



OPTIMAL PORTFOLIO STRUCTURING USING ROBUST STATISTICS

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Abstract

Emerging markets are known to have unique characteristics when compared to more developed markets. The direct use of standard mathematical models proposed and tested in more developed markets is not always recommended in emerging markets. Extreme events in emerging markets have already been verified to distort the results obtained when using standard mathematical models in several situations, including optimal portfolio structuring. One of the reasons for that is that extreme events and/or economic discontinuities (such as the Brazilian and the Argentinean devaluation crises) modify the financial environment in such a way that past data become of little use looking forward. In this work we concentrate on proposing a methodology to handle extreme events. Numerical examples taken from the Latin American stock markets are used to illustrate our proposal.

Key words: Emerging Markets; Portfolio Optimization; Robust Statistics.

Resumo

As características singulares dos mercados emergentes são bem conhecidas quando comparadas às dos mercados mais desenvolvidos. O uso de modelos matemáticos, propostos e testados nos mercados mais desenvolvidos, nem sempre pode ser recomendado para os mercados emergentes. Eventos extremos nos mercados emergentes distorcem os resultados obtidos usando modelos matemáticos em diversas situações, incluindo a otimização de carteiras de investimentos. Uma das razões para tal é que eventos extremos modificam os dados substancialmente (como após as desvalorizações cambiais no Brasil e Argentina no passado recente). Neste trabalho nos concentramos em uma proposta para otimizar carteiras quando da presença de eventos extremos. Exemplos numéricos retirados de mercados acionários emergentes são utilizados para ilustrar o potencial da proposta apresentada.

Palavras Chave: Mercados Emergentes; Otimização de Carteiras; Estatística Robusta.

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1. Introduction

Standard models for optimal portfolio structuring have been criticized because of their very restrictive modeling hypotheses (Chopra [1993], Gardner and Wuilloud [1995] and Michaud [1989] among others). Following that, several modeling improvements have been proposed, including the use of scenario analysis (Nemhauser *et al.* [1989]), network structures (Crum *et al.* [1979]), more general utility functions (Kallberg and Ziemba [1983]), incorporating investor's expectations (Black and Litterman [1991], Koskosidis and Duarte [1997]), asymmetric risk measures (Markowitz [1959], Harlow [1993]), large-scale models (Carino *et al.* [1994], Hiller and Eckstein [1993]) etc.

Emerging markets are known to have unique characteristics when compared to more developed markets. The direct use of the standard mathematical models proposed and tested in more developed markets is not always recommended in emerging markets. For example, extreme events (Embrechts *et al.* [1997]) in emerging markets have already been verified to completely distort the results obtained when using standard mathematical models (that is, those used by practitioners in more developed markets) in several situations, such as hedging using futures contracts (Duarte and Mendes [1998]), systematic risk estimation (Duarte and Mendes [1997]), Value-at-Risk computation (Duarte [1997]), volatility estimation (Mendes and Duarte [1999]), currency overlay (Duarte and Rajagopal [1999]), estimating term structures of interest rates (Almeida *et al.* [1998]), and optimal portfolio structuring (Reyna *et al.* [1999b]).

Practitioners working in the asset management industry in emerging markets have not yet incorporated optimization models in their routine. One of the reasons is that extreme events and/or economic discontinuities (such as the Real Economic Plan, the Mexican Crisis, the Brazilian and the Argentinean devaluation crises etc.) modify the financial environment in such a way that past data become of little use looking forward. Portfolio optimization models must allow these changes to be easily incorporated in the methodology or, otherwise, they will remain outside the routine of local practitioners. For instance, see Reyna *et al.* [1999a] for a methodology to incorporate macroeconomic discontinuities in a portfolio optimization methodology with an application to international diversification in Latin American emerging markets.

In this work we propose a methodology to handle extreme events based on the theory of robust statistics (Huber [1981]). To that end, we review in Section 2 some basic concepts from robust statistical theory, prove the main results needed throughout the article, and verify that the two most used portfolio optimization models by practitioners in international financial markets do not provide robust optimized portfolios. Section 3 describes portfolio optimization models that can provide robust optimized portfolios, addressing an important practical question: the computational implementation of these proposals. Section 4 provides two numerical examples from the Brazilian stock markets and one considering an international emergent market portfolio. Our conclusions are presented at Section 5.

