NEW COMPOUND NEIGHBORHOODS STRUCTURES FOR THE HETEROGENEOUS FIXED FLEET VEHICLE ROUTING PROBLEM

Puca Huachi Vaz Penna
Instituto do Noroeste Fluminense de Educação Superior - Universidade Federal Fluminense
Rua João Jazbik, s/no., Aeroporto, 28470-000, Santo Antônio de Pádua, RJ
Instituto de Computação - Universidade Federal Fluminense
Rua Passo da Pátria 156, Bloco E - 3ºandar, São Domingos, 24210-240, Niterói, RJ
ppenna@ic.uff.br
Thibaut Vidal
Instituto de Computação - Universidade Federal Fluminense
Rua Passo da Pátria 156, Bloco E - 3ºandar, São Domingos, 24210-240, Niterói, RJ
thibaut.vidal@cirrelt.ca
Luiz Satoru Ochi
Instituto de Computação - Universidade Federal Fluminense
Rua Passo da Pátria 156, Bloco E - 3ºandar, São Domingos, 24210-240, Niterói, RJ
satoru@ic.uff.br
Christian Prins
ICD-LOSI, Université de Technologie de Troyes BP 2060, 12 rue Marie Curie, 10010 Troyes Cedex, France
christian.prins@utt.fr

ABSTRACT

We investigate some large neighborhoods for Heterogeneous Fixed Fleet Vehicle Routing Problems (HFFVRP), combining reallocation and swap moves with a problem-tailored procedure for optimizing customer-to-vehicle assignment decisions. The assignment is either performed exactly by means of a primal-dual algorithm, or heuristically using the available vehicles to apply vehicle-type changes. The resulting large neighborhood are integrated into an iterated local search framework and compared with recently results found in the literature. Competitive results are produced for well-known HFFVRP benchmark instances.
KEYWORDS. Heterogeneous Fixed Fleet Vehicle Routing Problem, Metaheuristic, Iterated Local Search.

Main areas: MH - Metaheuristics, CO - Combinatorial Optimization.

RESUMO

Neste trabalho é desenvolvido uma nova estrutura de Vizinhança Grande para o Problema de Roteamento de Veículos com Frota Heterogênea Fixa (PRVFHF). Movimentos de realocação e troca são combinados com um procedimento para otimizar a atribuição de clientes aos tipos de veículos disponíveis. Esta atribuição é efetuada de modo exato por um algoritmo *Primal-Dual* ou heuristicamente fazendo uso dos veículos disponíveis durante o processo de associação de veículos as rotas. Esta Vizinhança Grande é integrada em uma meta-heurística baseada em *Iterated Local Search* e comparada com algortimos da literatura. Resultados competivos foram obtidos em instâncias bem conhecidas do PRVFHF.
PALAVRAS-CHAVE. Problema de Roteamento de Veículos com Frota Heterogênea Fixa, Meta-heurística, Iterated Local Search.

Áreas Principais: MH - Metaheurísticas, OC - Otimização Combinatória.

1 Introduction

The Vehicle Routing Problem (VRP) is one of the must studied problems in the field of Operations Research. Inspired by real world applications, several variants were proposed over the years. In this work, we study the Heterogeneous Fixed Fleet Routing Problem (HFFVRP), a generalization of the classical VRP in which customers are served by a heterogeneous fleet of vehicles with distinct capacities and costs. The objective is to determine the best fleet composition as well as the set of routes that minimize the travel costs in such a way that: (i) every route starts and ends at the depot; (ii) all the demands are satisfied; (iii) vehicle capacities are not exceeded; (iv) a customer is visited by only a single vehicle; (v) the sum of vehicle and route costs is minimized.

This problem is more realistic than the classical version and practical applications can be found in many distribution industries (Prins, 2002; Tarantilis and Kiranoudis, 2001, 2007). The problem is $\mathcal{NP}$-hard since any instance of classical VRP can be reduced to a HFFVRP instance in which all vehicles have the same characteristics. Two main variants are found in the literature. In the first one, named HFFVRP with Fixed and Dependent Cost (HFFVRPFD), the vehicles have both a fixed cost and a cost-per-mile associated to them. The other one, taking into account only the dependent vehicle costs, is know as HFFVRP with Dependent Cost (HFFVRPD). In this scope, the selection of an adequate vehicle type for each route is essential to obtain high-quality solutions. Some past works assume an unlimited fleet of vehicles of each type (Fleet Size and Mix – FSM, see Golden *et al.*, 1984). Here, our interest relies on the variant with a limited number of vehicles of each type, which lead to intricate assignment considerations. This problem is commonly called Heterogeneous Vehicle Routing Problem by some authors.

Most local search-based methods from the literature proposed independent neighborhood structures for optimizing visit sequences (routes), and route-to-vehicle assignments. In this paper, we introduce a new Compound Neighborhood Structure (CNS) for the problem. The CNS combines reallocation and swap moves with a problem-tailored procedure for optimizing the assignment. Thus, a relocate or swap move, which appeared as non-improving without any change of vehicle type, can lead to significant improvements when route-to-vehicle re-assignments are allowed. To efficiently optimize the assignment decisions during the search, two procedures are presented. The first one is based on a Primal-dual algorithm and the second one is a relaxed implementation, which only takes into account the routes involved in the move itself and the available set of vehicles during the assignment process.

The proposed structure is integrated in a metaheuristic framework based on Iterated Local Search with Variable Neighborhood Descent and Random Neighborhood Ordering (ILS-RVND) (Penna *et al.*, 2013). Extensive empirical studies are conducted to assess on the impact of the CNS with different assignment procedures. The results demonstrate the positive impact of the CNS on solution quality. This impact is even more significant when vehicle characteristics are notably different with no linear dependence between vehicles capacity and the costs. In addition, new best known solutions are generated for several well-known HFFVRP benchmark instances.

