IFG-TODIM: AN INTUITIONISTIC FUZZY TODIM FOR GROUP DECISION MAKING

Renato A. Krohling
Departamento de Engenharia de Produção &
Programa de Pós-graduação em Informática - PPGI
UFES - Universidade Federal do Espírito Santo
Av. Fernando Ferrari, 514, CEP 29075-910, Vitória, Espírito Santo, ES
krohling.renato@gmail.com

André G. C. Pacheco
Departamento de Informática
UFES - Universidade Federal do Espírito Santo
Av. Fernando Ferrari, 514, CEP 29075-910, Vitória, Espírito Santo, ES
pacheco.comp@gmail.com,

RESUMO

Nos últimos anos, muitos métodos de tomada de decisão multicritério (inglês: *multi-criteria decision making*, abreviado por MCDM) têm sido propostos. Um desses métodos conhecido como TODIM tem sido expandido para lidar com problemas de tomada de decisão incertos. Primeiramente usando números *fuzzy* e recentemente usando números *fuzzy intuicionista*. O método recentemente desenvolvido abreviado por IF-TODIM permite modelar a informação *fuzzy* através de sua função de pertinência e de não pertinência. Neste artigo, nós extendemos o método IF-TODIM para abordar problemas envolvendo um grupo de tomadores de decisão. Um estudo de caso ilustra a aplicação e os resultados mostram a viabilidade do novo método.

Palavras chave: Tomada de decisão multicritério, tomada de decisão em grupo, números fuzzy intuicionista, IF-TODIM.

ADM – Apoio à Decisão Multicritério

ABSTRACT

In the last few years, several Multi-criteria Decision Making (MCDM) methods have been proposed. One of these methods, known as TODIM (an acronym in Portuguese for Interative Multi-criteria Decision Making), has been extended to uncertain MCDM problems. Firstly, using fuzzy numbers and recently using intuitionistic fuzzy numbers. The recently developed intuitionistic fuzzy TODIM, for short, IF-TODIM, allows to model the fuzzy information by means of its membership and non-membership functions. In this paper, we extend the IF-TODIM to tackle decision making problems that take into account a group of decision makers. A case study illustrates the application and the results show the feasibility of the new method.

Keywords: Multi-criteria decision making (MCDM), group decision-making, intuitionistic fuzzy numbers, IF-TODIM.

MCDM – Multi-criteria Decision Making

1. Introduction

Multi-criteria Decision Making (MCDM) problems occur in different areas of science and engineering (Hwang and Yoon, 1981). Several research efforts have been made in order to develop new methods or to improve existing ones. Typical challenges for MCDM methods are uncertainty, risk, among others. One of these MCDM methods named TODIM (an acronym in Portuguese for Iterative Multi-criteria Decision Making), was proposed by Gomes and Lima (1992). In the original formulation of the TODIM, the rating of alternatives, which composes the decision matrix, is represented by crisp values. Despite its effectiveness and simplicity in concept, this method presents some shortcomings because of its inability to deal with uncertainty and imprecision inherent in the process of decision making. The TODIM method in its original formulation (Gomes and Rangel, 2009) was not able to handle uncertainty. Recently, Krohling and de Souza (2012) presented a fuzzy TODIM to tackle uncertain MCDM problems. A clear advantage of this method is its ability to treat uncertain information using fuzzy numbers.

Atanasov (1986) proposed a more general theory for fuzzy sets, known as intuitionistic fuzzy sets, which is described by a membership function and a non-membership function. In the last few years, intuitionistic fuzzy numbers have been applied to solve MCDM problems (Atanassov, Pasi, and Yager, 2005; Xu, 2007; Lin, Yuan and Xia, 2007; Liu, and Wang, 2007; Boran, Kurt and Akay, 2009; Wang and Zhang, 2009a; Wang and Zhang, 2009b; Wei and Wang, 2009; Wei, 2010; Chen and Li, 2011; Shen, Wang and Feng, 2011). Based on fuzzy TODIM (Krohling and de Souza, 2012) and intuitionistic fuzzy numbers (Atanasov, 1986), we recently develop the intuitionistic fuzzy TODIM method, for short, IF-TODIM to handle uncertain MCDM problems. The goal of this work is to extend the previous work on IF-TODIM to a group of decision makers to find the best alternative given the importance weights assigned to each of the decision makers.

The remainder of this paper is organized as follows. In section 2, some preliminary background on intuitionistic fuzzy numbers is provided. In section 3, a new intuitionistic fuzzy TODIM for group decision making, named IFG-TODIM, for short, is developed to tackle uncertain decision matrices modeled by intuitionistic trapezoidal fuzzy numbers. In section 4, a case study is presented to illustrate the method and the results show the feasibility of the approach. In section 5, some conclusions and directions for further research are given.

2. Preliminaries on intuitionistic fuzzy sets and intuitionistic fuzzy numbers

Next, some basic definitions of intuitionistic fuzzy sets and fuzzy numbers are provided. The reader interested in more detailed information is referred to (Atanassov, 1986; Dubois et al., 2005; Szmidt, and Kacprzyk, 2000; Grzegorzewski, 2004).