2. Robust Statistics Applied to Portfolio Optimization

Definition 1: The General Asset Management Model (GAMM) is defined in this work as

$$\begin{aligned}
 & \text{Maximize} && U(w) \\
 & \text{subject to:} && w = Rx \\
 & && e^T x = c_0 \\
 & && x \in \mathfrak{R}_+^n \\
 & && w \in \mathfrak{R}^m
 \end{aligned} \tag{1}$$

where n is the number of assets, m is the number of scenarios, $U(\cdot)$ is an utility function, x represents the composition of the portfolio, w is the wealth obtained under each scenario, c_0 is the initial wealth available for investing, $R = R^* + E$ (where R^* is a $m \times n$ matrix of returns for all m scenarios and each of the n assets under consideration, and E is a $m \times n$ matrix of 1's), and $e \in \mathfrak{R}^n$ is a vector of 1's.

Definition 2: If x^* is an optimal solution for the GAMM, it represents an optimal portfolio composition.

Definition 3: A Statistical Utility Function (SUF) is any utility function which can be put as a linear combination of real valued statistical estimators in the most parsimonious form. A SUF can be written as

$$U(\cdot) = \sum_{k=1}^l \beta_k T_k(\cdot) \quad (2)$$

where $\beta_k \in \mathfrak{R}$ and $T_k(\cdot)$ is a real-valued statistical estimator.

Definition 4. Given a sample $y = (y_1, y_2, \dots, y_m)$, we define a contaminated sample $y' = (y'_1, y'_2, \dots, y'_m)$ as a sample obtained by replacing any set of p (with $p < m$) observations in y by arbitrary values.

Definition 5. We define the maximum bias that can be caused by a contaminated sample on a real valued estimator $T(\cdot)$ as

$$b(p, T, y) = \sup_{y'} \|T(\underline{y}') - T(\underline{y})\| \quad (3)$$

where $\|\cdot\|$ is a norm, the “sup” is taken over all contaminated samples, and p is the number of contaminating values.

Definition 6. The finite sample replacement breakdown point is defined as

$$\varepsilon_m^*(T, \underline{y}) = \min_p \{b(p, T, \underline{y}) = \infty\} \quad (4)$$

where m is the sample size and p is the number of contaminating values.

The definition above of finite sample replacement breakdown point was given in Donoho and Huber [1983]. They define the breakdown point as the minimum proportion of observations of a sample that, if replaced by arbitrary values, can make the maximum bias of $T(\cdot)$ to be infinity, or $T(\cdot)$ not to exist.

Definition 7. Given $T(\cdot)$ and y' , we name as the Least Favorable Sample (LFS) as any sample y for which $b(p, T, y) = \infty$ for a given p .

Theorem 1. The breakdown point of a SUF equals the smallest breakdown point among all estimators composing the SUF. Proof: See Reyna [2001].

Definition 8. Given a random variable Y , with distribution F_Y , we define a contaminating process for Y as the process

$$Y(\varepsilon, H) \sim (1 - \varepsilon)F_Y + \varepsilon F_H \quad (5)$$

where F_H is the distribution of an arbitrary random variable H , and $\varepsilon \in (0,1)$.

For the purposes of this work one can consider the existence of an atypical observation for a financial time series as a contaminating process. That is, the existence of an atypical observation for the return (gain or loss) of an asset can be regarded as the process of replacing a typical observation by an arbitrary value.

At this point it is important to observe that the contaminating process is related to the elements of the scenario matrix R^* , but the result proved in Theorem 1 depends on the contamination of the elements in w . It is straightforward to verify that a contamination process for w is generated from a contaminating process for the columns of the scenario matrix R^* for any feasible solution of model (1).

As a consequence, when using a GAMM for structuring an optimal portfolio, if the SUF used is not robust, and if the scenarios used for the returns of the assets under consideration present outliers, the resulting optimal portfolio composition may be distorted by atypical observations.