The remainder of this paper is organized as follows. Section 2 reviews some works related to the HFFVRP. Section 3 presents the ILS-RVND framework and the proposed CNS. Section 4 displays the results of the proposed method and establishes a comparison with existing literature, while Section 5 concludes the paper.

2 Related Works

The HFFVRP was first formulated in Taillard (1999). The author developed an algorithm based on Adaptive Memory Procedure (AMP), Tabu Search (TS) and column generation which was also applied to solve FSM problems, where the fleet of available vehicles is unlimited.

An exact approach was developed by Baldacci and Mingozzi (2009) for the HFFVRP and FSM variants. The authors put forward a set-partitioning based algorithm that uses bounding procedures based on linear relaxation and Lagrangean relaxation. Their solution method is capable of solving instances with up to 100 customers and, to our knowledge, is the only exact approach proposed in the HFFVRP literature.

Some authors implemented heuristic procedures to tackle the problem. Prins (2002) dealt with the HFFVRP by implementing a heuristic that extends a series of VRP classical heuristics followed by a local search procedure based on the Steepest Descent Local Search and TS.

Tarantilis *et al*. (2003) and Tarantilis *et al*. (2004) solved the HFFVRPD by means of a threshold-accepting Simulating Annealing (SA) procedure, in which a worse solution is only accepted if it is within a given threshold. Li *et al*. (2007) put forward a record-to-record travel algorithm which, as the threshold method, exploits a deterministic variant of SA.

A Guided Tabu Search (GTS) was proposed by Tarantilis *et al*. (2008). The algorithm is based on a TS controlled by a continuous guiding mechanism, which modifies the objective function of the problem with the aim to obtain diversity during the search. A deterministic TS that makes use of different procedures for generating initial solutions was proposed by Brandão (2009). Li *et al*. (2010) proposed a multi-start AMP combined with Path Relinking and a modified TS to solve the HFFVRPFD. More recently, Brandão (2011) proposed a TS algorithm for the HFFVRP which includes additional features such as strategic oscillation, shaking and frequency-based memory.

Two Memetic Algorithms (MA) were developed by Prins (2009) to solve both HFFVRPD and FSM. The method relies on a giant tour solution representation and a *Split* procedure to determine the optimal fleet. This Split procedure consists of solving Shortest Path Problem with Resource Constraints (SPPRC) by Dynamic Programming. Duhamel *et al*. (2011) studied the impact of Split procedures embedded in a framework composed by a Greedy Randomized Adaptive Search Procedure (GRASP) with an Evolutionary Local Search (ELS) metaheuristic. A parallel version on this algorithm was recently proposed by Duhamel *et al*. (2013).

Subramanian *et al*. (2012) presented a hybrid algorithm with an Iterated Local Search (ILS) based heuristic and a Set Partitioning (SP) formulation, applied to the HFFVRP and FSM. The SP model was solved by means of a Mixed Integer Programming solver, which interactively calls the ILS heuristic during its execution.

Finally, Penna *et al*. (2013) introduced an algorithm based on the ILS metaheuristic which uses a Variable Neighborhood Descent procedure, with a random neighborhood ordering (RVND), in the local search phase. The authors solved all HFFVRP and FSM variants.

It should be noted that most of the previously-developed procedures exploit separate neighborhoods for optimizing the sequences and the vehicle types. Few attempts have been made in the literature to integrate more tightly together these two families of decisions. Our goal is to contribute on this topic and progress with new compound neighborhoods which optimize both aspects in a purposeful manner.

3 The Proposed Compound Neighborhood Structures

This section describes the proposed Compound Neighborhood Structures (CNS). To test the CNS, we use the algorithm presented by Penna *et al*. (2013), called ILS-RVND, composed by an iterated local search and a Variable Neighborhood Descent with Random neighborhood ordering (RVND) in the local search phase. This heuristic was successfully applied by Subramanian *et al*. (2012); Silva *et al*. (2012); Penna *et al*. (2012, 2013); Subramanian *et al*. (2013) to several VRP variants. The main components of the ILS-RVND heuristic, the neighborhoods, local search and perturbation mechanisms, are described in the following.

3.1 Local Search

ILS-RVND is multi-start ILS framework. The local search phase, called RVND, uses well-known VRP neighborhoods, 2-opt, 2-opt* and CROSS–exchanges of up to two consecutive customers. In CROSS it is possible to exchange {0,1,2} consecutive vertices with {0,1,2} consecutive vertices from the same or different routes. This neighborhood is similar to λ–interchanges scheme, with $\lambda = 2$. Reversals are allowed in the process.

These neighborhoods are exhaustively evaluated in random order, and only feasible and improving moves are applied. In most methods from the literature, the number of feasible moves is limited by the fleet composition, i.e., the current vehicle type associated with the routes. In our approach, these classical neighborhoods for VRP serve as a building block of a larger neighborhood structure, which combines relocate and swap moves with a problem-tailored procedure for optimizing customer-to-vehicle assignment decisions. The proposed CNS thus explores a wider search space, opening the way to new solution improvements.

The CNS works as follows. Firstly, a movement, of the classical neighborhood described above, is evaluated. If the move is feasible and improves the incumbent solution cost, then the current is updated. Otherwise, the selected move is tested together with the fleet reassignment.

Two approaches were considered in order to make the fleet reassignment. The first, uses an exact method based on the Primal-Dual Algorithm (section 3.1.1) to solve the Assignment Problem (AP) and find the optimal fleet composition according to the neighborhood tested. The second, uses a simple method that reassigns the fleet using only to available vehicles (3.1.2).

