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OPTIMAL VALUE-AT-RISK HEDGE WITH DERIVATIVES IN BRAZILIAN FINANCIAL MARKETS USING SIMULATION METHODS

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Abstract

Value-at-Risk has gained acceptance in world financial markets as the most appropriate risk measure. Among the methodologies used to estimate the Value-at-Risk of a portfolio, simulation methods (Historical or Monte Carlo) are the best choices, especially when portfolios with options or instruments with embedded options are considered. However, no portfolio optimization methodology in the finance literature incorporates the Value-at-Risk estimated using simulation methods as its risk measure nowadays. This work proposes a methodology for the computation of optimal Value-at-Risk hedges using simulation methods. The proposal is proved to be easy to understand, implement and use in the daily routine of financial institutions. Two examples from the Brazilian emerging stock and derivative markets are presented to illustrate its practical use.
Key words: Value-at-Risk; Optimal Hedge; Simulation Methods.

Resumo

O Value-at-Risk tornou-se nos mercados financeiros internacionais a medida de riscos de mercado mais utilizada. Dentre as metodologias usadas para estimar o Value-at-Risk de uma carteira, as de simulação (Histórica ou Monte Carlo) são as melhores alternativas, especialemtne quando derivativos com não-linearidades estão presentes, como opções. Entretanto, nenhuma metodologia de otimização de carteiras na literatura de finanças incorpora o Value-at-Risk estimado usando simulação como sua medida de risco. Neste trabalho propomos uma metodologia para a obtenção do hedge que minimiza o Value-at-Risk de uma carteira usando simulação. Esta proposta é fácil de implementar, entender e usar na rotina diária de instituições financeiras. Dois exemplos retirados do mercado brasileiro são apresentados com o propósito de ilustração.
Palavras-chave: Value-at-Risk; Hedge Ótimo; Métodos de Simulação

Introduction

A financial institution acting as a market maker, originator or trader must be careful to prevent losses in its derivatives books. For instance, the hedge of options portfolios may not be a simple task (Boyle and Emanuel [1980], Galai [1983] and Hull and White [1987]). Consider a financial institution that sold exotic options on a stock index. In this case, the option's sensitivity to the underlying (stock index) changes continuously as time passes. Also, the price of the exotic option is sensitive to other factors such as interest rates and the index's volatility. Since the exposures to risk factors such as the underlying stock index, interest rates and the index's volatility change continuously, it is necessary to rebalance the hedge frequently.

Liquidity and the availability of hedging instruments are two other important problems that need to be considered. Take the example of exotics sold on the Mexican stock index Indice de Precios y Cotizaciones (IPC), and assume the writer (of the option) wants to hedge his exposure using only instruments traded in American exchanges to minimize currency, operational and sovereign risks. ADRs on Mexican stocks (available in the New York Stock Exchange), futures on the IPC (available in the Chicago Mercantile Exchange) and options on ADRs on Telefonos de Mexico (available in the American Stock Exchange) can be used to control the delta, gamma, rho and kappa risks of the position. However, few among these instruments are liquid, and can be used in a dynamic hedging strategy: some ADRs and one (or at most two) options on ADRs on Telefonos de Mexico.

There are several approaches to derivatives hedging. Three of these approaches are covered positions, stop-loss strategies and dynamic strategies. Dynamic hedging strategies play an important role when managing the risks of options portfolios. We concentrate on dynamic hedging strategies in this work.

Value-at-Risk has gained acceptance in world financial markets as the most appropriate risk measure. Value-at-Risk measures the worst-case expected loss of a portfolio over a given holding period (say, one day) at a specified confidence level (say, 99%). For instance, a portfolio whose Value-at-Risk is \$10 million over a one-day holding period, with a 99% confidence level, would have only a 1% probability of suffering an overnight loss greater than \$10 million. Among the methodologies used to estimate the Value-at-Risk of a portfolio, simulation methods (historical and Monte Carlo) are the best choices, especially when portfolios with options or instruments with embedded options are analyzed (Jorion [1997]).