Definition 9. A real portfolio is a portfolio obtained from a GAMM with a non-contaminated sample.

Definition 10. The robust optimal portfolio is a portfolio obtained from a GAMM with a SUF with breakdown point greater than the contamination proportion existing in w .

Based on the theoretical structure defined above, we can prove that the optimized portfolio obtained from the Mean-Variance model (Markowitz [1959]) is not a robust portfolio. The Mean-variance model is obtained by substituting in (1)

$$U(w) = \frac{1}{m} \sum_{i=1}^m w_i - \frac{\lambda}{m} \sum_{i=1}^m \left(w_i - \frac{\sum_{i=1}^m w_i}{m} \right)^2 \quad (6)$$

where all scenarios are assumed to have equal probability ($= \frac{1}{m}$), w_i is the wealth under scenario i , and λ is the investor's risk aversion parameter.

Corollary 1. The optimal portfolio obtained from the Mean-Variance model is not a robust portfolio. Proof: See Reyna [2001].

The GAMM is being increasingly used by practitioners in international financial markets. Usually, the difference between the models is related only to the SUF chosen. One of the most used SUF is

$$U(w) = \frac{1}{m} \sum_{i=1}^m w_i - \frac{\lambda}{m} \sum_{i=1}^m \max\{0; c_0(1 + \text{MAR}) - w_i\} \quad (7)$$

where MAR is the investor's Minimal Acceptable Return in a downside risk portfolio optimization framework, as proposed by Harlow [1993], and all other parameters/variable have already been defined. The Mean-Downside Risk model is obtained by substituting (7) in (1).

Corollary 2. The optimal portfolio obtained from the Mean-Downside Risk model is not a robust portfolio. Proof: See Reyna [2001].

That is, the two of the most used portfolio optimization models by practitioners in international financial markets — the Mean-Variance model and the Mean-Downside Risk model — do not provide robust optimal portfolios. We need to search for a framework that allows us to “generate” portfolio optimization models which can provide robust optimal portfolios.

3. The GAMM and its Robust Version

Although we do not intend to present the most comprehensive framework for portfolio optimization models that can provide robust optimal portfolios in this article, Theorem 1 helps us in this task: in order to propose a GAMM which can provide robust optimal portfolios we need to modify the SUF, using to that end only statistical estimators that possess high breakdown points (for instance, 50%).

For instance, one can use a SUF such as

$$U(w) = \text{median}_{1 \leq i \leq m} \{w_i\} - \lambda \text{median}_{1 \leq i \leq m} \{\max\{0; c_0(1 + \text{MAR}) - w_i\}\} \quad (8)$$

We define a GAMM with a SUF such as (8) as the Median-Median Downside Risk (MMDR) model. This is an interesting model, as verified next.

Corollary 3. The optimal portfolio obtained from the MMDR model is a robust portfolio. Proof: See Reyna [2001].

3.1 Computational Implementation

A difficult problem in robust statistics estimation, and in particular in robust regression, is that of computing estimators based on minimization of the k^{th} smallest value of absolute residuals, because of their combinatorial complexity (Bocvek and Lachout [1995]). It can be verified that the MMDR model is a specific case of this kind of problems.

The MMDR can be formulated in a Mixed Integer Programming framework as

$$\begin{aligned}
 & \text{Maximize} && w_{\text{median}} - \lambda d_{\text{median}} \\
 & \text{subject to :} && w_{\text{median}} < w_i + M s_i \quad \forall i = 1, 2, \dots, m \\
 & && w_{\text{median}} > w_i - M + M s_i \quad \forall i = 1, 2, \dots, m \\
 & && d_{\text{median}} < d_i^- + M z_i \quad \forall i = 1, 2, \dots, m \\
 & && d_{\text{median}} > d_i^+ - M + M z_i \quad \forall i = 1, 2, \dots, m \\
 & && d^- + w = d^+ + c_0(1 + \text{MAR})e \\
 & && e^T s < \frac{m}{2} \\
 & && e^T z > \frac{(m+1)}{2} \\
 & && w = Rx \\
 & && e^T x = c_0 \\
 & && x_j \geq 0 \quad \forall j = 1, 2, \dots, n \\
 & && w_i, d_i^+, d_i^- \geq 0 \quad \forall i = 1, 2, \dots, m \\
 & && s_i, z_i \in \{0; 1\} \quad \forall i = 1, 2, \dots, m \\
 & && w_{\text{median}}, d_{\text{median}} \in \mathfrak{R} \tag{9}
 \end{aligned}$$