3.1.1 Primal-Dual Algorithm

For a given solution, the optimal fleet composition can be found by solving an Assignment Problem (AP) expressed in Equations (1-4). Let $\mathcal{R}$ be the set of routes and $\mathcal{P}$ be the set of available vehicles. The model relies on the binary decision variables x_{ij}, which take value 1 if and only if route i is associated to vehicle j. For each route i, let q_i be the load and d_i the distance associated to the route. For each vehicle k, let Q_k be the capacity, F_k the fixed cost and U_k the cost per distance unit. The cost of an assignment of a vehicle $j \in \mathcal{P}$ to a route $i \in \mathcal{R}$ is given by c_{ij}, where $c_{ij} = F_j + U_j \times d_i$ if $q_i \leq Q_j$ otherwise $c_{ij} = \infty$.

$$\text{Min} \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{P}} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j \in \mathcal{P}} x_{ij} = 1 \qquad \forall i \in \mathcal{R} \tag{2}$$

$$\sum_{i \in \mathcal{R}} x_{ij} = 1 \qquad \forall j \in \mathcal{P} \tag{3}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{R}, \forall j \in \mathcal{P}. \tag{4}$$

The objective function (1) minimizes the sum of the costs by choosing the best assignment of routes to vehicles. Constraints (2) state that a single route from the set $\mathcal{R}$ is associated to only one vehicle $j \in \mathcal{P}$. Constraints (3) requires that a single vehicle from the set $\mathcal{P}$ is assigned to only one route $i \in \mathcal{R}$. Constraints (4) define the domain of the decision variables. Note that AP requires $|\mathcal{R}| = |\mathcal{P}|$, if $|\mathcal{R}| < |\mathcal{P}|$ some fictitious routes needs to be created and assigned to vehicles with null costs. This AP can be solved in $\mathcal{O}(n^3)$ operations using the primal-dual algorithm (PDA) of McGinnis (1983).

3.1.2 Simple Fleet Reassignment

The previously-described PDA is exact but computationally expensive. Thus, we developed an alternative heuristic methods, called Simple Fleet Reassignment (SFR) procedure, which takes into account only the available vehicles and the routes involved in the move. First, a list with all available vehicles LV is created. Next, for routes r_1 and r_2, associated with the movement, the method finds in LV the vehicle type that best fits to these routes. Finally, if a better vehicle is found for these routes, the vehicle type assigned to routes r_1 and r_2 is updated and the solution cost is returned.

3.2 Perturbation Mechanisms

We rely on a set of four perturbation mechanisms for the HFFVRP. The first two perturbation operators, **Multiple-Swap(1,1)** – $P^{(1)}$ and **Multiple-Shift(1,1)** – $P^{(2)}$ were introduced in Penna *et al.* (2013). The number of movements in perturbations $P^{(1)}$ and $P^{(2)}$ are chosen at random from the interval $\{1, \ldots, 3\}$. We also add two new perturbations in order to tackle HFFVRP real-world based problems.

Split – $P^{(3)}$ – A route r is divided into smaller routes. Firstly, a route r with a vehicle type different from the one with the smallest capacity is randomly selected. Next, a new route is created, associated with a random available vehicle. The customers of r are sequentially transferred to this new route as long as the vehicle capacity is not violated and r is not empty. Another route with a random vehicle is created if the vehicle-capacity is exceeded. This process is repeated until r becomes empty. The newly generated routes are added to the solution s and the route r is removed.

Merge – $P^{(4)}$ – Two routes are concatenated into a new larger route. A route r_1 is randomly selected. The "closest route" r_2, in terms of euclidean distance between route barycenters, is selected. Routes associated with a vehicle with the largest capacity are not considered in the computation. Next, the customers of route r_1 are, sequentially, transferred to route r_3, followed by the customers of route r_2. The routes r_1 and r_2 are removed and the new route r_3 is added to the solution. The vehicle type associated with new route r_3 is selected within the subset of available vehicles, in such way that the vehicle capacity in not violated. The merge is canceled if no such vehicle exists. Finally, the Intra-route search procedures (Penna *et al.*, 2013) is executed on r_3 with the objective to find the best customers visit order in the route.

4 Computational Experiments

The algorithm ILS-RVND was coded in C++ (g++ 4.6.3) and executed on an Intel® Core$^{\text{TM}}$ i7 Processor 2.93 GHz with 8 GB of RAM memory running Ubuntu Linux 12.04 (kernel version 3.5). The approach was tested on two instance sets. The first set, proposed by Taillard (1999) and Brandão (2011), involves correlated vehicle costs and capacities, i.e., if vehicle types are considered in ascending order of capacities, the fixed costs and variable costs also increase (Figure 1(a)). The second, proposed by Duhamel *et al.* (2011) is based on road distances between major cities in different districts of France. The fleet composition is non-correlated in most problems (Figure 1(b)). Figure 1 shows the route cost per distance for each vehicle type for one instance of each set. In Figure 1(b), a route with a customer demand of 100 can be associated to vehicle type B, C or D. If the distance is smaller than 20 it is better use vehicle C. Otherwise, if the route distance is greater than 50, then vehicle D leads to smaller costs. This behavior does not happen when the fleet capacity and costs are correlated.