Optimization is an important mathematical technique when hedging dynamically options portfolios. However, no portfolio optimization methodology in the finance literature incorporates the Value-at-Risk estimated using simulation methods as its risk measure. This work proposes an optimization methodology for the computation of hedges that minimize the Value-at-Risk estimated using simulation methods. This optimization methodology is an essential part of the dynamic hedging strategy proposed in this work. The use of this dynamic hedging methodology is convenient because:

1) It "weights" the relative importance of the Greek risks (such as the delta risk, the gamma risk, the rho risk and the kappa risk) to achieve the optimal hedge. This allows the hedger to explore in an optimal way the correlation between different Greek risks when obtaining the optimal hedge. Also, since all Greek risks are considered together during the optimization phase, traditional dynamic hedging strategies (such as delta hedging and delta-gamma hedging) are only particular cases of our proposal. This is particularly important when there are not many liquid instruments available for hedging, as in emerging derivatives markets.
2) The Value-at-Risk of the portfolio is minimized using a scenario-based optimization methodology (Koskosidis and Duarte [1997]). There are several advantages when a scenario-based optimization methodology is compared with an analytic-based optimization methodology, such as Markowtiz's Mean-Variance (MV; Markowitz [1959]). One such an advantage is that scenario-based methodologies obviate the use of single-point forecasts for covariance and expected returns, as in the MV methodology. Also, the minimum variance hedging methodology (Johnson [1960]), which is derived from the MV methodology, can be obtained as a very particular case of our proposal, for a very specific choice of parameters (Chamberlain [1993], Epstein [1985], Kallberg and Ziemba [1983]). Another advantage is that any asymmetry of the portfolios' distribution of returns can be handled easily during the optimization phase. A third advantage is that since scenario-based methodologies handle multiple forecasts separately, they allow the hedger to "stress test" his final hedged portfolio during the optimization phase.
3) The Value-at-Risk of the portfolio, as measured by the risk management group, is minimized when obtaining the optimal hedge. The adoption of a hedging methodology that is consistent with the risk measurement methodology used by the risk management group is important because it reduces the chances of violating previously established

Value-at-Risk limits. Also, the proposal helps the integration of risk management and trading systems by establishing a common risk measurement methodology.

4) The hedge of the gamma, kappa and rho risks is very important in illiquid and very volatile markets, as emerging markets. This is illustrated in this work by two examples that use data from Latin American emerging stock and derivative markets.

Simulation Methods

There are basically two methodologies to estimate the Value-at-Risk of a portfolio: analytic and simulation (Jorion [1997]). Among the analytic methods, the two most used possibilities are the delta and the delta-gamma approximations. Among the simulation methods, the two possibilities are the historical and the Monte Carlo methods. For a same portfolio, the use of different methodologies can produce Value-at-Risk estimates substantially different.

Simulation methods combined with full valuation are the most appropriate for the risk analysis of portfolios presenting non-linearities (such as options and instruments with embedded options). A simple illustration is given in Exhibit 1 where the Value-at-Risk (for one week, at a 99% confidence level) are reported for a one-year at-the-money American put option on one futures contract of the São Paulo Stock Exchange Index. (Futures contracts on the São Paulo Stock Exchange Index are negotiated in the Bolsa de Mercadorias e Futuros, São Paulo, Brazil.) Two positions are considered: long the put option and short the put option. Two methodologies for estimating the Value-at-Risk are used: the delta equivalent analytic method, and the Monte Carlo simulation method with full valuation and ten thousand scenarios.

One observes that while the delta equivalent analytic approach provides equal Value-at-Risk estimates for the long and short positions, the Monte Carlo simulation methodology estimates differ substantially. Exhibit 2 illustrates graphically why this is the case for the Monte Carlo simulation methodology: the smoothed estimate for the probability density of the option's expected return is asymmetric. This leads to a substantial difference when estimating the 1% percentile of the distributions in Exhibit 2 (in absolute value), which correspond to their Value-at-Risk at a 99% confidence level. On the other hand, the delta equivalent analytic methodology assumes that the distribution of the put option's expected return presents the same shape as that of its underlying (the futures contract), which is symmetric. The consequence of this modeling hypothesis is illustrated in Exhibit 1: the Value-at-Risk is the same for both positions (long and short). This is unacceptable from the modeling point of view.

The use of portfolio optimization techniques (which include the optimal hedging problem as a particular case) that take into account asymmetries in the expected returns of derivatives portfolios is necessary. The dynamic hedging methodology described in the next section satisfies this requirement.

