



## **Modelo de Otimização Fuzzy para a Localização de Ambulâncias Pré-Hospitales na cidade de Cali, Colômbia**

### **Fuzzy Optimization Model for the Location of Prehospital Care Ambulances in the city of Cali Colombia**

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#### **RESUMO**

Em 2015, as autoridades em Cali, Colômbia, relataram mais de 309 mortes em acidentes de trânsito, onde 70% dos casos envolviam motocicletas. Esses números geram preocupação nas autoridades municipais, o que leva ao estudo de alternativas para a melhoria da atenção pré-hospitalar de emergência. Com base na sentença anterior, esta pesquisa apresenta um modelo de localização para determinar o número de ambulâncias necessárias para lidar com as emergências relatadas pela agência de trânsito da cidade e sua localização geográfica. O modelo proposto baseia-se no problema de localização de cobertura máxima esperada (MEXCLP), cujo objetivo é maximizar a demanda satisfeita ponderada pela disponibilidade do serviço, que é calculada através da estimativa global do nível médio de ocupação de cada ambulância, com base em A distribuição geográfica e temporal da demanda histórica. Finalmente, propõe-se o modelo Fuzzy SCLP (Set Covering Location Problem) para determinar o número mínimo de bases necessárias na cidade.

**PALAVRAS CHAVE.** Cuidados pré-hospitales, Ambulância Localização, MEXCLP, SCLP Fuzzy.

**Tópicos:** SA – PO na Área de Saúde, PM – Programação Matemática, SE – PO em Serviços, OA – Outras aplicações em PO.

#### **ABSTRACT**

In 2015, authorities in Cali, Colombia, reported more than 309 deaths in traffic accidents, where 70% of the cases involved motorcycles. These figures generate concern in the city authorities, which leads to the study of alternatives for improving prehospital emergency care. Based on the previous sentence, this research presents a localization model to determine the number of ambulances needed to deal with emergencies reported by the city traffic bureau and its geographical location. The proposed model is based on the expected maximum coverage location



problem (MEXCLP), whose objective is to maximize the satisfied demand weighted by the availability of the service, which is calculated through the global estimate of the average occupancy level for each ambulance, based on the geographical and temporal distribution of historical demand. Finally, the SCLP (Set Covering Location Problem) Fuzzy model is proposed to determine the minimum number of bases required in the city.

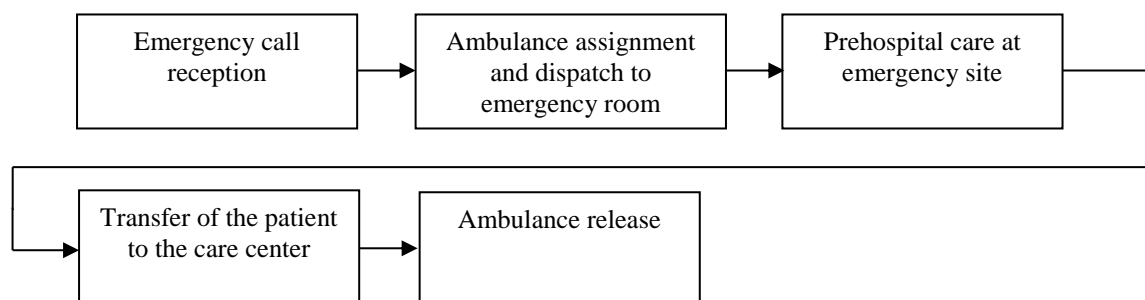
**KEYWORDS.** Prehospital care, Ambulance location, MEXCLP, SCLP Fuzzy.

**Paper topics:** Operation Research in the Health Area, Mathematical Programming, Operation Research in Services, Other Applications in Operation Research.

## 1. Introduction

The emergency medical response service is responsible for providing timely care to the user or patient who is a victim of an illness or accident. In this case, the ambulance is required for the transfer of the patient to the hospital with medical assistance or to provide care in the same place where he has had the accident. The response time is considered as the time from the end of the emergency call to the time the ambulance arrives at the place; while the level of preparedness calculates the ability of the ambulance to provide medical service within the response time [Peleg & Pliskin, 2004]. Medical Emergency Services (EMS) are systems that are responsible for the stabilization and pre-hospital transportation of patients with medical emergency. One of the constant concerns of the SEM is to improve the response time to the occurrence of an event, since this is a very important performance measure to determine the quality of prehospital care of the SEM and for the health of patients [Peleg & Pliskin, 2004]. In addition, some studies have shown that there is a direct relationship between decreased response time and decreased mortality [Sánchez & García, 2010]. Generally, the response time is defined as the time interval between the time the service request is received and the time the SEM vehicle arrives at the scene of the incident [Van Buuren et al, 2012]. Many variables in the operation of the SEM that affect the response time have been identified, and some of them are the location of ambulances.