Definition 1: Let X be the universe of discourse. An intuitionistic fuzzy set $\tilde{A}$ is characterized by a subset of X defined by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$, where $\mu_A : X \to [0;1]$ and $\nu_A : X \to [0;1]$ with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1 \mid \forall x \in X$. The numeric values $\mu_A(x)$ and $\nu_A(x)$ stands for the degree of membership and the degree of non-membership of x in A, respectively (Atanassov, 1986).

Definition 2: An intuitionistic trapezoidal fuzzy number $\tilde{a}$ is defined by $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$ with membership function given by (Wang and Zhang, 2009; Wei, 2010):

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} \cdot \tilde{\mu}_{\tilde{a}}, & a_2 > x \ge a_1 \\ \tilde{\mu}_{\tilde{a}}, & a_3 \ge x \ge a_2 \\ \dfrac{a_4 - x}{a_4 - a_3} . \tilde{\mu}_{\tilde{a}}, & a_4 \ge x > a_3 \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

while the non-membership function is given by:

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x) + \tilde{\nu}_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & a_2 > x \geq a_1 \\ \tilde{\nu}_{\tilde{a}} & a_3 \geq x \geq a_2 \\ \dfrac{(x - a_3) + \tilde{\nu}_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & a_4 \geq x > a_3 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The values $\tilde{\mu}_{\tilde{a}}$ and $\tilde{\nu}_{\tilde{a}}$ represent the maximum value of degree of membership and degree of non-membership of $\tilde{a}$, respectively. For instance, consider the intuitionistic trapezoidal fuzzy number (ITFN) $\langle VG; 0.6, 0.3\rangle = \langle 0.5, 0.75, 0.95, 1;\ 0.6, 0.3\rangle$. In this case, a decision maker not only assess the rating of an alternative by using the linguistically defined trapezoidal fuzzy number (TFN) *VG (Very Good)* but also provides the degree of membership and non-membership, 0.6 and 0.3 respectively.

Definition 3: Let $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$, be two intuitionistic trapezoidal fuzzy numbers and $k > 0$ be a scalar number, then the operation with these fuzzy numbers are defined as follows (Wang and Zhang, 2009; Wei, 2010):

1. Addition (+)

$$\tilde{a}(+)\tilde{b} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})\ (+)\ (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$$
$$= (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3, a_4 + b_4; \tilde{\mu}_{\tilde{a}} + \tilde{\mu}_{\tilde{b}} - \tilde{\mu}_{\tilde{a}} \cdot \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{a}} \cdot \tilde{\nu}_{\tilde{b}}).$$

2. Multiplication (.)

$$\tilde{a}(\cdot)\tilde{b} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})\ (\cdot)\ (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$$
$$= (a_1 \cdot b_1,\ a_2 \cdot b_2,\ a_3 \cdot b_3, a_4 \cdot b_4; \tilde{\mu}_{\tilde{a}} \cdot \tilde{\mu}_{\tilde{b}}, \tilde{\mu}_{\tilde{a}} + \tilde{\mu}_{\tilde{b}} - \tilde{\nu}_{\tilde{a}} \cdot \tilde{\nu}_{\tilde{b}}).$$

3. Multiplication by a scalar number k

$$k\tilde{a} = k\tilde{a} = k \cdot (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}}) = (ka_1,\ ka_2,\ ka_3, ka_4; 1 - (1 - \tilde{\mu}_{\tilde{a}})^k, \tilde{\nu}_{\tilde{a}}{}^k).$$

4. Exponentiation

$$\tilde{a}^k = (a_1{}^k,\ a_2{}^k,\ a_3{}^k, a_4{}^k; \tilde{\mu}_{\tilde{a}}{}^k, 1 - (1 - \tilde{\nu}_{\tilde{a}})^k).$$

Definition 4 Let an intuitionistic trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$, then its expected value is calculated as $I(\tilde{a}) = [(a_1 + a_2 + a_3 + a_4) \cdot (1 + \tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}})]/8$. In addition, definitions for $S(\tilde{a}) = I(\tilde{a}) \cdot (\tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}})$ and $H(\tilde{a}) = I(\tilde{a}) \cdot (\tilde{\mu}_{\tilde{a}} + \tilde{\nu}_{\tilde{a}})$ are presented, which are known as score function and accuracy function, respectively (Wang and Zhang, 2009).

Definition 5: Let two intuitionistic trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$, then (Wang and Zhang, 2009):

If $S(\tilde{a}) > S(\tilde{b})$ then $\tilde{a} > \tilde{b}$.

If $S(\tilde{a}) = S(\tilde{b})$ and If $H(\tilde{a}) = H(\tilde{b})$ then $\tilde{a} = \tilde{b}$.

If $S(\tilde{a}) = S(\tilde{b})$ and $H(\tilde{a}) > H(\tilde{b})$ then $\tilde{a} > \tilde{b}$.

