variable  $\tilde{k}_{aw}$  is the scaled deviation of  $\tilde{Q}_{aw}$  from its nominal value and is defined by  $\tilde{k}_{aw} = (\tilde{Q}_{aw} - Q_{aw})/QD_{aw}$ , belonging thus to the interval  $[-1,1]$ .



The parameter  $\Gamma_{wt}$  is introduced in order to adjust the model robustness against the conservatism of the solution. It is also known as the budget of uncertainty which reflects the decision-maker's attitude towards uncertainty. As  $\Gamma_{wt}$  is an integer in this problem, it is interpreted as the maximum number of the uncertain parameters that can deviate from their nominal values and it may vary from 0 (the deterministic case, when no uncertainty is considered) to  $|K|$  (the worst case, when all deviations assume their highest value). Table 3 presents the additional parameters and variables that are used to develop the robust formulation.

**Table 3 - Additional parameters and variables (adapted from Carvalho et al., 2016)**

$\Gamma_{wt}$	Parameter to adjust the model robustness
$QD_{aw}$	Deviation in the processing time of activity $a$ at work centre $w$
$k_{aw}$	Scaled deviation
$\Pi_{wt}$	Robustness variable
$p_{awt}$	Auxiliary robustness variable

To build the robust counterpart of the model, it is necessary to modify the formulation of the workload constraint (1) to incorporate uncertainty. Therefore, we consider that the sum of the activities durations and their deviation must be equal to or smaller than the internal capacity plus subcontractation minus the already committed workload. This is presented in constraint (2), which refers to a bilevel programming problem. The upper level problem determines the optimal value for variables  $x_{at}^*$  so that the overall costs are minimized. This problem is subject to the worst-case response observed by the random parameters from within the fixed capacity periods.

$$\sum_a Q_{aw}x_{at} + \text{Max}_k \{ \sum_a QD_{aw}x_{at}k_{aw} \mid \sum_a k_{aw} \leq \Gamma_{wt}; k_{aw} \in [0,1] \} - \sum_{a|Q_{aw}>0} s_{at} \leq (RHE_{wt} + OHE_{wt})NE_{wt} - WH_{wt} \quad \forall w, \forall t \mid t \leq FC \quad (2)$$

Applying the robust optimization framework developed by Bertsimas and Sim (2004), an auxiliary problem is formulated (3-5). Its objective is to maximize the sum of all deviations over the set of all admissible realizations of the uncertain parameters, given an optimal decision  $x_{at} = x_{at}^*$ .

$$\text{Max}_k \sum_a QD_{aw}x_{at}^*k_{aw} \quad (3)$$

Subject to

$$\sum_a k_{aw} \leq \Gamma_{wt} \quad (4)$$

$$k_{aw} \leq 1 \quad \forall a \quad (5)$$

If  $\Gamma_{wt} = 0$ , the  $k_{aw}$  for all  $a$  are forced to 0, so that parameters  $\tilde{Q}_{aw}$  are equal to their mean value  $Q_{aw}$  and there is no protection against uncertainty. On the other hand, when  $\Gamma_{wt} = K$ , the  $k_{aw}$  for all  $a$  are forced to 1 (in this particular problem) and constraint (4) is completely protected against uncertainty, which yields a very conservative solution. For values in between 0 and  $K$ , the decision-maker can make a trade-off between the protection level of the constraint and the degree of conservatism of the solution. Following the same rationale of Bertsimas and Sim (2004), the dual of model (3)-(5) is stated next:

$$\text{Min}_{p,\pi} \Gamma_{wt} \pi_{wt} + \sum_a p_{awt} \quad (6)$$

Subject to



$$\pi_{wt} + p_{awt} \geq QD_{aw}x_{at}^* \quad \forall a \quad (7)$$

$$\pi_{wt} \geq 0 \quad (8)$$

$$p_{awt} \geq 0 \quad \forall a \quad (9)$$

This dual problem has two dual variables ( $\pi_{wt}$ ,  $p_{awt}$ ) that are associated to constraints (4) and (5), respectively. By strong duality, as model (3)-(5) is feasible, convex and bounded for all  $\Gamma_{wt} \in [0, |K|]$ , then its dual problem (6)-(9) is also feasible and their optimal objective function values coincide (i.e., no duality gap exists). Substituting (6)-(9) in constraint (2), the robust single-level counterpart is obtained, as presented through constraints (10)-(13).