Optimal Value-at-Risk Hedge

The optimal Value-at-Risk methodology is a dynamic hedging strategy that minimizes the Value-at-Risk of a portfolio at each rebalancing period. That is, for a given initial portfolio and set of hedging instruments, the methodology can be used to rebalance the hedged portfolio on a continuous basis. It updates the optimal number of contracts to be bought/sold according to the latest price fluctuations in the market, minimizing the Value-at-Risk of the hedged portfolio (initial portfolio plus hedging instruments).

The optimization model used in the dynamic hedging methodology is based on the following modeling and operational principles:

1) The use of the optimal Value-at-Risk hedging methodology requires that scenarios be generated for each hedging instrument and for the portfolio. These scenarios should incorporate the latest price fluctuations in the market. They can be generated using

standard simulation methods for Value-at-Risk estimation (Monte Carlo or historical), or incorporate investors' opinions as described in Koskosidis and Duarte [1997].

2) Different hedging instruments (stocks, options, futures, etc.) on different underlying assets can be used for hedging. The model allows the user to require that only round lots be bought/sold. Remember that since odd-lot trading is prohibitively expensive for use in dynamic hedging strategies, especially in emerging markets, the amount of each hedging instrument to be bought/sold should preferably avoid odd-lot trading. As an example we mention that while the estimated cost of a "round trip" (purchase and later sale) of a round lot of stocks by an American institutional investor is about 2.1% in Argentina and 1.7% in Brazil, these costs can easily double (even for liquid stocks) when odd-lot trading is used in these two countries.
3) Since the model is designed to be used continuously during trading hours, it is reliable, user-friendly, computationally efficient and easy to maintain, to avoid operational risk.

The rigorous mathematical formulation of the optimization problem is presented in the Appendix. Also, implementation details important for those interested in reproducing our proposal in their daily routine are discussed in the Appendix.

The next sections present two case studies, which illustrate the use of the optimal Value-at-Risk hedging methodology in practice. The two examples are taken from the Brazilian stock and derivatives markets.

Case Study: Hedging Put Options on Futures Contracts

Two exchanges offer options in Brazil: the Bolsa de Mercadorias & Futuros (stock indexes, foreign exchange and short-term interest rates), and the São Paulo Stock Exchange (stocks).

Among the stock options available in the São Paulo Stock Exchange a couple of years ago, only at-the-money and close-to-expiration options on Telebras PN were liquid and could be used in dynamic hedging strategies. Brazilian risk managers had few liquid options (usually two) available to use in dynamic hedging strategies. Under these circumstances, the hedger had to "weight" the relative importance of the Greek risks. The optimization model given in the appendix allows this "weighting" to be done in an optimal way.

Besides options, futures contracts on the São Paulo Stock Exchange Index are used as hedging instruments in the Brazilian stock market (Duarte and Mendes [2001]). These futures contracts are available for negotiation in the Bolsa de Mercadorias & Futuros. Although six series are always listed at the Bolsa de Mercadorias & Futuros, only those next-to-expire are liquid to be used in a dynamic hedging methodology. Futures contracts are also combined with indexed portfolios in hedging strategies in Brazil in order to exploit arbitrage opportunities between stock and derivatives markets (Duarte [1997]).

In this (and the next) case study we shall use the only three liquid hedging instruments available in Brazilian derivatives markets:

1) Next-to-expire futures contracts on the São Paulo Stock Exchange Index.
2) One at-the-money and close-to-expiration call option on Telebras PN.
3) One at-the-money and close-to-expiration put option on Telebras PN.

The following five characteristics of Brazilian financial markets illustrate why the gamma, the kappa and the rho risks should not be neglected for the period of time under consideration:

1) The Brazilian stock market is much more volatile than more developed stock markets. For example, the GARCH volatility of the São Paulo Stock Exchange Index was consistently larger than the GARCH volatility of the New York Stock Composite Index during the years of 1995, 1996 and 1997; see Exhibit 3.
2) The same is true when the Brazilian fixed-income market is compared to the American and European fixed-income markets. For example, the average GARCH yield volatility of

the one-month rates of Brazilian CDs was approximately eight times larger than its corresponding value for the one-month LIBOR US dollar during the years of 1995, 1996 and 1997. This illustrates the importance of the rho risk for options on local stocks and stock indexes.