Definition 6: Let two intuitionistic trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$, then the distance between them is calculated as (Wang and Zhang, 2009):

$$\begin{aligned} d(\tilde{a},\tilde{b}) = \frac{1}{8}[& \left|(1+\tilde{\mu}_{\tilde{a}}-\tilde{\nu}_{\tilde{a}})\cdot a_1-(1+\tilde{\mu}_{\tilde{b}}-\tilde{\nu}_{\tilde{b}})\cdot b_1\right| \\ & +\left|(1+\tilde{\mu}_{\tilde{a}}-\tilde{\nu}_{\tilde{a}})\cdot a_2-(1+\tilde{\mu}_{\tilde{b}}-\tilde{\nu}_{\tilde{b}})\cdot b_2\right| \\ & +\left|(1+\tilde{\mu}_{\tilde{a}}-\tilde{\nu}_{\tilde{a}})\cdot a_3-(1+\tilde{\mu}_{\tilde{b}}-\tilde{\nu}_{\tilde{b}})\cdot b_3\right| \\ & +\left|(1+\tilde{\mu}_{\tilde{a}}-\tilde{\nu}_{\tilde{a}})\cdot a_4-(1+\tilde{\mu}_{\tilde{b}}-\tilde{\nu}_{\tilde{b}})\cdot b_4\right|]. \end{aligned} \tag{3}$$

For instance, consider $\tilde{a} = (VL; 0.8, 0.1) = (0.1, 0.2, 0.3, 0.4; 0.8, 0.1)$ and $\tilde{b} = (EH; 0.7, 0.2) = (0.7, 0.8, 0.9, 0.95; 0.7, 0.2)$, where *VL* and *EH* are linguistic definitions of the trapezoidal fuzzy numbers, *Very Low* and *Extremely High*, respectively. The distance between them is $d(\tilde{a},\tilde{b}) = 0.4156$.

3. Intuitionistic Fuzzy Multi-criteria Decision Making

Intuitionistic trapezoidal fuzzy numbers are effective for solving decision-making problems, where the available information is imprecise.

Let us consider the fuzzy decision matrix *A*, which consists of a*lternatives* and *criteria*, described by:

$$A = \begin{array}{c} \\ A_1 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \ldots & C_n \end{array} \\ \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{pmatrix} \end{array}$$

where $A_1, A_2, \cdots, A_m$ are alternatives, $C_1, C_2, ..., C_n$ are criteria, and $\tilde{x}_{ij}$ are intuitionistic trapezoidal fuzzy numbers that indicates the rating of the alternative A_i with respect to criterion C_j. The weight vector $W = (w_1, w_2 ..., w_n)$ composed of the individual weights $w_j (j = 1, ..., n)$ for each criterion C_j satisfying $\sum_{j=1}^{n} w_j = 1$.

In the following section, the method is presented.

IFG-TODIM: An intuitionistic fuzzy TODIM for group decision making

The group decision-making framework proposed by Zhang & Lu (2003) integrates the following properties: decision makers may have different weights; decision makers can express fuzzy preferences for alternative solution; decision makers can give different judgments on selection criteria; and to each group member (decision maker) is assigned a weighting. The final group decision is made through aggregating group members' preferences on alternative under their weights and judgments on selection criteria. In a previous work, it was developed a Fuzzy TOPSIS for group decision making (Krohling & Campanharo, 2011). In this paper, based on that work we extend the recently developed intuitionistic Fuzzy TODIM (Krohling, Pacheco, & Siviero, 2013) for group decision making. So, it is possible to take into account the preferences of the decision makers.

Let us consider a group decision making problem, which consists of L members (DM) that participate in the decision-making process as given by $G = \{M_1, M_2, \ldots, M_L\}$. As we have a group of L decision makers, the weight vector with respect to each group member is described by $W^l = (w_1^l, w_2^l \ldots, w_n^l)$ with $l = 1, 2, \ldots, L$, where each w_j^l represents $\sum_{j=1}^{n} w_j^l = 1$ by the group member M_l, which satisfies $0 \leq w_l^j \leq 1$. We assume also that each group member (DM) has a degree of importance described by $0 \leq \propto_l \leq 1$, $\sum_{l=1}^{L} \propto_l = 1$.

The IF-TODIM method is applied to the intuitionisitic fuzzy decision matrix for each one of the L decision makers. The results are then aggregated to create a new decision matrix, with the results from the previous method. The TODIM method is then applied to the resulting decision matrix with the assigned importance weights to each of the L decision makers. After that, the ranking of each alternative through the final normalized values are obtained from the application of the TODIM method. The proposed method is illustrated in Fig. 1.