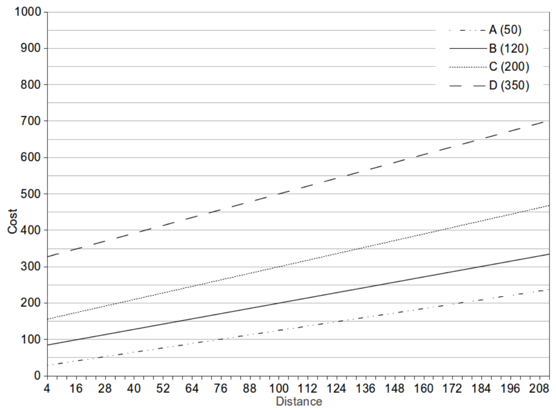
Three versions of the algorithm were implemented in this study:

- **MS-ILS**: Multi-start ILS-RVND without the CNS;
- **MS-ILS-SFR**: MS-ILS with CNS and the Simple Fleet Reassignment procedure;
- **MS-ILS-PDA**: MS-ILS with CNS and the Primal-Dual procedure.

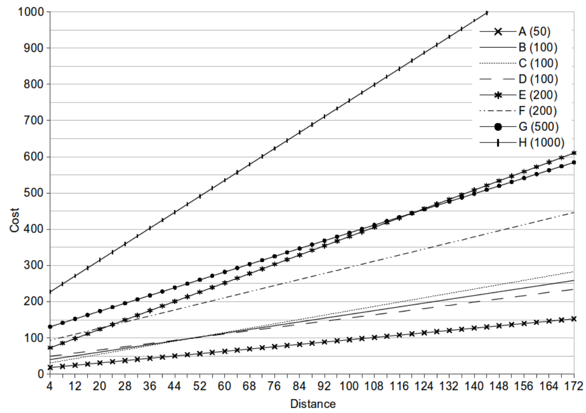
These three method variants have been tested with and without the Merge ($P^{(4)}$) neighborhood, leading to overall six versions of the proposed algorithm.

Each version was executed 10 times for each instance and the number of multi-start iterations of the ILS-RVND parameter was set to 100 to achieve similar run times than existing literature. The results are presented in Tables 1 and 2. In these two tables, **Gap** denotes the gap between the average solution, on 10 runs, found by each version of the algorithm and the best known solution of the literature. **Time** corresponds to the average time, in seconds, of these runs. The best average gap for each version is highlighted in boldface.

Table 1 displays the results on the 8 HFFVRPD and HFFVRPFD benchmark instances of Taillard (1999), and the HFFVRPD instances from Brandão (2011). As illustrated by the results, all versions attain a similar performance in terms of average gap, but the version MS-ILS-PDA



(a) Inst. # 17 (Taillard, 1999)



(b) Inst. # 06 (Duhamel *et al.*, 2011)

Figure 1: Correlated and Non-Correlated Instances

without the Merge perturbation slightly outperformed the other five. In this set of instances, the use of the Merge procedure generated worse solutions. This behavior was expected, as this perturbation creates routes associated to vehicles with larger capacities and consequently higher costs. In terms of the average computational time, it can be verified that the CNS increases the algorithm execution time, since the solution space increases considerably.

Table 1: Comparative results for all versions of the algorithm on Correlated Instances (Taillard, 1999; Brandão, 2011)

Problem type	n	$P^{(1)}+P^{(2)}+P^{(3)}$						$P^{(1)}+P^{(2)}+P^{(3)}+P^{(4)}$					
		MS-ILS		MS-ILS+SFR		MS-ILS+PDA		MS-ILS		MS-ILS+SFR		MS-ILS+PDA	
		Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)
HFFVRPFD[1]	50 – 100	0.37	30.54	0.35	32.02	**0.34**	1639.91	**0.34**	27.36	0.36	39.51	0.39	1448.92
HFFVRPD[1]	50 – 100	0.30	30.19	0.29	30.96	**0.28**	1900.57	0.34	26.56	0.29	38.06	0.31	1621.06
HFFVRPD[2]	100 – 199	**0.21**	271.40	**0.21**	384.50	**0.21**	50779.58	**0.21**	267.04	**0.21**	389.86	0.32	51331.29
Average		0.29	110.71	0.28	149.16	**0.27**	18106.69	0.29	106.98	0.29	155.81	0.34	18133.76

[1]: Taillard (1999); [2]: Brandão (2011).

Table 2 describes the results on the more realistic HFFVRPFD benchmark instances of Duhamel *et al.* (2011). This set contains 96 instances, ranging from 20 to 256 customers, and with 3 to 8 types of vehicles. This set of instances is divided into four subsets, a "small" subset containing 15 instances with less than 100 customers, 38 instances with 100 to 150 customers, 31 instances with 150 to 200 customers, and finally 12 instances and with more than 200 customers. Due the computational complexity of the CNS using the PDA, it was only possible to test it with the small instance subset. MS-ILR-SFR with the Merge perturbation appears to outperform all other versions in terms of average solution gap. This shows the impact of the CNS proposed and the importance of merging routes when the vehicle costs are non-correlated.

It should be noted that algorithms with the CNS outperformed the version without in both set of instances. The Merge perturbation has a positive impact only when the fleet of vehicles has non-correlated costs. Several new best known solutions have been generated during these tests for the instances of Duhamel *et al.* (2011). These solutions are presented along with the detailed results of MS-ILS-SFR in Tables 3 to 6. A comparison is established with the best-known solutions obtained by different versions of the GRASPxELS of Duhamel *et al.* (2010, 2011, 2013). For these experiments the number of multi-start iterations of the algorithm was set to 200.