3) The implied volatility by liquid Brazilian stock options varies much more than the implied volatility by liquid stock options negotiated in American and European exchanges. Remember that the variation of this implied volatility can be used to measure the kappa risk. The implied volatility of at-the-money liquid calls negotiated in the São Paulo Stock Exchange on Telebras PN are depicted in Exhibit 4. One observes that the implied volatility varies between 20% and 100% for the three years covered in Exhibit 4. This illustrates the importance of considering the kappa risk for options on local stocks and stock indexes.
4) The gamma of plain-vanilla calls/puts is larger for those options at-the-money and close-to-expiration. These are the only liquid options that can be used in dynamic hedging strategies in Brazilian derivatives markets. Ignoring the gamma risk of these options can lead to an underestimation of the market risk of a portfolio with options, as illustrated in Duarte and Maia [1997]. This shows the importance of the gamma risk.

Not considering the Greek risks (other than delta risk) when hedging in the Brazilian stock and derivatives markets results in model risk. We strongly recommend using more sophisticated dynamic hedging strategies, instead of simply delta hedging, when managing the market risk of derivatives portfolios in Brazil.

As a first illustration, let us assume that a financial institution sold one thousand one-year at-the-money American put options on futures contracts on the São Paulo Stock Exchange Index. This is the same option which the distribution of returns is depicted in Exhibit 2. Ten thousand scenarios generated using Monte Carlo simulation with full valuation (as in Exhibit 1) are used. The Value-at-Risk (weekly, 99%) for the unhedged portfolio with these one thousand options is R$ 3,892,360.21; see Exhibit 5.

The results of hedging methodologies using the three hedging instruments mentioned above, separately, are given in Exhibit 5. For example, the Value-at-Risk of the hedged portfolio obtained using only futures contracts and the minimum variance hedging methodology (Johnson [1960]) remains at R$ 1,609,079.27 . Exhibit 5 shows that the optimal Value-at-Risk hedging methodology provides better results when compared to the minimum variance. The optimal Value-at-Risk hedging methodology also provides better results when compared to the results obtained using only one option with the delta hedge methodology.

Although the use of several hedging instruments provides better hedges, one must be careful to select which instruments should be combined. For instance, the combination of futures contracts and the put option on Telebras PN improves substantially the hedge, bringing the Value-at-Risk of the portfolio to only R$ 1,053,836.42, almost one fourth of its initial exposure. However, the inclusion of the call option on Telebras PN presents only a minor effect on the hedge performance, bringing the Value-at-Risk from R$ 1,053,836.42 to R$ 989,310.43.

Since the optimal Value-at-Risk hedging methodology proposed admits a very reliable and efficient implementation from the computational point of view (see Appendix), the hedger can easily experiment with several possibilities in a short time interval to obtain the most suitable combination of hedging instruments in his opinion.

The next example addresses the question of combining hedging instruments for the optimal Value-at-Risk hedge of a portfolio with stocks.

Case Study: Hedging a Portfolio with Stocks

For the same period of time of the previous example, a R$ 60 million portfolio composed by six liquid Brazilian stocks is considered in this case study; see Exhibit 6. The same three hedging instruments considered in the previous case study are used. The Monte

Carlo simulation methodology is used to generate ten thousand scenarios for this case study. A graphical representation of the portfolio's market exposure reduction, measured using the Value-at-Risk (weekly, 95%), using different combinations of hedging instruments, is depicted in Exhibit 7. This case study illustrates how the hedging methodology proposed can be used to select the most satisfying combination of hedging instruments.

The unhedged portfolio (only stocks) presents a Value-at-Risk of R$ 5,330,744.00 . The optimal Value-at-Risk methodology using futures contracts on the São Paulo Stock Exchange Index produces the most significant reduction in the portfolio's market exposure when using only one hedging instrument: a Value-at-Risk of R$ 814,612.00 . Combining hedging instruments pairwise produces another substantial reduction in the portfolio's market exposure: in the best case, the Value-at-Risk is brought down to R$ 527,767.00, when futures contracts are combined with at-the-money put options on Telebras PN. Finally, the use of all three hedging instruments together reduces the portfolio's Value-at-Risk only marginally to R$ 479,810.00 . Once more including at-the-money call options on Telebras PN does not produce much better results than using only futures contracts and at-the-money put options on Telebras PN.

Choosing the most satisfying combination of hedging instruments can be very easily and efficiently done using the optimization model given in the appendix, as illustrated in Exhibit 7. For instance, it took less than two minutes to generate all the results in Exhibit 7, as explained in the Appendix.