$$\sum_a Q_{aw}x_{at} + \Gamma_{wt}\pi_{wt} + \sum_a p_{awt} - \sum_{a|Q_{aw}>0} s_{at} \leq (RHE_{wt} + OHE_{wt})NE_{wt} - WH_{wt} \quad \forall w, \forall t | t \leq FC \quad (10)$$

$$\pi_{wt} + p_{awt} \geq QD_{aw}x_{at} \quad \forall w, \forall t | t \leq FC, \forall a \quad (11)$$

$$\pi_{wt} \geq 0 \quad \forall w, \forall t | t \leq FC \quad (12)$$

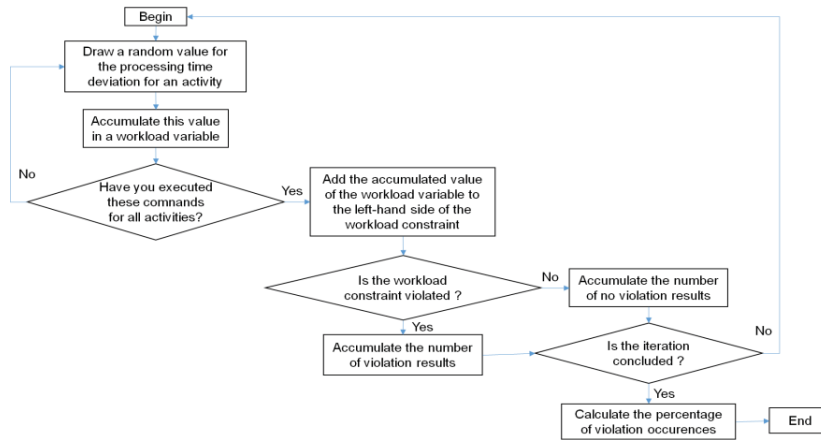
$$p_{awt} \geq 0 \quad \forall w, \forall t | t \leq FC, \forall a \quad (13)$$

Finally, the complete robust optimization model proposed herein consists of the deterministic model with the original workload constraint (1) being replaced by constraints (10) to (13) to consider uncertainty in the production activities processing time. The robust model is detailed in Carvalho et al. (2016). This model minimizes the overall variable production costs and guarantees that if up to  $\Gamma_{wt}$  coefficients change their values within the permitted interval (i.e.,  $[Q_{aw} - QD_{aw}, Q_{aw} + QD_{aw}]$ ), then the solution of the robust optimization model will remain stable. In other words, the solution of this model is a robust solution.

## 5.2 Probability bounds for constraint violation

It is possible to estimate constraint violation probability bounds for the solutions obtained with the robust model. In this research, the theoretical bounds proposed by Bertsimas and Sim (2004) were not adopted for two reasons. The first one refers to the fact these authors' approach refers to a situation where there is a single constraint to be violated as opposed to the studied problem where multiple constraints are being assessed. The second reason is that processing times distributions are often asymmetric (Juan et al., 2014) and Bertsimas and Sim (2004)'s approach assumes symmetrical probability distributions.

In this sense, Monte Carlo simulation was applied to estimate constraint violation probability bounds (see Figure 1). As simulation requires knowledge of the probability distribution on the uncertainty set and this knowledge is unclear, random values for the processing time deviations ( $QD_{aw}$ ) were drawn from two different distributions. In this sense, a normal distribution was first applied. Additionally, to explore the effects of using a nonsymmetrical distribution, random values were generated considering a lognormal distribution (characterized by being non-negative, asymmetrical and skewed rightwards).



**Fig. 1: Monte Carlo simulation process**

Within this simulation process, the assessed plan is executed based on the random values drawn from these distributions. More specifically, the adjusted workload constraint (14) is checked, considering that there is now an extra term referring to the processing time deviation. In this analysis, variables  $x_{at}$  and  $s_{at}$  assume the values from the original assessed plan, whereas  $Q_{aw}$ ,  $RHE_w$ ,  $OHE_w$ ,  $NE_w$  and  $WH_{wt}$  are all parameters.

$$\sum_a Q_{aw} x_{at} + \sum_a QD_{aw} x_{at} - \sum_a s_{at} \leq (RHE_{wt} + OHE_{wt}) NE_{wt} - WH_{wt} \quad (14)$$

This process is repeated for thousands of times and the results of all iterations are aggregated to calculate the percentage of violation occurrences. In this sense, for each assessed production plan (which refers to a specific  $\Gamma$  parameter), it is possible to estimate the probability of constraint violation, that is, the probability of the plan to absorb these deviations within the fixed capacity periods, without amplifying effects to the following periods. This measures the solution robustness of the production plans that are subject to deviations in the processing times.