In the tables presented hereafter, **Inst. #** denotes the number of the test-problem, n is

Table 2: Comparative results for all versions of the algorithm on Non-Correlated Instances (Duhamel *et al.*, 2011)

		$P^{(1)} + P^{(2)} + P^{(3)}$						$P^{(1)} + P^{(2)} + P^{(3)} + P^{(4)}$					
Problem type	*n*	MS-ILS		MS-ILS+SFR		MS-ILS+PDA		MS-ILS		MS-ILS+SFR		MS-ILS+PDA	
		Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)
HFFVRPFD	20 –95	0.18	30.52	0.58	40.95	0.57	23579.44	0.19	29.22	**0.17**	42.37	0.18	23064.81
HFFVRPFD	102 – 147	0.58	164.04	0.98	255.80	–	–	0.55	169.19	**0.53**	252.49	–	–
HFFVRPFD	152 – 196	0.65	542.13	0.84	820.84	–	–	0.67	538.01	**0.64**	772.70	–	–
HFFVRPFD	203 – 256	0.25	1352.96	0.95	2197.56	–	–	0.21	1244.18	**0.19**	1935.74	–	–
Average		0.42	522.41	0.84	828.79	–	–	0.41	495.15	**0.38**	750.83	–	–

the number of customers, **BKS** represents the best known solution reported in the literature, **Best Sol.** indicates the best solution, **Gap** denotes the gap between the best solution found by the MS-ILS-SFR and the best known solution, **Avg. Sol.** and **Avg. Time** represents the average solution and the average time, in seconds, of the 10 runs, respectively. Finally, **Avg. Gap** corresponds to the gap between the average solution found by the MS-ILS-SFR and the best known solution.

Table 3: MS-ILS-SFR Results on subset 1 of Duhamel *et al.* (2011) Instances

			GRASPxELS[1]		MS-ILS-SFR				
Inst. #	n	BKS	Best Sol.	Avg. Time	Best Sol.	Gap	Avg. Sol.	Avg. Time	Avg. Gap
HVRP_01_DLP	92	9210.14	**9210.14**	52.29	**9210.14**	0.00%	9211.23	269.10	0.01%
HVRP_08_DLP	84	4591.75	4598.49	304.85	**4591.75**	0.00%	4596.86	92.61	0.11%
HVRP_10_DLP	69	2107.55	**2107.55**	24.83	**2107.55**	0.00%	2107.55	118.69	0.00%
HVRP_11_DLP	95	3367.41	3370.47	264.61	**3367.41**	0.00%	3371.39	244.22	0.12%
HVRP_36_DLP	85	5684.61	5759.34	104.39	5684.62	0.00%	5702.85	267.37	0.32%
HVRP_39_DLP	77	2923.72	2934.55	182.11	**2921.36**	-0.08%	2934.11	170.75	0.36%
HVRP_43_DLP	86	8737.02	8764.75	219.91	**8707.94**	-0.33%	8742.78	178.10	0.07%
HVRP_52_DLP	59	4027.27	4029.42	39.97	**4027.27**	0.00%	4029.21	62.89	0.05%
HVRP_55_DLP	56	10244.34	10247.86	190.76	**10244.34**	0.00%	10247.84	27.85	0.03%
HVRP_70_DLP	78	6685.24	6689.61	120.60	**6684.56**	-0.01%	6691.86	125.31	0.10%
HVRP_75_DLP	20	452.85	**452.85**	0.02	**452.85**	0.00%	452.85	1.63	0.00%
HVRP_82_DLP	79	4766.74	4774.26	144.51	**4766.74**	0.00%	4771.18	107.78	0.09%
HVRP_92_DLP	35	564.39	**564.39**	20.63	**564.39**	0.00%	564.39	14.02	0.00%
HVRP_93_DLP	39	1036.99	**1036.99**	27.39	**1036.99**	0.00%	1037.77	17.03	0.07%
HVRP_94_DLP	46	1378.25	1378.66	15.68	**1378.25**	0.00%	1378.25	27.47	0.00%
Average			0.17%	114.17		-0.03%		114.99	0.09%

[1]: GRASPxELS with DFS Split (Duhamel *et al.*, 2011)

5 Concluding Remarks

This article introduces new large neighborhood searches techniques, with a Compound Neighborhood Structure (CNS), as well as several shaking operators, for the Heterogeneous Fixed Fleet Vehicle Routing Problem (HFFVRP). The CNS combines relocate and swap moves with fleet assignment re-optimizations. Two approaches were studied to operate the assignment, an exact method based on the Primal-Dual Algorithm (PDA), and a Simple Fleet Reassignment (SFR) heuristic using only available vehicles. These techniques have been integrated into a multi-start Iterated Local Search framework.

Empirical experiments, on classic benchmark instances for the proposed problems, demonstrate that the PDA version is capable to find good solutions in terms of average gap, but with an unacceptable computational time. This version is still useful to assess on the "maximum impact" of taking the optimal assignment decision at each step. In addition, the version of the proposed