Conclusion

We presented a dynamic hedging methodology that minimizes the Value-at-Risk of portfolios using simulation techniques. This methodology is based on a reliable, computationally efficient, easy to implement and to use optimization framework. Two examples from the Brazilian stock and derivatives markets were presented to illustrate its practical use.

The methodology outlined is particularly useful when hedging portfolios in illiquid and very volatile markets, such as emerging derivatives markets. For those professionals working in these markets, we mention that this methodology is quite general in order to cover the most important market risks present in their derivatives portfolios, but also easy to implement in practice, provided an optimization software for mixed integer programming probems is available.

The methodology has not been developed to work under stress situations. However, since it relies on simulation methods incorporating the current market information into the methodology through the use of scenarios, it can be adapted to stress situations: it is enough in these cases to properly generate stress scenarios, using the latest for the purpose of hedging. It is straightforward to observe from the Appendix that this brings no extra computational cost for the methodology when used in practice. Also, if the hedger desires to incorporate his opinion in the scenarios in order to influence the optimal hedge, it is suffcient the generate scenarios in accordance to this last interest.

Our objective when presenting thetwo examples was to motivate the use of our proposal, showing that it can be made operational easily. The use of a larger number of derivatives, other derivatives (such as exotic options or strutured produtcs), multiperiod hedging strategies etc., although possible, will not add any other information for the interested practitioner than that shown by our two numerical examples. Customizing the methodology for the particular needs of a hedger is out of the scope of this article, although it is not difficult.

Appendix

The simplest possible formulation for the scenario-based optimization model that minimizes the Value-at-Risk is

$$
\begin{aligned}
&\textit{Minimize} \quad && |V| \\
&\textit{Subject to:} && R_j = \Delta p_j + \sum_{i=1}^{n} \Delta a_{ij} H_i \quad \forall\, j = 1,2,\ldots,m \\
& && V \le R_j + \Omega F_j \quad \forall\, j = 1,2,\ldots,m \\
& && \sum_{j=1}^{m} \omega_j F_j \le \alpha \\
& && V, R_1, R_2, \ldots, R_m \in \Re \\
& && H_1, H_2, \ldots, H_n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\} \\
& && F_1, F_2, \ldots, F_m \in \{0, 1\}
\end{aligned}
\tag{A1}
$$

This model assumes that:

1) m scenarios are available.
2) The expected return (profit or loss) of the portfolio (to be hedged) according to the j^{th} scenario is denoted by Δp_j.
3) There are n hedging instruments available.
4) The expected return (profit or loss) of one round lot of the i^{th} hedging instrument according to the j^{th} scenario is denoted by Δa_{ij}.
5) The optimal number of round lots to be bought/sold of the i^{th} hedging instrument is denoted by H_i.
6) In the model, R_j denotes the expected return of the hedged portfolio (initial portfolio plus all hedging instruments bought/sold) according to the j^{th} scenario.
7) The parameter Ω should be set to a very "large" number (for instance, in the two numerical illustrations given it was equal to 10^{10}), playing an equivalent role to the "Big-M" of optimization models (Luenber-j6.157m0/e12[(e.)]TJ0.0009 Tc 00088 Tw -0.5254 -1.245 Td[8

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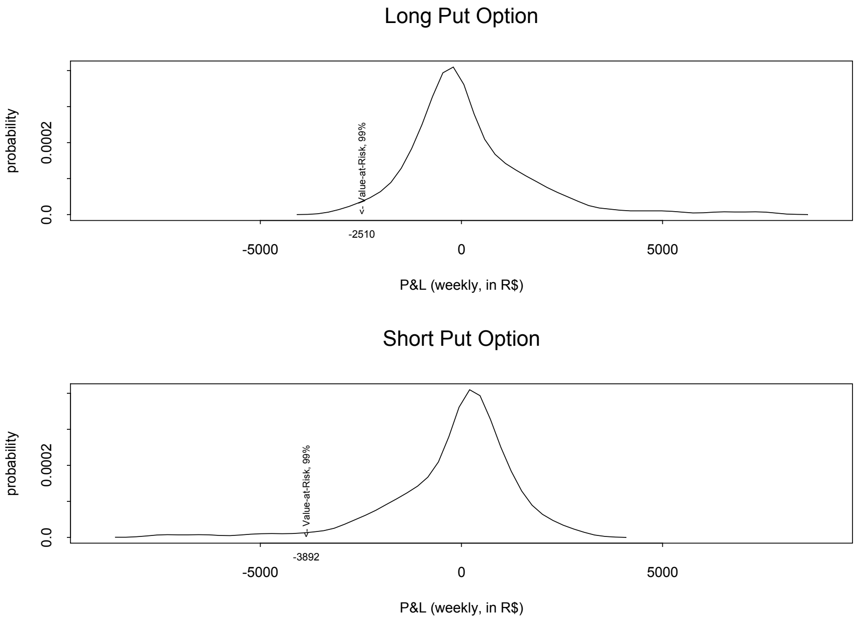
Exhibit 1. Value-at-Risk for Two Positions Using Two Methodologies