The Traffic Operations Center in Cali and the Controlling and Emergency Center are responsible for receiving emergency calls. By simply dialing 1-2-3, the emergency call will be received by one of the reception operators in any center. The system identifies the telephone number and displays information on a city digital map. The operator asks basic questions contained in the system about the emergency, to get the necessary information to address the emergency. Once all the information is gathered, the system automatically identifies and sends dispatchers of the center that should address this incident, taking into account the type of emergency and its location. Subsequently the prehospital care is done at the place of occurrence and the patient is moved to the most appropriate care center. Once the emergency has been resolved, the operator is responsible for closing it and storing the information on a database of reported incidents (see Figure 1).



**Fig. 1.** Emergency care process in the city of Cali, Colombia.



## 2. Literature Review

The location of vehicles that are used for the provision of emergency services is important to ensure that the highest number of incidents is dealt with within a set time. There are several models, which are used to solve the problem of ambulance localization. The Location Set Covering Model (LSCM) is one of the first models of graphs proposed for the location of this type of vehicle, which aims to minimize the number of vehicles being required to cover all points of demand [Toregas et al. 1971]. On the other hand, such authors as [Baker & Taylor. 1989], [Eaton et al, 1985] and [Eaton et al. 1986] address the Maximal Covering Location Problem (MCLP), which considers the available number of vehicles to be assigned. In addition, this model improves the formulation of the problem by assigning a relative weight or demand to each demand point, aspect that in the present investigation is considered unlike the other works.

Also, such authors as [Daskin, 1983] use the model MEXCLP (Maximal Expected Covering Location Problem) which includes the calculation of the probability that a vehicle is occupied, using an assumption of independence in the operation of vehicles in the system. Additionally, a heuristic for the location of vehicles is proposed, and the effect of the change in the number of available vehicles, on the coverage of demand, is analyzed. In addition [Sasaki et al, 2010] used genetic algorithms for vehicle location in an EMS, together with a forecast of future demand growth to evaluate the potential location of emergency vehicles under a projected scenario. However, unlike the present work, the availability of the service of a neighborhood or locality is not included when a certain number of ambulances are possessed within a coverage radius. Finally, [Villegas et al. 2012] proposes a mathematical model based on the guidelines of the MEXCLP to find the number of ambulances that are needed to meet the demand of victims of traffic accidents. A second model is used, which combines the elements of the Set Covering Location Problem (SCLP) and the MEXCLP to minimize the number of stations where the ambulances, determined in the previous model, will be located.

There are variations of this model such as the TIMEXCLP, which includes the effect of the variation of the speed of movement throughout the day, and this is evaluated under a simulation environment [Repede, 1994], as well as the adjusted AMEXCLP or MEXCLP [Batta, 1989] in which an adjustment factor is applied to the objective function to take into account the fact that emergency vehicles do not operate independently, but unlike the present work, it does not involve the assignment of a relative weight or demand to each locality or place of attention. Another variation proposed for the SCLM and MCLP models takes into account that more than one vehicle is often required to assist an emergency report at a point of demand [Batta & Mannur, 1990]. This variation is known as MLLSCP or Multi Level Location Set Covering, and this is also discussed in [Church & Gerrard, 2003].

On the other hand, in [Alsalloum & Rand, 2003] and [Alsalloum & Rand, 2007] a programming model by objectives to determine the minimum number of vehicles required for emergency care in Riyadh City, Saudi Arabia, as well as their location, with a probabilistic component in the coverage of demand point is proposed [Pawlak & Skowron, 2007]. Also, [Sasaki et al, 2010] employs genetic algorithms for the current vehicle location in an EMS, together with a forecast of future demand growth to assess the potential location of emergency vehicles under the projected scenario in Niigata prefecture, Japan. However, the latter authors do not consider the calculation of the occupancy of the system, which is one of the most relevant elements of the model in this research.

Other author as [Van der Zee, 2016] addresses the problems of ambulance dispatching and ambulance redeployment, that is, deciding on which ambulance to send to an accident and choosing a base for the ambulance to return to after it finishes service. The goal in these problems is to minimize the fraction of late arrivals. As an alternative to the well-known closest-idle policy,



they propose a dispatching policy that makes a weighted choice between distance to the accident and the coverage an idle ambulance currently provides. However, dynamic models are more recent. They consider the possibility to relocate the ambulances during the day. One of these models is presented by [Jagtenberg et al. 2015] and [Daskin & Stern, 1981].