where w_{median} represents the median of the wealth obtained under the hypotheses of each scenarios, d_{median} represents the median of the downside risk measure, s, z, d^+ and d^- are all auxiliary variables, M is a “big constant” chosen in accordance to the “Big M ” method (Luenberger [1984]), and all other parameters/variables have been defined before.

The idea behind the formulation in (9) is that the process of calculating the median of a sample is equivalent to sorting out the sample and getting the value that is in the middle of the sorted sample. This means that we will have fifty percent of the sample below and fifty percent above the median: that is why we need the binary variables s and z , in order to “control” the proportion of the sample below and above w_{median} and d_{median} .

From the optimization point of view, model (9) is a difficult problem to solve, because of its computational complexity. This complexity is directly related to the number of scenarios (m) and assets (n) used in the optimization process: the larger these values, the more difficult it becomes to obtain the robust optimized portfolio. For implementation details please refer to Reyna [2001].

4. Numerical Examples

In this section we provide three numerical examples:

- a) The first example is based on the hypothesis that we know the true probability distribution function of all assets’ returns, and we use a contamination process for this distribution.
- b) The second example uses historical data, with a specific atypical event. The objective is to “simulate” the real world and to illustrate the usefulness of our proposal in practice.
- c) The third example considers the task of building a portfolio in global emerging stock markets.

In the first two numerical experiments we use daily data from the Brazilian equity market from December 1st, 1998, to May 7th, 1999. This period was chosen because it covers the Brazilian currency devaluation crisis of January, 1999. The assets considered for the analyses were the ten most traded stocks in the São Paulo Stock Exchange during that period of time (presented below in alphabetical order):

- a) Bradesco PN.
- b) Banco do Brasil PN.
- c) Cemig PN.
- d) Eletrobrás ON.

- e) Eletrobrás PNB.
- f) Petrobrás PN.
- g) Telebrás ON.
- h) Telebrás PN.
- i) Telesp PN.
- j) CVRD PNA.

The riskless asset is represented by local short-term interest rates.

In the third numerical example, we used monthly data from five emerging stock markets:

- a) Brazil.
- b) Argentina.
- c) Mexico.
- d) Indian.
- e) Hong Kong.

The data used covers a period of time that goes from 1997 to 2004.

4.1 The First Numerical Example

The main assumption in this first example is that we know the probability distribution of all assets considered for investing: a Multivariate Normal distribution. This family of distributions needs only two parameters for its full characterization: the mean vector (μ) and the covariance matrix (Σ). For estimating these parameters we use all historical data available. We also rely on a robust estimation technique: the Minimum Covariance Determinant (Rousseeuw and Zomeren [1990]). This procedure provides the robust estimators for μ and Σ .

4.1.1 Scenario Generation

With the estimates just obtained we generate a two-month sample (that is, forty-two business days) of returns for all assets under consideration using a Monte Carlo method. We generate one hundred sets of two-months samples, which we refer to from now on as “clean scenarios.” At the end of the scenario generation process we end up with one hundred “clean scenarios.”

4.1.2 Contaminating Process

The next step consists of contaminating the “clean scenarios” using outliers. To that end, our choice was to use the outliers found in the historical data. These outliers can be identified using a robust distance concept (Rousseeuw [1984]). According to this distance these historical outliers are ordered and introduced in the “clean scenarios” according to the degree of contamination desired. In the numerical results presented below the samples contained 5%, 25%, 40% or 50% of contamination. After the contamination the “clean scenarios” become the “contaminated scenarios.”