Table 4: MS-ILS-SFR Results on subset 2 of Duhamel *et al*. (2011) Instances

			GRASPxELS[1]		MS-ILS-SFR				
Inst. #	n	BKS	Best Sol.	Avg. Time	Best Sol.	Gap	Avg. Sol.	Avg. Time	Avg. Gap
HVRP_03_DLP	124	10738.28	11320.58	512.10	**10730.10**	-0.08%	10778.32	672.52	0.37%
HVRP_05_DLP	116	10903.63	10963.62	488.63	**10896.33**	-0.07%	10957.05	267.32	0.49%
HVRP_06_DLP	121	11692.85	11792.94	367.91	11711.35	0.16%	11793.50	388.63	0.86%
HVRP_07_DLP	108	8095.88	8130.50	306.09	**8089.21**	-0.08%	8146.61	245.33	0.63%
HVRP_12_DLP	112	3543.99	**3543.99**	71.46	**3543.99**	0.00%	3544.35	535.69	0.01%
HVRP_13_DLP	119	6696.43	6713.14	303.37	**6696.43**	0.00%	6705.85	367.78	0.14%
HVRP_16_DLP	129	4156.97	4161.61	180.91	**4156.97**	0.00%	4158.38	798.51	0.03%
HVRP_17_DLP	105	5362.83	5370.05	172.82	5369.31	0.12%	5386.07	248.22	0.43%
HVRP_2A_DLP	113	7793.16	7885.93	298.92	**7793.16**	0.00%	7803.15	448.00	0.13%
HVRP_2B_DLP	107	8464.69	8537.31	303.14	8482.79	0.21%	8514.03	537.57	0.58%
HVRP_21_DLP	126	5141.49	5154.38	330.23	**5139.84**	-0.03%	5158.58	407.99	0.33%
HVRP_25_DLP	143	7206.64	7228.54	518.28	7217.26	0.15%	7227.27	1722.18	0.29%
HVRP_26_DLP	126	6446.31	6481.93	350.71	**6423.70**	-0.35%	6456.02	1044.76	0.15%
HVRP_28_DLP	141	5531.06	5542.76	343.06	5533.01	0.04%	5543.92	525.38	0.23%
HVRP_30_DLP	112	6313.39	6321.69	201.39	6322.39	0.14%	6338.65	424.74	0.40%
HVRP_31_DLP	131	4091.52	4103.88	308.39	4091.81	0.01%	4111.96	1173.78	0.50%
HVRP_34_DLP	136	5758.089	5800.12	405.62	5786.98	0.50%	5805.19	784.08	0.82%
HVRP_40_DLP	132	11123.56	11172.98	614.92	**11122.32**	-0.01%	11145.14	808.47	0.19%
HVRP_41_DLP	135	7616.17	7679.32	325.80	**7572.07**	-0.58%	7647.19	969.42	0.41%
HVRP_47_DLP	111	16206.14	16222.94	333.85	**16175.22**	-0.19%	16267.44	391.41	0.38%
HVRP_48_DLP	111	21318.04	21413.92	371.30	21330.75	0.06%	21403.22	519.67	0.40%
HVRP_51_DLP	129	7721.47	7780.88	315.60	7766.77	0.59%	7794.97	760.96	0.95%
HVRP_53_DLP	115	6434.83	6470.49	418.17	**6434.83**	0.00%	6448.41	510.67	0.21%
HVRP_60_DLP	137	17037.39	17067.85	444.32	**17037.23**	0.00%	17084.71	647.13	0.28%
HVRP_61_DLP	111	7295.67	7300.10	108.21	**7292.03**	-0.05%	7296.92	518.61	0.02%
HVRP_66_DLP	150	12830.82	13319.73	442.89	**12828.34**	-0.02%	12856.39	1315.84	0.20%
HVRP_68_DLP	125	8976.53	9135.23	269.63	**8935.89**	-0.45%	8987.60	650.43	0.12%
HVRP_73_DLP	137	10195.33	10243.66	598.34	10196.04	0.01%	10211.26	535.98	0.16%
HVRP_74_DLP	125	11586.87	11732.54	246.66	11592.72	0.05%	11607.20	536.07	0.18%
HVRP_79_DLP	147	7259.54	7314.89	473.69	7274.18	0.20%	7291.25	1138.42	0.44%
HVRP_81_DLP	106	10700.47	10715.28	83.71	**10689.77**	-0.10%	10696.60	384.85	-0.04%
HVRP_83_DLP	124	10019.15	10019.83	332.47	10029.60	0.10%	10045.38	635.09	0.26%
HVRP_84_DLP	105	7227.88	7269.55	206.41	7236.49	0.12%	7244.57	343.76	0.23%
HVRP_85_DLP	146	8779.76	8874.31	382.98	8797.92	0.21%	8834.98	786.67	0.63%
HVRP_87_DLP	108	3753.87	**3753.87**	104.11	**3753.87**	0.00%	3759.05	431.80	0.14%
HVRP_88_DLP	127	12402.85	12443.41	632.22	12448.38	0.37%	12474.00	409.80	0.57%
HVRP_89_DLP	134	7106.84	7135.36	245.63	**7105.47**	-0.02%	7118.99	682.49	0.17%
HVRP_90_DLP	102	2346.13	2360.83	15.36	2347.50	0.06%	2351.82	331.87	0.24%
			0.71%	327.09		0.03%		629.00	0.33%

[1]: GRASPxELS with DFS Split (Duhamel *et al*., 2010)