However, [Hwang et al. 2004] proposed a set-covering model using the concept of fuzzy set theory to define “fuzzy covers”. The proposed fuzzy set-covering model can be reduced to a nonlinear integer programming problem which is easily solvable with modern software. This model is a nature extension of the classical set-covering model, and is able to handle uncertainty.

Also, [Guzmán et al. 2016] provides useful information that helps to identify some opportunities for the application of fuzzy approaches to the covering location problems. In this paper, the literature associated with the covering location problems addressing uncertainty under a fuzzy approach is reviewed. Specifically, the papers related to the most commonly applied models such as set covering location problem, maximal covering location problem, and hub covering location problem are examined.

Likewise, [Leknes et al. 2017] present a new mixed integer model for the problem of location of ambulance stations and the allocation of ambulances to these stations especially suitable for regions with heterogeneous demand and multiple performance measures. The model decides on the location, allocation of stations and ambulances, calculates the service and arrival rates for each station and the probabilities that a call is served by a particular station.

Finally, [Anasari et al. 2017] propose a new approximate hypercube spatial queueing model that allows for multiple servers to be located at the same station as well as multiple servers to be dispatched to a single call. A simulation study validates the accuracy of the queueing approximations. Computational results suggest that the models are effective in evaluating the performance of emergency systems.

### **3. Methodology**

The methodology to be developed is part of the identification of basic information related to the Emergency Medical System connected to the number of traffic accidents that occur in a neighborhood or municipality, maximum coverage distance and the required time of ambulance care, as well as the results of such parameters of interest as the index of quality and occupation of the system.

Subsequently a whole linear programming model is formulated from the given information, and by using a mathematical programming language, the respective model files, data and commands are designed. The programming language used in this research was AMPL (A Mathematical Programming Language). AMPL integrates a modeling language for describing optimization data, variables, objectives, and constraints; a command language for debugging models and analyzing results; and a scripting language for manipulating data and implementing optimization strategies.

Based on the results of the MEXCLP model, we obtain input information for the Fuzzy SCLP model; Where the performance function and the optimal number of ambulances coming from the first model (MEXCLP) becomes a limitation and input information for the second model (Fuzzy SCLP). Finally, an analysis of the results is performed from the NEOS Server for Optimization platform (see figure 2).

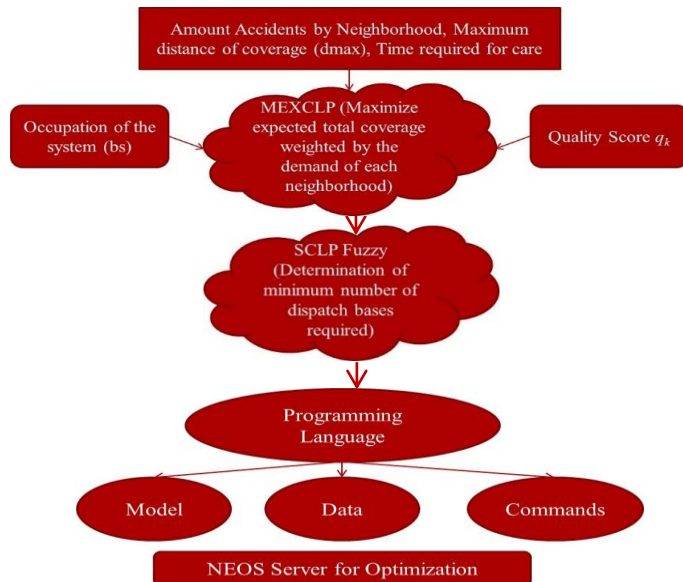


Fig. 2. Application Methodology

#### 4. Mathematical model

For the development of this research, a linear optimization model is used, whose structure is explained below.

##### 4.1 Model Assumptions

The assumptions defined for the construction of the mathematical model were as follows:

- (a) The dispatch bases are known;
- (b) The neighborhoods or areas of attention are known and only those of the city urban perimeter are considered;
- (c) A homogeneous fleet of ambulances is considered, that is to say, that they are equipped with the same resources and that they can address the same types of emergency,
- (c) Relocation of ambulances is not considered and
- (d) The demand of each neighborhood in Cali is given in minutes and is known.

##### 4.1 Main Sets

$BAR$ : Set of Neighborhoods in Cali indexed by  $i$

$AMB$ : Set of Potential Ambulance indexed by  $k$

$BAS$ : Set of Dispatch Bases indexed by  $j$

$BMAX\{BAR\} \subseteq BAS$ : Set of Dispatch Bases that are able to assist the neighborhood within the maximum response time indexed by  $j$

##### 4.2 Parameters

$dem_i$ : Demand from each neighborhood  $i$  and Cali (minutes).