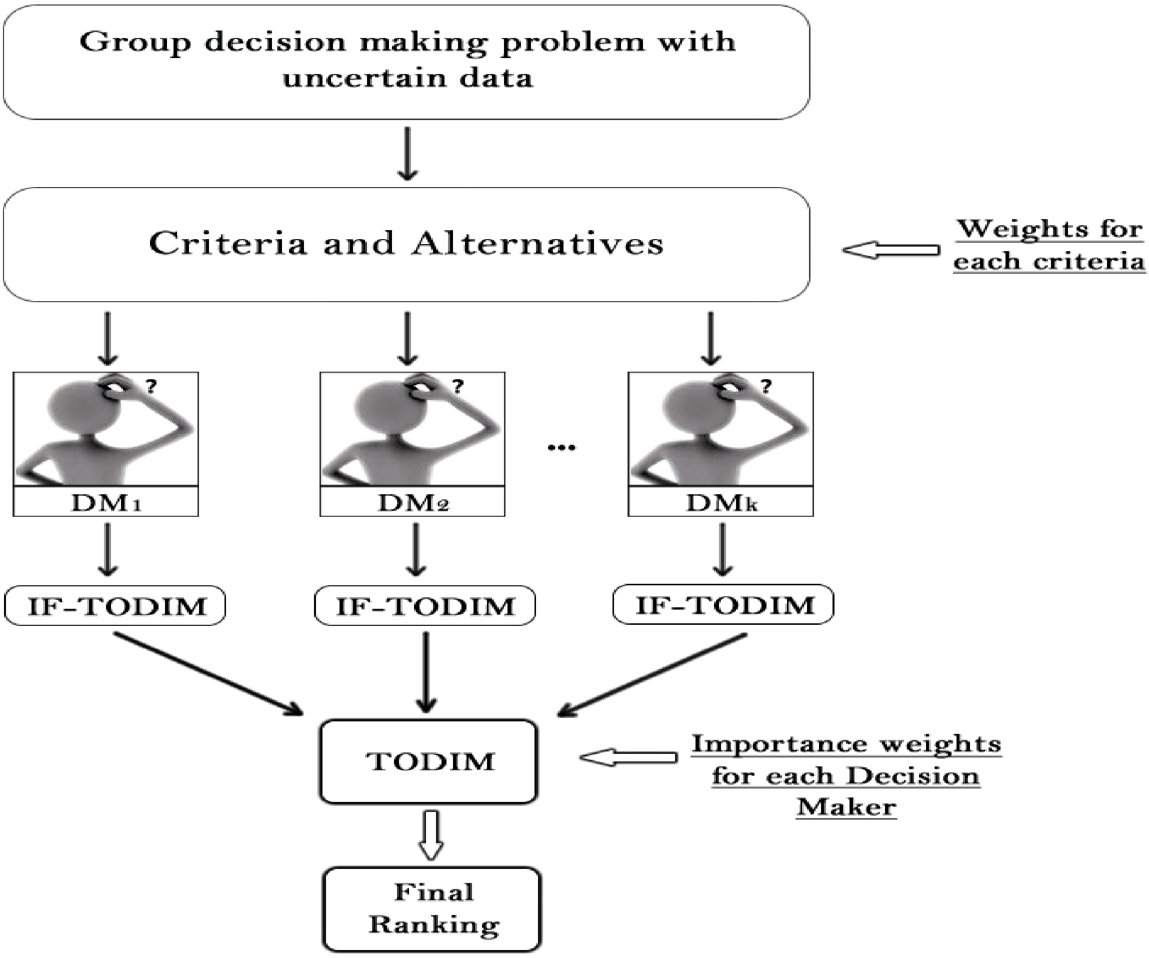

Figure 1. Illustration of the IFG-TODIM for group decision making.

The steps to calculate the best alternatives are described in the following:

Step 1: The criteria are normally classified into two types: *benefit* and *cost*. The intuitionistic trapezoidal fuzzy-decision matrix $\tilde{A} = \left[\tilde{x}_{ij}\right]_{mxn}$ with $i = 1, \ldots, m$, and $j = 1, \ldots, n$ is normalized, which results the correspondent fuzzy decision matrix $\tilde{R} = \left[\tilde{r}_{ij}\right]_{mxn}$. The fuzzy normalized value $\tilde{r}_{ij}$ is calculated as:

$$r_{ij}^{\,k} = \frac{\max(a_{ij}^{\,4}) - a_{ij}^{\,k}}{\max_i (a_{ij}^{\,4}) - \min_i a_{ij}^{\,1}} \text{ with } k=1,2,3,4 \qquad \text{for cost criteria}$$

$$r_{ij}^{\,k} = \frac{a_{ij}^{\,k} - \min(a_{ij}^{\,1})}{\max_i (a_{ij}^{\,4}) - \min_i a_{ij}^{\,1}} \text{ with } k=1,2,3,4 \qquad \text{for benefit criteria} \tag{4}$$

We denote a_k of $a_{ij} = \{(a_1, a_2, a_3, a_4)\}$ as $a_{ij}^{\,k}$, e.g., $a_{ij}^{\,3} = a_3$.

Step 2: Calculate the dominance of each alternative $\tilde{R}_i$ over each alternative $\tilde{R}_j$ using the following expression:

$$\delta(\tilde{R}_i, \tilde{R}_j) = \sum_{c=1}^{m} \phi_c(\tilde{R}_i, \tilde{R}_j) \qquad \forall (i, j) \tag{5}$$

where

$$\phi_c(\tilde{R}_i, \tilde{R}_j) = \begin{cases} \sqrt{\dfrac{w_{rc}}{\sum_{c=1}^{m} w_{rc}}} \cdot d(\tilde{r}_{ic}, \tilde{x}_{jc}) & \text{if } (\tilde{r}_{ic} > \tilde{r}_{jc}) \\ 0, & \text{if } (\tilde{r}_{ic} = \tilde{r}_{jc}) \\ \dfrac{-1}{\theta} \sqrt{\dfrac{\left(\sum_{c=1}^{m} w_{rc}\right)}{w_{rc}}} \cdot d(\tilde{r}_{ic}, \tilde{x}_{jc}) & \text{if } (\tilde{r}_{ic} < \tilde{r}_{jc}) \end{cases} \tag{6}$$