Table 5: MS-ILS-SFR Results on subset 3 of Duhamel *et al.* (2011) Instances

			GRASPxELS[1]		MS-ILS-SFR				
Inst. #	n	BKS	Best Sol.	Avg. Time	Best Sol.	Gap	Avg. Sol.	Avg. Time	Avg. Gap
HVRP_02_DLP	181	11790.35	12102.01	325.86	**11718.86**	-0.61%	11746.01	1693.43	-0.38%
HVRP_04_DLP	183	10808.31	11276.45	726.38	**10787.03**	-0.20%	10810.12	1709.04	0.02%
HVRP_09_DLP	167	7619.19	7647.59	450.18	7651.33	0.42%	7662.47	876.89	0.57%
HVRP_14_DLP	176	5644.92	5679.80	448.59	5667.82	0.41%	5686.48	2498.16	0.74%
HVRP_15_DLP	188	8236.4	8301.63	520.82	8268.18	0.39%	8282.90	2223.92	0.56%
HVRP_24_DLP	163	9101.47	9183.78	609.82	9118.01	0.18%	9165.96	1721.61	0.71%
HVRP_29_DLP	164	9143.69	9147.39	424.95	**9142.86**	-0.01%	9155.45	1362.74	0.13%
HVRP_33_DLP	189	9421.01	9543.17	602.72	9437.30	0.17%	9468.07	2276.72	0.50%
HVRP_35_DLP	168	9574.71	9640.80	458.96	9592.43	0.19%	9643.93	1162.89	0.72%
HVRP_37_DLP	161	6858.23	6921.19	383.70	6870.11	0.17%	6880.68	1552.82	0.33%
HVRP_42_DLP	178	10902.84	11713.90	316.85	**10855.73**	-0.43%	10910.71	3695.65	0.07%
HVRP_44_DLP	172	12197.46	12418.00	447.32	12237.42	0.33%	12272.48	1851.99	0.62%
HVRP_45_DLP	170	10484.23	10519.25	450.59	10496.88	0.12%	10558.01	2117.97	0.70%
HVRP_50_DLP	187	12374.04	12508.77	646.87	12385.32	0.09%	12421.13	4091.66	0.38%
HVRP_54_DLP	172	10393.23	11511.62	364.47	**10370.09**	-0.22%	10401.12	2725.20	0.08%
HVRP_56_DLP	153	31090.71	31292.81	394.08	**31090.53**	0.00%	31187.32	1135.99	0.31%
HVRP_57_DLP	163	44818.18	45152.42	638.93	44850.05	0.07%	44927.45	1361.30	0.24%
HVRP_59_DLP	193	14282.59	14367.14	676.23	14309.48	0.19%	14323.59	2973.86	0.29%
HVRP_63_DLP	174	19951.76	20241.72	693.90	19994.01	0.21%	20178.68	983.73	1.14%
HVRP_64_DLP	161	17162.39	17157.37	512.03	**17135.16**	-0.16%	17151.71	791.95	-0.06%
HVRP_67_DLP	172	10937.67	11854.61	336.67	**10915.60**	-0.20%	10945.40	1494.50	0.07%
HVRP_69_DLP	152	9162.78	9276.93	508.55	9167.18	0.05%	9228.99	939.94	0.72%
HVRP_71_DLP	186	9870.22	9960.84	639.69	9891.50	0.22%	9952.33	1293.91	0.83%
HVRP_72_DLP	186	5905.58	5976.54	197.11	5883.33	-0.38%	5933.51	2238.37	0.47%
HVRP_76_DLP	152	12018.26	12098.66	685.64	**12018.22**	0.00%	12064.35	1240.58	0.38%
HVRP_77_DLP	190	6930.44	6991.59	636.46	**6929.67**	-0.01%	6967.54	2803.83	0.54%
HVRP_78_DLP	190	7035.01	7069.82	471.38	7039.90	0.07%	7082.69	1936.95	0.68%
HVRP_80_DLP	171	6816.89	6839.96	229.66	6825.46	0.13%	6837.43	1529.94	0.30%
HVRP_86_DLP	153	9030.68	9076.63	383.30	9053.41	0.25%	9069.47	908.09	0.43%
HVRP_91_DLP	196	6377.48	6437.14	544.07	6381.13	0.06%	6406.12	3234.78	0.45%
HVRP_95_DLP	183	6181.6	6244.13	322.61	**6175.62**	-0.10%	6234.62	907.11	0.86%
Average			1.74%	485.43		0.04%		1849.53	0.43%

[1]: GRASPxELS with DFS Split (Duhamel *et al.*, 2010)

Table 6: MS-ILS-SFR Results on subset 4 of Duhamel *et al.* (2011) Instances

			GRASPxELS[1]		MS-ILS-SFR				
Inst. #	n	BKS	Best Sol.	Avg. Time	Best Sol.	Gap	Avg. Sol.	Avg. Time	Avg. Gap
HVRP_18_DLP	256	9702.75	9797.61	1216.10	**9668.17**	-0.36%	9687.95	6127.10	-0.15%
HVRP_19_DLP	224	11702.77	11805.34	1009.87	11702.98	0.00%	11730.10	2587.90	0.23%
HVRP_22_DLP	239	13068.03	13162.90	835.87	13103.51	0.27%	13134.34	2423.58	0.51%
HVRP_23_DLP	203	7750.27	7809.20	802.30	7760.62	0.13%	7784.68	2657.22	0.44%
HVRP_27_DLP	220	8469.19	8520.74	995.85	**8436.55**	-0.39%	8450.14	3424.38	-0.22%
HVRP_32_DLP	244	9417.62	9537.48	1131.44	**9412.56**	-0.05%	9453.66	5771.40	0.38%
HVRP_38_DLP	205	11242.95	11439.58	421.50	**11217.53**	-0.23%	11253.53	2612.66	0.09%
HVRP_46_DLP	250	24674.26	24805.27	1475.05	**24428.54**	-1.00%	24558.48	7371.30	-0.47%
HVRP_49_DLP	246	16377.69	16417.30	990.34	**16219.41**	-0.97%	16262.98	8693.86	-0.70%
HVRP_58_DLP	220	23397.76	23530.10	1028.25	23504.15	0.45%	23587.32	2640.37	0.81%
HVRP_62_DLP	225	23149.61	23434.56	828.76	**22952.06**	-0.85%	23123.16	3220.24	-0.11%
HVRP_65_DLP	223	13053.8	13077.63	635.64	**13013.89**	-0.31%	13042.00	4347.73	-0.09%
			0.81%	947.58		-0.27%		4145.20	0.05%

[1]: GRASPxELS with classical Split (Duhamel *et al.*, 2010)

algorithm with Simple Fleet Reassignment (MS-ILS-SFR) was tested on 96 benchmark instances up to 256 customers and based on real-world data created by Duhamel *et al*. (2011). MS-ILS-SFR finds improved results on 33 instances, and equal results on 17 instances. The proposed CNS contributes significantly to the search performance when the fleet costs are uncorrelated, a situation which often arises in practice. The proposed shaking techniques, Split and Merge, also lead to solutions of higher quality. Hence, these techniques are a good alternative address fleet optimization on routing problems with heterogeneous fleets. Several research avenues remain open, e.g., to improve the CNS computational efficiently, and better exploit the capabilities of route-to-vehicle assignment procedures during heuristic searches.