## 6 Application

This section presents the main findings obtained with the application of the robust optimization model to solve a real-world planning problem. The idea is to evaluate the behaviour of the model for different levels of protection against the constraint violation by varying the robust parameter  $\Gamma$ . The data from the real-world ETO problem are described in detail in Carvalho et al. (2016). The planning horizon consists of 18 periods, where the three initial ones correspond to the fixed capacity periods. Twenty-five projects were considered comprising 250 production activities besides the already committed workload (which corresponds to 89,750 h). The regular and overtime capacities represent 150 h and 25 h per period per employee respectively. There are 284 employees allocated initially for all five workcentres considered. Additionally, historical data on former projects were used to estimate the processing times and the maximum expected deviations in the studied setting. For instance, these deviations refer to approximately 50% of the processing time, which was considered an appropriate value by the company's managers.

The results obtained from the application of the robust optimization approach generated different production plans. It can be noted that under uncertainty, as  $\Gamma$  increases, the model tends to postpone more and more workload from the fixed capacity periods to the flexible capacity periods. In a sense, a capacity buffer is created by explicitly planning "idle" time on a work centre during the fixed capacity periods. That is, time is reserved in case uncertainties occur. In this sense, production is postponed and the increasing need of production capacity in the future periods leads to capacity change costs (i.e.,

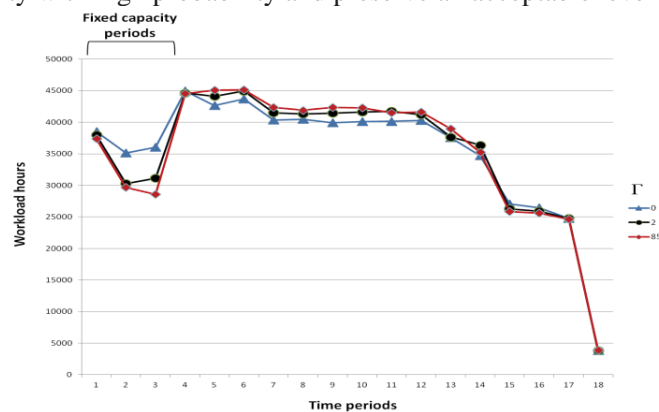




costs related to hiring more personnel). Consequently, the robust solutions are more expensive than the deterministic one ( $\Gamma = 0$ ).

Making a parallel with the knapsack problem, during the fixed capacity planning periods, each work centre's capacity is equivalent to the fixed-size knapsack. The production workload corresponds to the items that are chosen to fill up the knapsack. When considering uncertainty, less workload is allocated to the work centre in a given period as the processing time deviation is also allocated to the work centre. This deviation does not appear explicitly in the production plan, as it is an "idle" time which corresponds to a capacity buffer.

Comparing the production plans presented in Figure 2, in the deterministic solution, the plan does not consider the capacity buffer and the workload is distributed along the periods of time considering only the constraints of the deterministic model. On the other hand, in the fully protected situation ( $\Gamma = 85$ ), the plan is conservative, as it assumes that all 85 activities (among the 250 activities, 85 may be processed within the fixed capacity periods) will be penalized with the maximum processing time deviation. A capacity buffer is therefore dimensioned to address this "extra" demand. Comparing the two situations, the former represents a levelled and smoother distribution of workload while the latter suggests considerable increase of workload from the fixed to the flexible periods. Intermediate values of  $\Gamma$  guarantee stability with high probability and preserve an acceptable level of performance.



**Fig. 2: Workload distributions (for different values of  $\Gamma$ ) along the planning horizon (adapted from Carvalho et al., 2016)**

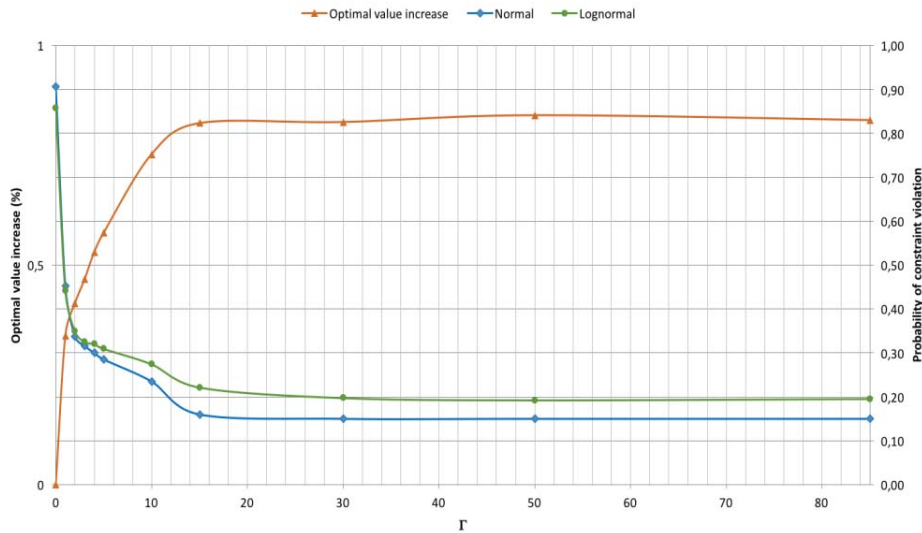
The trade-offs between robustness and the total expected solution cost may also be analysed through the results obtained. Figure 3 displays the optimal value increase in total cost (%) and the probability bound of workload constraint violation (%) as a function of  $\Gamma$ . The probability bounds were calculated through Monte Carlo simulation, as described in Subsection 5.2, where two distinct probability distributions, a normal and a lognormal, were tested.