The term $\phi_c(\tilde{R}_i, \tilde{R}_j)$, denoted by partial dominance, represents the contribution of the criterion c to the function $\delta(\tilde{R}_i, \tilde{R}_j)$ when comparing the alternative i with alternative j. The values $\tilde{r}_{ic}$ and $\tilde{r}_{ij}$ are the rating of the alternatives i and j, respectively with respect to criterion c. The value w_{rc} represents the weight of criterion c divided by the weight of the reference r, i.e., $w_{rc} = \frac{w_c}{w_r}$, whereas the latter is the criterion that has the greater weight. The term $d(\tilde{r}_{ic}, \tilde{r}_{jc})$ stands for the distance between the two intuitionistic fuzzy numbers $\tilde{r}_{ic}$ and $\tilde{r}_{jc}$, calculated by Eq. (3). Three cases can occur in Eq. (6): i) if $(\tilde{r}_{ic} > \tilde{r}_{jc})$, it represents a gain; ii) if $(\tilde{r}_{ic} = \tilde{r}_{jc})$, it is nil; and iii) if $(\tilde{r}_{ic} < \tilde{r}_{jc})$, it represent a loss. Definitions (4) and (5) are used in each case. The parameter θ represents the attenuation factor of the losses. The final matrix of dominance is obtained by summing up the partial matrices of dominance for each criterion.

Step 3: Calculate the global value of the alternative i by normalizing the final matrix of dominance according to the following expression:

$$\xi_i = \frac{\sum \delta(i, j) - \min \sum \delta(i, j)}{\max \sum \delta(i, j) - \min \sum \delta(i, j)} \tag{7}$$

The final matrix of dominance for each group member is then aggregated to form a new crisp decision matrix as given by:

$$C = \begin{pmatrix} {}^{1}\xi(A_1) & \cdots & {}^{L}\xi(A_1) \\ \vdots & \ddots & \vdots \\ {}^{1}\xi(A_m) & \cdots & {}^{L}\xi(A_m) \end{pmatrix} \quad (8)$$

From this stage on our method continues by applying the standard TODIM method to the resulting decision matrix given by (8) in order to identify the matrix of dominance and the ranking of the alternatives.

Step 4: For the decision matrix *C*, we now have associated an importance weight to each group member α_l, for $l = 1,...,L$. Within the decision matrix *C*, we calculate the dominance of each alternative A_i over each alternative A_j using the following expression:

$$\delta_G(A_i, A_j) = \sum_{c=1}^{L} \phi_l(A_i, A_j) \qquad \forall (i, j) \quad (9)$$

where

$$\phi_l(A_i, A_j) = \begin{cases} \sqrt{\dfrac{\alpha_l (x_{ic} - x_{jc})}{\sum_{c=1}^{L} \alpha_l}} & \text{if } (x_{ic} - x_{jc}) > 0 \\ 0 & \text{if } (x_{ic} - x_{jc}) = 0 \\ \dfrac{-1}{\theta} \sqrt{\dfrac{\left(\sum_{c=1}^{m} \alpha_l\right)(x_{jc} - x_{ic})}{\alpha_l}} & \text{if } (x_{ic} - x_{jc}) < 0 \end{cases} \quad (10)$$

The term $\phi_l(A_i, A_j)$ represents the contribution of the group member *l* to the function $\delta_G(A_i, A_j)$ when comparing the alternative *i* with alternative *j*. The parameter θ represents the attenuation factor of the losses, which can be tuned according to the problem at hand. In expression 10) it can occur 3 cases: i) if the value $(x_{ic} - x_{jc})$ is positive, it represents a gain; ii) if the value $(x_{ic} - x_{jc})$ is nil; and iii) if the value $(x_{ic} - x_{jc})$ is negative, it represent a loss. The final matrix of dominance is obtained by summing up the partial matrices of dominance for each group member.

Step 5: Calculate the global value of the alternative by normalizing the final matrix of dominance according to the following expression:

$$\xi_G = \frac{\sum \delta_G(i, j) - \min \sum \delta_G(i, j)}{\max \sum \delta_G(i, j) - \min \sum \delta_G(i, j)} \quad (11)$$

Sorting the values ξ_G provides the rank of each alternative. The best alternatives are those that have higher value ξ_G.

4. Experimental Results

This case study, which is used as benchmark, was investigated by Shen, Wang and Feng (2011). The authors also developed a group decision making method using intuitionistic trapezoidal fuzzy numbers. In this case, the problem consists of an information system made up of 5 sub-systems which are the alternatives A_i with $i = 1,...,5$. The risks of each sub-system are evaluated according to four criteria C_j with $j = 1,...,4$. The weight vector associated to each criterion is given by $W = (w_1, w_2, w_3, w_4) = (0.2, 0.1, 0.3, 0.4)$. The goal is to find out the most unsafe sub-system for a specific application. There are three decision makers (DM) involved in the assessment of the five sub-systems according to the four criteria. The intuitionistic trapezoidal fuzzy decision matrix for each decision maker k described by $\widetilde{R}_k = (\tilde{r}_{ij}^{(k)})_{5x4}$ with $k = 1,2,3$ is given in Table 1. The importance weight of each decision maker used is $\alpha = (0.35, 0.4, 0.25)$. First, the IF-TODIM is applied to each one of the three intuitionistic fuzzy decision matrix, which results the 3 columns listed Table 2. Next, the application of the original TODIM to the 3 columns of the matrix in Table 2 results the final ranking as given in the 4th column of the same matrix.

