On the complexity of (k, ℓ) graph sandwich problems

Fernanda Couto
COPPE/UFRJ, Brasil
UFRJ, Rio de Janeiro–RJ, Brasil
nandavdc@gmail.com

Luerbio Faria
DCC/UERJ
UERJ, Rio de Janeiro–RJ, Brasil
luerbio@cos.ufrj.br

Sulamita Klein
IM/COPPE/UFRJ, Brasil
UFRJ, Rio de Janeiro–RJ, Brasil
sula@cos.ufrj.br

Fábio Protti
IC/UFF
UFF, Rio de Janeiro–RJ, Brasil
fabio@ic.uff.br

Loana T. Nogueira
IC/UFF
UFF, Rio de Janeiro–RJ, Brasil
loana@ic.uff.br

Abstract

In 1995 Golumbic, Kaplan and Shamir defined GRAPH SANDWICH PROBLEMS as follows:

GRAPH SANDWICH PROBLEM FOR PROPERTY Π (Π-SP)
Input: Two graphs $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.
Question: Is there a graph $G = (V, E)$ satisfying the property Π such that $E^1 \subseteq E \subseteq E^2$?

The graph G is called *sandwich graph* for the pair (G^1, G^2). Note that making $E^1 = E = E^2$ we have the RECOGNITION PROBLEM FOR THE PROPERTY Π. Thus, we can easily see that graph sandwich problems are natural generalizations of recognition problems and, because of that, if the recognition problem for some property Π is NP-complete, so it will be the corresponding graph sandwich problem. In this work we will deal with two special properties: 'to be a strongly chordal-(k, ℓ) graph " and "to be a chordal-(k, ℓ) graph".

A graph is (k, ℓ) if its vertex set can be partitioned into at most k independent sets and into at most ℓ cliques. Brandstadt et al. have proved that this problem is NP-complete for $k \geq 3$ and $\ell \geq 3$, and polynomially solvable otherwise. It is also known that the sandwich problem for (k, ℓ) graphs is NP-complete for $k + \ell \geq 3$ [Dantas et al.] and polynomial otherwise [Golumbic et al.].

A graph is *chordal* if it has no C_k as an induced subraph, $k \geq 4$. A graph is *strongly chordal* if it is chordal and if every even cycle of length at least 6 has an odd chord, i.e.,

a chord between non-consecutive vertices apart from each other by an odd distance in the cycle. The recognition problem for chordal (resp. strongly chordal) graphs is polynomially solvable [Rose et al.] (resp. [Farber]) but the graph sandwich problems for both properties are NP-complete [Golumbic et al.; Faria et al.].

Hell et al. have proved that the recognition problem for chordal-(k, ℓ) graphs is polynomially solvable for all k, ℓ, what allows us to study the complexity of the corresponding graph sandwich problem.

Golumbic et al. proved that CHORDAL-$(1,1)$-SP is in P and R. Sritharan communicated that STRONGLY CHORDAL-$(1,1)$-SP is NP-complete.

In this work we present the result that CHORDAL-(k, ℓ)-SP is NP-complete: for $k \geq 1$ and $\ell \geq 2$, and for $k \geq 2$ and $\ell \geq 1$. We also state that STRONGLY CHORDAL-(k, ℓ)-SP is NP-complete for $k \geq 1$ and $\ell \geq 1$ (refer Figure 1).

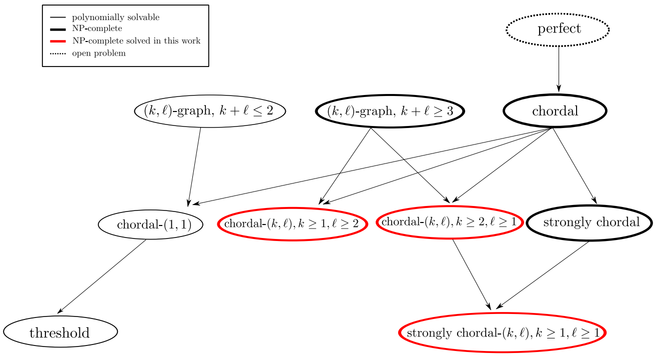

Figure 1: Our contribution to Golumbic et al.'s diagram presented in the seminal paper of graph sandwich problems.

Keywords: Sandwich problems, strongly chordal-(k, ℓ), chordal-(k, ℓ).