References

Baldacci, R. and Mingozzi, A. (2009), A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming*, v. 120, p. 347–380.

Brandão, J. (2009), A deterministic tabu search algorithm for the fleet size and mix vehicle routing problem. *European Journal of Operational Research*, v. 195, p. 716–728.

Brandão, J. (2011), A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. *Computers & Operations Research*, v. 38, p. 140–151.

Duhamel, C., Lacomme, P. and C., P. A GRASPxELS with depth first search split procedure for the hvrp. Relatório Técnico LIMOS/RR-10-08, LIMOS, 2010.

Duhamel, C., Gouinaud, C., Lacomme, P. and Prodhon, C. A multi-thread GRASPxELS for the heterogeneous capacitated vehicle routing problem. Talbi, E.-G. (Ed.), *Hybrid Metaheuristics*, volume 434 of *Studies in Computational Intelligence*, p. 237–269. Springer Berlin Heidelberg, 2013.

Duhamel, C., Lacomme, P. and Prodhon, C. (2011), Efficient frameworks for greedy split and new depth first search split procedures for routing problems. *Computers & Operations Research*, v. 38, n. 4, p. 723 – 739.

Golden, B. L., Assad, A. A., Levy, L. and Gheysens, F. G. (1984), The fleet size and mix vehicle routing problem. *Computers & Operations Research*, v. 11, p. 49–66.

Li, F., Golden, B. and Wasil, E. (2007), A record-to-record travel algorithm for solving the heterogeneous fleet vehicle routing problem. *Computers & Operations Research*, v. 34, p. 2734 – 2742.

Li, X., Tian, P. and Aneja, Y. (2010), An adaptive memory programming metaheuristic for the heterogeneous fixed fleet vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, v. 46, n. 6, p. 1111 – 1127.

McGinnis, L. F. (1983), Implementation and testing of a primal-dual algorithm for the assignment problem. *Operations Research*, v. 31, n. 2, p. 277 – 291.

Osman, I. H. (1993), Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem. *Ann. of Oper. Res.*, v. 41, n. 1-4, p. 421–451.

Penna, P. H. V., Subramanian, A. and Ochi, L. An iterated local search heuristic for open vehicle routing problems. *Anais XVI CLAIO/XLIV SBPO – Congreso Latino-Iberoamericano de Investigación Operativa/ Simpósio Brasileiro de Pesquisa Operacional*, p. 1 – 11, Rio de Janeiro – Brasil, 2012.

Penna, P. H. V., Subramanian, A. and Ochi, L. S. (2013), An iterated local search heuristic for the heterogeneous fleet vehicle routing problem. *Journal of Heuristics*, v. 19, n. 2, p. 201–232.

Potvin, J. Y. and Rousseau, J. M. (1995), An exchange heuristic for routing problems with time windows. *Journal of the Operational Research Society*, v. 46, p. 1433 – 1446.

Prins, C. (2002), Eficient heuristics for the heterogeneous fleet multitrip VRP with application to a large-scale real case. *Journal of Mathematical Modelling and Algorithms*, v. 1, p. 135–150.

Prins, C. (2009), Two memetic algorithms for heterogeneous fleet vehicle routing problems. *Engineering Applications of Artificial Intelligence*, v. 22, n. 6, p. 916–928.

Silva, M. M., Subramanian, A., Vidal, T. and Ochi, L. S. (2012), A simple and effective metaheuristic for the minimum latency problem. *European Journal of Operational Research*, v. 221, n. 3, p. 513 – 520.

Subramanian, A., Penna, P. H. V., Ochi, L. S. and Souza, M. J. F. *Meta-Heurísticas em Pesquisa Operacional*, Capítulo Um Algoritmo Heurístico Baseado em Iterated Local Search para Problemas de Roteamento de Veículos, p. 167 – 191. Omnipax, 2013.

Subramanian, A., Penna, P. H. V., Uchoa, E. and Ochi, L. S. (2012), A hybrid algorithm for the heterogenous fleet vehicle routing problem. *European Journal of Operational Research*, v. 221, p. 285 – 295.

Taillard, E., Badeau, P., Gendreau, M., Guertin, F. and JY, P. (1997), A tabu search heuristic for the vehicle routing problem with soft time windows. *Transportation Science*, v. 31, p. 170–186.

Taillard, E. D. (1999), A heuristic column generation method for heterogeneous fleet. *RAIRO (Recherche opérationnelle)*, v. 33, p. 1–14.

Tarantilis, C. D. and Kiranoudis, C. T. (2001), A meta-heuristic algorithm for the efficient distribution of perishable foods. *Journal of Food Engineering*, v. 50, p. 1–9.

Tarantilis, C. D. and Kiranoudis, C. T. (2007), A flexible adaptive memory-based algorithm for real-life transportation operations: Two case studies from dairy and construction sector. *European Journal of Operational Research*, v. 179, p. 806–822.

Tarantilis, C. D., Kiranoudis, C. T. and Vassiliadis, V. S. (2003), A list based threshold accepting metaheuristic for the heterogeneous fixed fleet vehicle routing problem. *Journal of the Operational Research Society*, v. 54, p. 65–71.

Tarantilis, C. D., Kiranoudis, C. T. and Vassiliadis, V. S. (2004), A threshold accepting metaheuristic for the heterogeneous fixed fleet vehicle routing problem. *European Journal of Operational Research*, v. 152, p. 148–158.

Tarantilis, C. D., Zachariadis, E. E. and Kiranoudis, C. T. (2008), A guided tabu search for the heterogeneous vehicle routeing problem. *Journal of the Operational Research Society*, v. 59, p. 1659–1673.