As we can notice, the final ranking obtained is $A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$. In this case, the sub-system A_2 is the most critical one. We carried out the experiments for two different values of $\theta = 1$ and $\theta = 2.5$. One observes that the parameter θ has a small influence on the value ξ_i, but it does not change the final ranking of the alternatives. The results obtained for $\theta = 1$ and $\theta = 2.5$ is listed in Table 2 and Table 3, respectively. The ranking of the alternatives and the prospect function for $\theta = 1$ and $\theta = 2.5$ are depicted in Fig. 2 and Fig. 3, respectively.

Table 1: Intuitionistic fuzzy decision matrices for 3 decision makers.

$$\widetilde{R}_1 = \begin{bmatrix} (0.5\ 0.6\ 0.7\ 0.8;\ 0.5,0.4) & (0.1\ 0.2\ 0.3\ 0.4;\ 0.6,0.3) & (0.5\ 0.6\ 0.8\ 0.9;\ 0.3,0.6) & (0.4\ 0.5\ 0.6\ 0.7;\ 0.2,0.7) \\ (0.6\ 0.7\ 0.8\ 0.9;\ 0.7,0.3) & (0.5\ 0.6\ 0.7\ 0.8;\ 0.7,0.2) & (0.4\ 0.5\ 0.7\ 0.8;\ 0.7,0.2) & (0.5\ 0.6\ 0.7\ 0.9;\ 0.4,0.5) \\ (0.1\ 0.2\ 0.4\ 0.5;\ 0.6,0.4) & (0.2\ 0.3\ 0.5\ 0.6;\ 0.5,0.4) & (0.5\ 0.6\ 0.7\ 0.8;\ 0.5,0.3) & (0.3\ 0.5\ 0.7\ 0.9;\ 0.2,0.3) \\ (0.3\ 0.4\ 0.5\ 0.6;\ 0.8,0.1) & (0.1\ 0.3\ 0.4\ 0.5;\ 0.6,0.3) & (0.1\ 0.3\ 0.5\ 0.7;\ 0.3,0.4) & (0.6\ 0.7\ 0.8\ 0.9;\ 0.2,0.6) \\ (0.2\ 0.3\ 0.4\ 0.5;\ 0.6,0.2) & (0.3\ 0.4\ 0.5\ 0.6;\ 0.4,0.3) & (0.2\ 0.3\ 0.4\ 0.5;\ 0.7,0.1) & (0.5\ 0.6\ 0.7\ 0.8;\ 0.1,0.3) \end{bmatrix}$$

$$\widetilde{R}_2 = \begin{bmatrix} (0.4\ 0.5\ 0.6\ 0.7;\ 0.4,0.3) & (0.1\ 0.2\ 0.3\ 0.4;\ 0.6,0.2) & (0.4\ 0.5\ 0.7\ 0.8;\ 0.2,0.5) & (0.3\ 0.4\ 0.5\ 0.6;\ 0.1,0.6) \\ (0.5\ 0.6\ 0.7\ 0.8;\ 0.6,0.2) & (0.4\ 0.5\ 0.6\ 0.7;\ 0.6,0.1) & (0.3\ 0.4\ 0.6\ 0.7;\ 0.6,0.1) & (0.4\ 0.5\ 0.6\ 0.8;\ 0.3,0.4) \\ (0.1\ 0.2\ 0.3\ 0.4;\ 0.5,0.3) & (0.1\ 0.2\ 0.4\ 0.5;\ 0.4,0.3) & (0.4\ 0.5\ 0.6\ 0.7;\ 0.4,0.2) & (0.2\ 0.4\ 0.6\ 0.6;\ 0.5,0.2) \\ (0.3\ 0.4\ 0.5\ 0.6;\ 0.8,0.1) & (0.1\ 0.2\ 0.3\ 0.5;\ 0.5,0.2) & (0.1\ 0.2\ 0.4\ 0.6;\ 0.2,0.3) & (0.5\ 0.6\ 0.7\ 0.8;\ 0.1,0.5) \\ (0.1\ 0.2\ 0.3\ 0.4;\ 0.5,0.1) & (0.2\ 0.3\ 0.4\ 0.5;\ 0.3,0.2) & (0.1\ 0.2\ 0.3\ 0.4;\ 0.6,0.2) & (0.4\ 0.5\ 0.6\ 0.7;\ 0.4,0.2) \end{bmatrix}$$