In the deterministic solution, the minimum cost value is not increased and there is a high probability of constraint violation. A lower probability of constraint violation and a higher cost are obtained with the more conservative values of  $\Gamma$ . In other words, increasing the protection (i.e., increasing the value of  $\Gamma$ ), the probability of constraint violation decreases and the minimum cost value increases. This is known as the *price of the robustness*.

Moreover, the intermediate solutions represent little variation between the two distributions in terms of the probability bound of constraint violation. For both analysed distributions, the probability of constraint violation significantly decreases for  $\Gamma = 15$ . For the maximum protection case ( $\Gamma = 85$ ), there is less chance of constraint violation for the probability distributions analysed. This corresponds to Soyster's approximation of the worst-case scenario where all uncertain parameter assumes its most adverse value. As the processing time deviation is relatively high (i.e., 50% of the processing time) in this problem and as both distributions are supported on infinite intervals, even for high values of



$\Gamma$ , there may be cases where the random values drawn from these distributions represent constraint violation (i.e., the minimum values obtained for the probability of constraint violation do not reach zero).



**Fig. 3: Optimal value increase and probability bound of constraint violation as a function of  $\Gamma$  (adapted from Carvalho et al., 2016)**

## 7 Conclusion

The ETO production system is characterized by high-complex and uncertain situations. In general, in the tactical planning level of ETO organizations, there is a lack of information as projects are gradually detailed along their execution. On the other hand, decisions must be made before complete information is available. To address this issue, a proactive planning approach is adopted to include information about process uncertainty into the tactical capacity plans of a real-world ETO industry setting. More specifically, a robust optimization framework, proposed by Bertsimas and Sim (2004), is used to incorporate the variability relative to the production activities processing times in a tactical capacity planning model. In this study, real data is used to evaluate the behaviour of the proposed robust model for different levels of protection against the degree of conservatism of the solutions generated.

Although this paper provides evidence from a single firm, which limits the extent to which the findings can be generalised, its key contribution is to confirm and provide insights of the applicability of the model for the studied setting. In the first place, the proposed model does not require that the probability distributions of the activities processing times are known. This is crucial in the studied setting which lacks this type of information when planning at the tactical level. In this sense, robust optimization seems to be a suitable approach for developing the proposed solution. Moreover, as this approach is based on linear programming, the model's solution time are small (which is very convenient for practical application) and it can be solved by widely available off-the-shelf software packages. And finally, as the robust formulation permits the adjustment of the decision-maker's attitude towards uncertainty, different planning situations may be addressed. For instance, according to the company's managers, this characteristic permits the adoption of a more conservative standpoint in cases involving innovative projects and a less conservative one in situations where projects are similar to former ones. Furthermore, one relevant insight of the performance of the model when varying the protection level refers to the postponement of production activities. This can be interpreted as a capacity buffer that was dimensioned (i.e., the size of this buffer is optimally dimensioned by the model according to the decision maker's risk aversion behaviour) to protect the plan against uncertainty. For instance, in the ETO context it may be more economical to employ a capacity buffer, rather than to build an inventory



buffer to cope with uncertainty. In this context, it may not be feasible to have an inventory buffer due to all possible combinations of products or to the lack of data on future demand. In this sense, creating this buffer seems to be the suitable measure to provide flexibility in a proactive planning stance. To conclude, this research could be extended in other ways. For instance, the proposed model could be expanded by incorporating other types of uncertainties to assess the effects of additional variability on robust plans. In addition, as the ETO context is subject to different sources of uncertainty, a more rigorous and systematic approach could be developed to measure variability in this context.

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