4.1.3 Models

We compare two portfolio optimization models:

- a) The first one is the Mean-Downside Risk model, mentioned before. We shall refer to this model as the “classical model.”
- b) The second one is the MMDR model, presented above. We shall refer to this model as the “robust model.”

In both cases we set $\lambda = 1$.

4.1.4 Steps

The two steps of this first numerical experiment are:

- a) To run both models for the 100 contaminated samples for all different levels of contamination (that is, those with 5%, 25%, 40% and 50% of outliers).
- b) To compare the portfolios’ optimal compositions obtained for different samples, assessing the effect of the contamination proportion on the two models.

4.1.5 Results

Figure 1 and Figure 2 compare the portfolio allocation in one of the stocks considered (Cemig) for each of the one hundred 5% “contaminated scenarios,” when using the two models mentioned above. Figure 3 and Figure 4 provide the similar results for each of the one hundred 25% “contaminated scenarios.”

Clearly, while in the case of the “classical model” the optimal allocation varies arbitrarily between 0% and 100% for the different scenarios used, in the case of the “robust portfolio” it remains fairly stable for the two levels of contamination depicted. Also notice that the allocation provided by the “robust model” varies slightly more in the case of the 25% “contaminated scenarios” when compared to the 5% “contaminated scenarios.”

We now observe the effect of higher contamination proportions. Figure 5 shows for a randomly chosen sample, for the “classical model,” the behavior of the optimal allocation for two stocks (Cemig and CVRD) as the percentage of contamination increases (from 0%, to 5%, to 25%, to 40%, and finally to 50%). Figure 6 shows the corresponding result obtained when using the “robust model,” instead of the “classical model.”

We can note a more stable pattern of the optimal weights obtained from the “robust model” when compared to the “classical model” as the contamination proportion increases to the value of the breakdown point (that is, 50%).

The six figures exhibited so far clearly show that the optimal allocation obtained from the “classical model” varies much more than those obtained from the “robust portfolio” as the contamination proportion increases. As a consequence, we can expect the optimal allocation proposed by a “robust model” to remain more stable during/following extreme events. Remember that this was exactly our objective when proposing this new approach to optimal portfolio structuring.

4.2 The Second Numerical Experiment

Using the data available, in this experiment a window of 42 days is used to define the historical scenarios. This window is shifted every day so that, at each step, a new sample is obtained by the incorporation of a new observation, and the withdraw of the oldest one. A total of 40 samples were generated.

We chose the initial sample to contain the days corresponding to the Brazilian currency devaluation. The objective of this numerical example is to understand the practical effect of atypical events in the optimal allocations proposed by the two models.

For all 40 windows we optimize both models. At each step, the next-day returns (thus, out of the sample) are used to obtain the marked-to-the-market returns for the optimal portfolios proposed by the “robust model” and by the “classical model.” By moving the window, and following the portfolios’ recommendations for the optimal allocation, a set of 40 out-of-sample portfolios returns were obtained.

4.2.1 Results

Figure 7 depicts the out-of-sample returns obtained for the two models. The first comment is that the returns posted by the two models are not much different, except for the period around the end of March, 1999: that is the period when the effects of the Brazilian devaluation start leaving the historical scenarios used for the optimizations. That is, around the end of March, 1999, almost two months after the devaluation, the optimal allocations provided by the “robust model” tend to change smoothly, this not being the case for the “classical model”. As a consequence, since the stock markets returns during that period remained fairly stable, the returns posted by the “robust model” change smoothly too, this not being the case, again, for the “classical model.” These comments corroborate the results obtained in the first numerical example.

Finally, in Table 1 we observe that the mean and the accumulated out-of-sample returns obtained with the “robust model” were higher than the ones obtained with the “classical model,” for basically the same level of out-of-sample volatility. These results illustrate a better ex-post performance for the “robust model” when compared to the “classical model.”

4.3 The Third Numerical Example

As in the two previous cases, we relied on historical scenarios in this numerical example. We used a window of three years to generate monthly-return scenarios to the models. The two models (“robust” and “classical”) were run and evaluated out-the-sample, at the end of each month, when rebalancing was allowed. This way, we tried to replicate the decision making process on an institutional investor as time passes, allowing changes in the portfolios on at the end of each month.