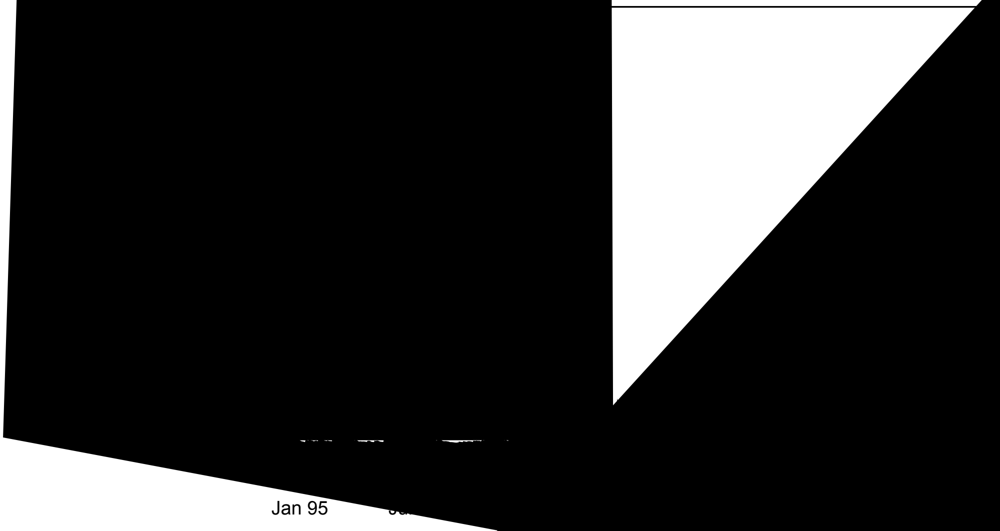
Position	Methodology	Value-at-Risk (weekly, 99%) *
Long Put Option	Delta Equivalent Analytic	R$ 3,177.60
Short Put Option	Delta Equivalent Analytic	R$ 3,177.60
Long Put Option	Monte Carlo Simulation	R$ 2,510.42
Short Put Option	Monte Carlo Simulation	R$ 3,892.36

* The Brazilian currency is the Real, R$. The exchange rate was 1.15 R$/US$ in this day.

Exhibit 2. Histogram of the P&L of Two Positions



[redacted] of Two Stock Indexes



[redacted]

Exhibit 5. Optimal Value-at-Risk Hedge of One Thousand Put Options on Futures Contracts

Hedging Methodology	Hedging Instruments	Value-at-Risk (weekly, 99%)
None	None	R$ 3,892,360.21
Minimum Variance	Futures Contracts	R$ 1,609,079.27
Optimal Value-at-Risk	Futures Contracts	R$ 1,574,832.14
Delta Hedge	Call Option on Telebras PN	R$ 2,781,513.24
Optimal Value-at-Risk	Call Option on Telebras PN	R$ 2,439,920.03
Delta Hedge	Put Option on Telebras PN	R$ 1,905,989.51
Optimal Value-at-Risk	Put Option on Telebras PN	R$ 1,772,793.55
Optimal Value-at-Risk	Futures Contracts and Put Option on Telebras PN	R$ 1,053,836.42
Optimal Value-at-Risk	Futures Contracts, Put Option on Telebras PN and Call Option on Telebras PN	R$ 989,310.43

Exhibit 6. Portfolio in the Brazilian Stock Market

Stocks	Portfolio Composition
Telebras PN	40%
Petrobras PN	20%
Vale PN	10%
Eletrobras PNB	10%
Cemig PN	10%
Usiminas PN	10%

Exhibit 7. Effects of Combining Different Derivatives for the Optimal Value-at-Risk Hedge