$q_k$ : Index of quality that determines the availability of the service in a neighborhood when it has  $k$  ambulances within the coverage radius.

$K$ : Number  $k$  of ambulances.

$p$ : Number of potential ambulances to locate (Servers).

$p'$ : Number of potential required ambulances (Servers).

$mendeg_{jk}$ : Degree of membership expected by the ambulance  $k$  from the dispatch base  $j$

$mbrt_k$ : Degree of total membership expected by ambulance  $k$



#### 4.4 Decision Variables

$Y_i$ : 1, if the neighborhood  $i \in BAR$  is covered by the ambulance  $k \in AMB$ ; 0 otherwise.

$X_j$ : Number of ambulances located in the dispatch base  $j \in BAS$

$W_j$ : 1, if the dispatch base is located on the site  $j \in BAS$ ; 0 otherwise.

#### 4.5 MEXCLP Model

$$Z_{max} = \sum_{i \in BAR} \sum_{k \in AMB} dem_i * q_k * Y_{ik} \quad (1)$$

Subject to:

$$\sum_{j \in BMAX} X_j \geq \sum_{k \in AMB} K * Y_{ik}; \quad \forall i \in BAR \quad (2)$$

$$\sum_{k \in AMB} Y_{ik} \leq 1; \quad \forall i \in BAR \quad (3)$$

$$\sum_{j \in BAS} X_j \leq p \quad (4)$$

$$X_j \in \mathbb{Z}^+; \quad \forall j \in BAS \quad (5)$$

$$Y_{ik} \in \{0, 1\}; \quad \forall i \in BAR, k \in AMB \quad (6)$$

#### 4.6 SCLP Fuzzy Model

$$Z_{min} = \sum_{j \in BAS} W_j \quad (7)$$

Subject to:

$$\sum_{i \in BAR} \sum_{k \in AMB} dem_i * q_k * Y_{ik} = Z_{min1} \quad (8)$$

$$\sum_{j \in BMAX} X_j \geq \sum_{k \in AMB} K * Y_{ik}; \quad \forall i \in BAR \quad (9)$$

$$\sum_{k \in AMB} Y_{ik} \leq 1; \quad \forall i \in BAR \quad (10)$$

$$\sum_{j \in BAS} X_j = p' \quad (11)$$

$$X_j \leq p' * W_j; \quad \forall j \in BAS \quad (12)$$

$$\sum_{j \in BAS} (W_j * [-\log(1 - memdeg_{jk})]) \geq -\log(1 - mebrt_k) \quad (13)$$

$$W_j \in \{0, 1\}; \quad \forall j \in BAS \quad (14)$$

$$X_j \in \mathbb{Z}^+; \quad \forall j \in BAS \quad (15)$$



$$Y_{ik} \in \{0, 1\}; \forall i \in BAR, k \in AMB \quad (16)$$

In the model MEXCLP, the equation (1) is the performance function whose purpose is to maximize the expected total coverage weighted by the demand of each neighborhood. The constraints (2) and (3) determine the level of availability offered to each neighborhood, where (2) ensures that the binary variable equals 1 when a neighborhood has  $k$  ambulances within a coverage radius and (5) ensures that only one quality index is considered for each neighborhood. Constraint (4) establishes the number of ambulances to be located. Finally, restrictions (5) and (6) represent the logical or non-negativity constraints.

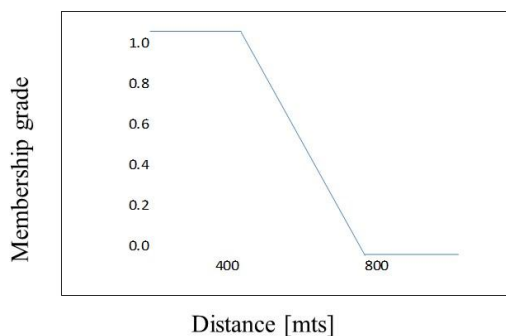
Moreover, the model SCLP Fuzzy, the equation (7) minimize number of dispatch bases required. The constrain (8) guarantees the expected coverage. Constrains (9) and (10) determine the level of availability offered to each neighborhood. Constrain (11) preserves the number of required ambulances. Constrain (12) ensures that only ambulances are assigned to open bases. Constrain (13) is the one that specifies the fuzzy coverage, using a membership function (equation 19) with a minimum coverage radius ( $s$ ) and a maximum coverage radius ( $r$ ). Finally, restrictions (14), (15) and (16) represent the logical or non-negativity constraints.