Figure 8 exhibits the out-the-sample results for both models. We observed that the results obtained for the robust portfolio proved to be slightly worse when the market was up, but were much better when the market was down. A simple calculation pointed out that the accumulated return obtained with the “robust model” was higher than the one obtained with the “classical model” (36% against 19%), and that the standard deviation of returns for the “robust model” was lower when compared to that of the “classical model” (6,5% against 7,7%).

Table 2 and Table 3 depict the portfolio composition at the end of four months covered by the simulation process. Note that the robust portfolio composition is more “stable” through time, illustrating that changes in the allocation suggested by the “robust model” are smoother than those suggested by the “classical model”.

5. Conclusion

Asset managers need to make strategic and tactical investment decisions: the strategic decisions are related to the process of elaborating a long term investment policy; the tactical decisions are related to the process of elaborating short term decisions. Although the proposal presented in this article can handle both types of decisions, it is more suited to strategic decisions.

The strategic asset allocation is more concerned with the “essence” of assets return-risk behavior, instead of atypical events that the financial markets may occasionally present, distorting the short-term performance of assets. Since robust procedures are typically concerned with the pattern followed by the majority of the data, this characteristic is one of the main incentives for using the proposed robust optimal portfolios for strategic asset allocation. For instance, it is not reasonable for institutional investors to alter all their investment policy (say, devised for an investment horizon of decades) just because of a few atypical observations.

The optimization computations required by our proposal present a higher level of computational difficulty when compared to the computations required by the portfolio structuring methodologies used by practitioners nowadays. More research on proposing more efficient computational solutions for large-scale mixed integer programming models is needed. Until new developments are made for large-scale mixed integer programming, our proposal will remain useful only when applied to small portfolio structuring models, as in our numerical examples.

Finally, the concepts developed in this article can be easily adapted to other applications out of financial markets, such as robust crew scheduling, robust production planning, and robust assignment, to mention only three possibilities.

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Figure 1. CEMIG's Optimal Allocation Using the "Classical Model"

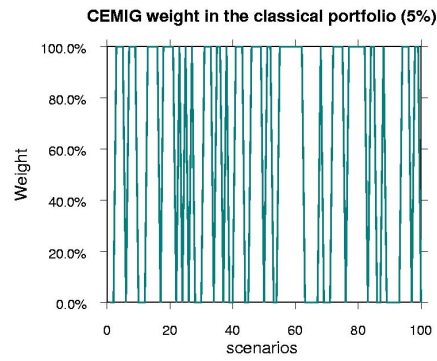


Figure 2. CEMIG's Optimal Allocation Using the "Robust Model"

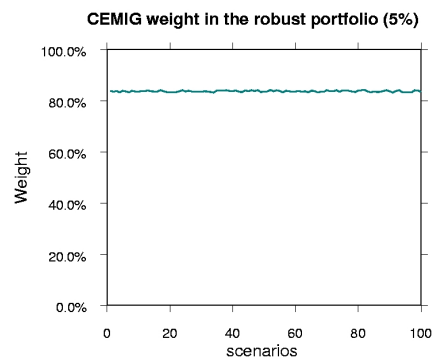


Figure 3. CEMIG's Optimal Allocation Using the "Classical Model"

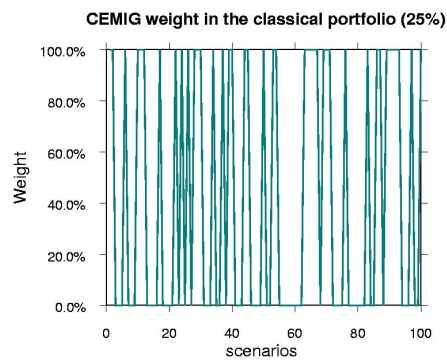


Figure 4. CEMIG's Optimal Allocation Using the "Robust Model"