$$\widetilde{R}_3 = \begin{bmatrix} (0.6\ 0.7\ 0.8\ 0.9;\ 0.4,0.5) & (0.2\ 0.3\ 0.4\ 0.5;\ 0.5,0.4) & (0.6\ 0.7\ 0.9\ 1.0;\ 0.2,0.7) & (0.5\ 0.6\ 0.7\ 0.8;\ 0.1,0.8) \\ (0.7\ 0.8\ 0.9\ 1.0;\ 0.6,0.4) & (0.6\ 0.7\ 0.8\ 0.9;\ 0.6,0.3) & (0.5\ 0.6\ 0.8\ 0.9;\ 0.6,0.3) & (0.6\ 0.7\ 0.8\ 1.0;\ 0.3,0.6) \\ (0.2\ 0.3\ 0.5\ 0.6;\ 0.5,0.5) & (0.3\ 0.4\ 0.6\ 0.7;\ 0.4,0.5) & (0.6\ 0.7\ 0.8\ 0.9;\ 0.4,0.4) & (0.4\ 0.6\ 0.8\ 1.0;\ 0.5,0.4) \\ (0.4\ 0.5\ 0.6\ 0.7;\ 0.7,0.2) & (0.2\ 0.4\ 0.5\ 0.6;\ 0.5,0.4) & (0.2\ 0.4\ 0.6\ 0.8;\ 0.2,0.5) & (0.7\ 0.8\ 0.9\ 1.0;\ 0.1,0.7) \\ (0.3\ 0.4\ 0.5\ 0.6;\ 0.5,0.3) & (0.4\ 0.5\ 0.6\ 0.7;\ 0.3,0.4) & (0.3\ 0.4\ 0.5\ 0.6;\ 0.6,0.2) & (0.6\ 0.7\ 0.8\ 0.9;\ 0.4,0.4) \end{bmatrix}$$

Table 2: Ranking of the alternatives using θ = 1.

Alternatives	DM 1	DM 2	DM 3	Final Ranking
	${}^{1}\xi_i$	${}^{2}\xi_i$	${}^{3}\xi_i$	${}^{G}\xi_i$
A_1	0	0	0	0
A_2	1.0000	1.0000	1.0000	1.0000
A_3	0.2969	0.1241	0.4714	0.4237
A_4	0.4192	0.2986	0.3907	0.5169
A_5	0.3561	0.3469	0.4840	0.5853

Table 3: Ranking of the alternatives using θ = 2.5.

Alternatives	DM 1	DM 2	DM 3	Final Ranking
	ξ_1	ξ_2	ξ_3	${}^{G}\xi_i$
A_1	0	0	0	0
A_2	1.0000	1.0000	1.0000	1.0000
A_3	0.3191	0.1831	0.5021	0.3950
A_4	0.3737	0.2762	0.3419	0.4037
A_5	0.3454	0.3722	0.4985	0.5066

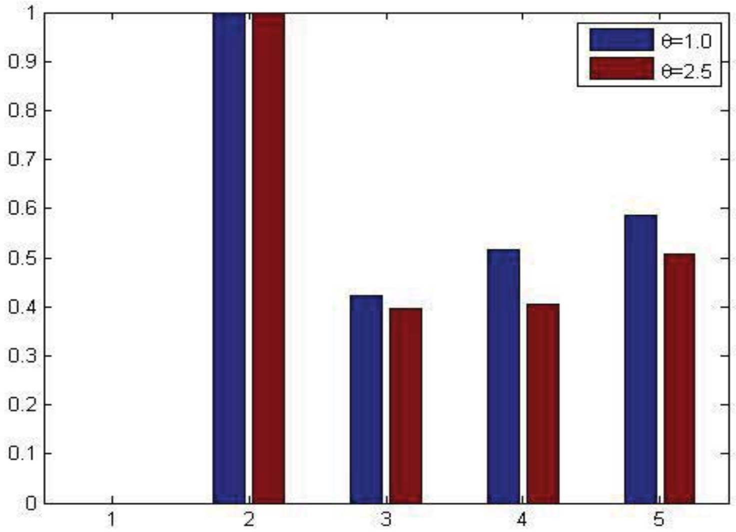

Figure 2. Ranking of the alternatives for $\theta = 1$ and $\theta = 2.5$.

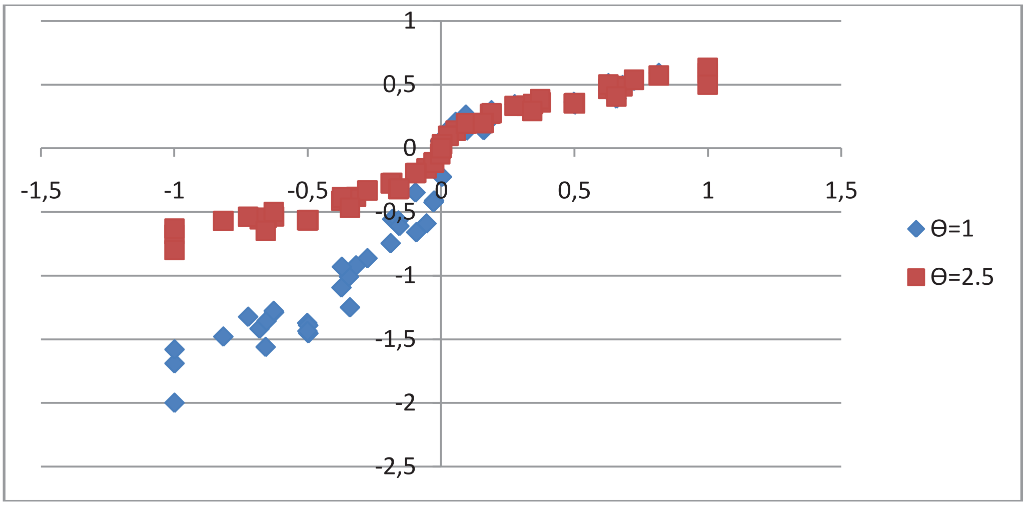

Figure 3. Prospect function value for $\theta = 1$ and $\theta = 2.5$.