From the duration of the shift  $t$  and the total aggregate demand, the occupation of the system is estimated, which is one of the most relevant elements of the model, which is given by the following equation:

$$bs = \frac{\text{Total Demand (time)}}{\#\text{servers} * \text{shift duration (time)}} \quad (17)$$

Finally, based on obtaining this value, the quality index  $q_k$  is calculated from the following expression:

$$q_k = 1 - bs^k \quad (18)$$



$$memdeg_{jk} = \begin{cases} 1 & \text{if } d_{jk} \leq s \\ \frac{r - d_{jk}}{r - s} & \text{if } s \leq d_{jk} \leq r \\ 0 & \text{if } d_{jk} \geq r \end{cases} \quad (19)$$

$$s = 400 \text{ mts}$$

$$r = 800 \text{ mts}$$

## 5. Case study

In order to conduct this research, information provided by the city's traffic center in 2015 was taken as a basis. A statistical analysis of the information provided was performed, determining the number of incidents per neighborhood. An important factor to be considered within the model is the time required for addressing events. It was found that more than 75% of the events reported, in which there was record of attention, time is approximately 60 minutes. Based on this information, it was decided to use a time period of 60 minutes per event when determining the care load generated by each neighborhood; there were only available records for 25 of them. Another important factor of the model is the maximum coverage distance ( $d_{max}$ ). In this case, the desired maximum response time was 15 minutes, which implies a maximum distance of 7.50 km when using an average speed of 30 km / h and shift  $t$ , which is equal, in this case, to 8 hours (6:00 a.m. - 2:00 p.m.). Finally, based on historical information, a minimum coverage radius of 400 meters and a maximum coverage radius of 800 meters is used.



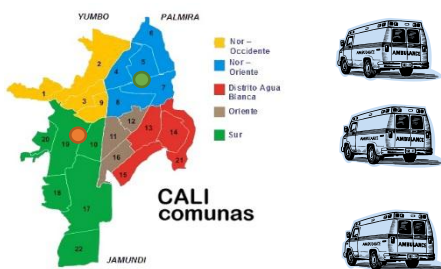
## 6. Analysis of Results

The computational results obtained by the mathematical model, using the computer language AMPL (A Mathematical Programming Language), and using the CPLEX solver of the platform NEOS SERVER for OPTIMIZATION, will be presented and the mathematical model will be forced to decide the number of ambulances that are going to be located in Cali; this issue will allow validation of the results obtained in the optimization instance. Table 1 shows the result achieved for the model, whose optimum result corresponds to the opening of ten ambulances, generating a total coverage of the ambulance network of 1392 units of time (minutes); where the first column of table 1 denotes the total aggregate demand in hours; column 2 is the number of servers or ambulances in the system and the calculation of the occupation and availability of the system in columns 3 and 4 respectively.

**Table 1.** Model ambulance localization results in Cali, Colombia.

Demand (hours)	Number of serves (p)	Occupation (bs)	Availability (qk)
25	4	78,13%	62,75%
25	5	62,50%	90,46%
25	6	52,08%	98,00%
25	7	44,64%	99,65%
25	8	39,06%	99,95%
25	9	34,72%	99,99%
25	10	31,25%	100,00%

The model also suggests a uniform distribution of ten ambulances in two dispatch bases, that is, five ambulances would be assigned to each base.



**Fig. 3.** Location of dispatch bases

## 7. Conclusions and Future Research

In the configuration of the ambulance network, an important result that contributes to the decision-making process is related to the location of ambulances to cover the total demand of each neighborhood or area of attention; thus, generating an optimal decision-making at a strategic level. It can be seen from the results that the location of ambulances plays an important role in the logistic decision-making in the city under study; however, it should be taken into account that this selection is based on the total coverage of the population. Likewise, the diffuse coverage to determine the quantity and location of dispatch bases in the city under study considered as well.

Therefore, for future research, different criteria that support the location of ambulances should be included. Otherwise, it would be important to include restrictions of capacity and availability of vehicle resources and consider different types of ambulances or vehicles to be used in the network. Although it is true that the development of a network of ambulances generates positive impacts, both in the attention of the accident rate, reduction of the mortality rate, among others, these are issues that were not taken into account; if they were to be considered, that could





imply a multiobjective problem. Finally, the best configuration of the ambulance network is obtained with the location of ten vehicles in the city; however, it would be interesting to extend this case study to other neighborhoods in the city and include operational decisions regarding routing of the vehicle care units. Based on the size of the example used in this research, the results were obtained in an efficient computational time. Finally, ambulance relocation and its impact on patient care are suggested for future research.

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