Combining *k*-Opt Improvement Procedure and Tabu-Search to Solve the Symmetric TSP via MILP Formulation

Celso Gustavo Stall Sikora, Adalberto Sato Michels, Thiago Cantos Lopes, Leandro Magatão
Federal University of Technology - Paraná (UTFPR)
Graduate Program in Electrical and Computer Engineering (CPGEI)
Av. Sete de Setembro, 3165, Curitiba-PR, Brasil, 80230-901
sikora@alunos.utfpr.edu.br, amichels@alunos.utfpr.edu.br,
thiagolopes@alunos.utfpr.edu.br, magatao@utfpr.edu.br

ABSTRACT

The symmetric travelling salesman problem (TSP) is a classical and well-known optimisation problem. Given n cities, the salesman travels to each city exactly once by the shortest Hamiltonian cycle in a graph (minimum-length tour). This route can be decided by using exact algorithms or heuristic procedures. Among the solution procedures for exact methods, integer-programming formulations running on modern personal computers are able to solve the majority of randomly generated problems up to 70 cities in a reasonable time. Heuristic procedures are largely used to find near optimal solutions in a reduced computational processing time. Generally, these heuristic procedures are divided into two steps: tour construction and improvement. The first step can be executed by adding the nearest neighbour to the route, by making greedy decisions, or simply constructing the path sequentially for an initial answer. Once the tour has been generated, its improvement might be performed by using 2-opt, 3-opt, or any *k*-opt local searches. Moreover, tabu-searches may be implemented in order to avoid the algorithm to get stuck in a minimum local solution.

Although TSPs have been widely approached in several ways, to the best knowledge of the authors, their tour improvement step has never been addressed by a Mixed-Integer Linear Programming (MILP) formulation that simultaneously takes into account *k*-opt moves and a tabu-search procedure. This paper proposes a local search MILP model and apply it to the TSP. Nonetheless, this formulation can be extended to a broad variety of combinatorial optimisation problems. Firstly, scenarios with 30 to 100 cities are randomly generated (10 of each, resulting in 80 scenarios). Sequential routes (1-2-3-...-n-1) are given to these scenarios as an initial answer and these constructed tours are improved by doing a tabu-search and one *k*-opt move per execution. Each scenario is solved using k = 2, 3, 5, 7, 10 and 15, totalling 480 instances. A universal solver is employed to iteratively solve the MILP generated models.

The total computational processing time is set to 900 seconds and the results for both objective value and CPU time are compared to those that could have been obtained by using the integer programming formulation directly since from the beginning. From 30 to 70 cities, the integer formulation is better suited for most cases. However, on average, 5-opt and 7-opt yield the best solutions for the k values among the proposed formulation: 0.27% and 0.14% of gap, respectively. In the instances of 80 cities, the proposed formulation obtained better solutions or the same solution in a reduced CPU time in 5 out of 10 cases. Lastly, the 100 cities cases' results are inconclusive due to time limit exceeding (900 seconds). Further investigation using tour construction heuristics for the initial answer appears to be essential in order to solve larger instances and make the proposed formulation competitive.

KEYWORDS. Travelling Salesman Problem; Local Search; *k*-opt.

Main Area: OC – Combinatorial Optimisation