According to the results obtained, the best alternative is A2. However, the uncertainty of the decision matrices (the membership and non-membership degrees) may affect the final ranking of the alternatives as recently investigated by Krohling, Pacheco, and Siviero (2013). The method can be applied to other MCDM problems with a finite number of alternatives, criteria and decision makers.

4. Conclusions

In this paper, based on previous work on intuitionistic fuzzy TODIM, we extend the approach for group decision making, for short, IFG-TODIM. This approach takes into account the uncertainty of the decision matrices and the importance weight of the decision makers to find the best alternative in a multi-criteria decision making problem. In this work, we have applied the proposed method to a case study, where the decision matrices are represented by intuitionistic fuzzyy matrices and the results indicated the effectiveness of the IFG-TODIM method when considering several decision makers with different importance weights. The method is currently being expanded to other applications.

Acknowledgements: R.A. Krohling would like to thank the financial support of the Brazilian agency CNPq under Grant No. 303577/2012-6.

References

Atanassov. K. T. (1986). Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, *20*, 87-96.

Atanassov, K., Pasi, G., & Yager, R. (2005). Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making. *International Journal of Systems Science*, *36*(14), 859-868.

Bellman, R.E., & Zadeh, L.A. (1970). Decision-making in a fuzzy environment. *Management Science*, *17*, 141-164.

Boran, F. E., Kurt, M., & Akay, D. (2009). A multi-criteria intuitionistic fuzzy group decision making for selection of supplier with TOPSIS method. *Expert Systems with Applications*, *36*(8), 11363-11368.

Chen, Y., & Li, B. (2011). Dynamic multi-attribute decision making model based on triangular intuitionistic fuzzy numbers. *Scientia Iranica*, *18*(2), 268-274.

Dubois, D., Gottwald, S., Hajek, P., Kacprzyk, J., Prade, H. (2005). Terminological difficulties in fuzzy set theory - the case of intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *156*(3), 485-491.

Dubois, D., Prade, H. (1980). *Fuzzy Sets and Systems: Theory and Applications*. New York: Academic Press.

Gomes, L.F.A.M., & Lima, M.M.P.P. (1992). TODIM: Basics and application to multicriteria ranking of projects with environmental impacts. *Foundations of Computing and Decision Sciences*, *16* (4), 113-127.
Gomes, L.F.A.M., & Rangel, L.A.D. (2009). An application of the TODIM method to the multicriteria rental evaluation of residential properties. *European Journal of Operational Research 193*, 204-211.
Grzegorzewski P. (2004). Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. *Fuzzy Sets and Systems*, *148*, 319-328.
Hwang, C. L., & Yoon, K. *Multiple Attribute Decision Making*. Berlin: Springer-Verlag, 1981.
Kahneman, D., Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*, 263-292.
Krohling, R.A., & Campanharo, V.C. (2011). Fuzzy TOPSIS for group decision making: A case study for accidents with oil spill in the sea. *Expert Systems with Applications*, *38*, 4190-4197.
Krohling, R. A., & de Souza, T.T.M. (2012). Combining prospect theory and fuzzy numbers to multi-criteria decision making. *Expert Systems with Applications*, *39*, 11487-11493.
Krohling, R. Pacheco, A.G.C., & Siviero, A.L.T. (2013). IF-TODIM: An intuitionistic fuzzy TODIM for decision making. Relatório Técnico, PPGI/UFES, 2013.
Lin, L., Yuan, X. H., & Xia, Z.Q. (2007). Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets. *Journal of Computer and System*, *73*(1), 84-88.
Liu, H. W., & Wang, G. J. (2007). Multi-criteria decision-making methods based on intuitionistic fuzzy sets. *European Journal of Operational Research*, *179*(1), 220-233.
Salminen, P. (1994). Solving the discrete multiple criteria problem using linear prospect theory. *European Journal of Operational Research*, *72*(1), 146-54.
Shen, L., Wang, H., & Feng, X. (2011). Some arithmetic aggregation operators within intuitionistic trapezoidal fuzzy setting and their application to group decision making, MSIE, 1053-1059.
Szmidt, E., Kacprzyk, J. (2000). Distances between intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *114*, 505-518.
Tversky, A., Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*(4), 297-323.
Wang J. Q., & Zhang, Z. (2009). Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems. *Journal of Systems Engineering and Electronics*, *20*, 321-326.
Wang, J. Q., Zhang, Z. (2009). Multi-criteria decision-making method with incomplete certain information based on intuitionistic fuzzy number. *Control and Decision*, *24*(2), 226-230.
Wei, G. W. (2010). Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making. *Journal of Computers*, *5*, 345-351.
Wei, G., & Wang, H. ITFWG operator and its application to decision making, *Chinese Control and Decision Conference (CCDC)*, 5904-5909, 2009.
Xu, Z. (2007). Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making. *Fuzzy Optimization and Decision Making*, *6*, 109-121.
Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, *8*, 338-353.