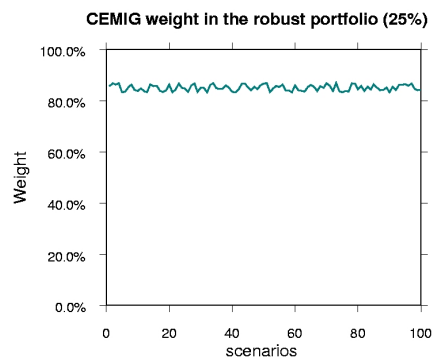


Figure 5. Optimal Allocation for Two Stocks Using the “Classical Model”

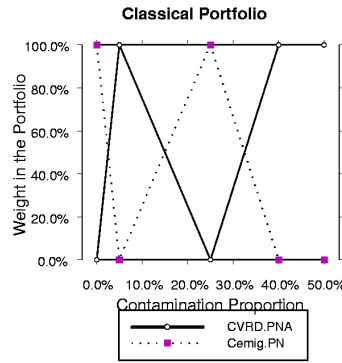


Figure 6. Optimal Allocation for Two Stocks Using the “Robust Model”

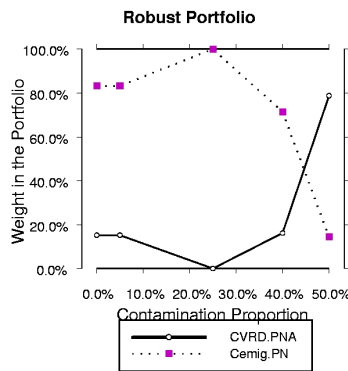


Figure 7. Out-of-sample Returns for Two Models

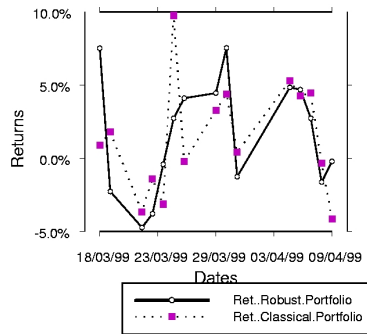


Figure 8. Out-of-sample Returns for Two Models

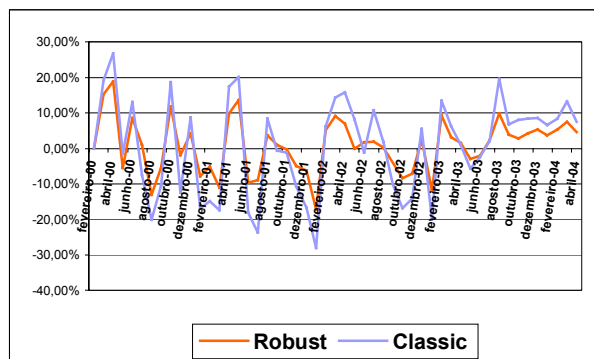


Table 1. Statistics for Out-of-Sample Returns of Two Models

| | “Classical Model” | “Robust Model” |
|--------------------|--------------------------|-----------------------|
| Mean Return | 1.453% | 1.633% |
| Accumulated Return | 22.936% | 26.167% |
| Standard Deviation | 0.038% | 0.039% |

Table 2. “Robust Model” Composition

| | Brazil | Argentina | Mexico | India | Hong Kong |
|-----------|---------------|------------------|---------------|--------------|------------------|
| Dec.-1999 | 68% | 0% | 20% | 0% | 12% |
| Dec.-2000 | 66% | 0% | 25% | 0% | 10% |
| Dec.-2001 | 39% | 9% | 14% | 0% | 38% |
| Dec.-2002 | 27% | 0% | 73% | 0% | 0% |

Table 3. “Classical Model” Composition

| | Brazil | Argentina | Mexico | India | Hong Kong |
|-----------|---------------|------------------|---------------|--------------|------------------|
| Dec.-1999 | 0% | 0% | 0% | 100% | 0% |
| Dec.-2000 | 0% | 0% | 0% | 0% | 100% |
| Dec.-2001 | 0% | 0% | 100% | 0% | 0% |
| Dec.-2002 | 0% | 0% | 100% | 0% | 0% |