A CONFIDENCEMEWMA CONTROL CHART FOR GAUSSIAN MEAN VECTORS

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Abstract: *In this work we show that for normal distributions theHotelling´s T and MEWMA distances are straight related to the Bhattacharyya distance. This distance provides interesting information concerning on the upper bound ofthe misclassification error probability, which is very difficult to compute precisely. Therefore, the purpose of this simulation study is to monitor the mean vector of Gaussian process by means of a more informative control chart based on probabilities. A comparison study shows that although this propose do not increase the power of change detection as measured by the average run length, this useful information being bounded in the 0-1 rangeindicates the overlap degree between the in- and out-of-control processes. The simulated results demonstrated that the confidence control chart MEWMA based is easy to calibrate and revealsa new approach in the monitoring of Gaussian mean vectors.*

Keywords: Gaussian point processes, mean vectors, statistical process control, noncentrality parameter, multivariate exponentially moving average, Bhattacharyya distance.

1. Introduction

In many industrial problems theprobability of misclassification is a subject of great interest, but the calculation is a difficult task even when the observed datais normal. Therefore, the idea of monitoringa process by calculating its probability to be in- or out-of-control is usually discarded. Recent advanced statistical techniqueswith applications to the $\bar{\mathbf{X}}$ and the $\mathbf{S}^2$ control chartincludes the univariate case (Faraz and Saniga, 2012), and the multivariate case, where a recent work also covers the global process monitoring by controlling the mean vector and covariance matrix simultaneously (Niaki and Memar, 2009).

Considering the process control of mean vectors only, the most utilized method to monitor big shifts is the Hotelling's T control chart (Hotelling, 1947), while in the case of smaller shifts the multivariate exponentially weighted moving average (MEWMA) control chart (Lowry et al., 1992) is more popular for being simpler to implement when compared to its most famous concurrent, themultivariate cumulative sums (MCUSUM) control chart (Crosier, 1988). Although the methodology utilized in this work may be extend for the multivariate global process monitoring by means of probability measures, as an initial propose we only consider the process control of multivariate mean vectors.The main problem related to the monitoring of mean vectors can be addressed to the emptiness property of the hyperspaces (Jimenez and Landgrebe, 1998). In a general context, the impact of the limited sample data to estimate a large quantity of parameters is also known as *the curse of dimensionality*, which has been proved by Hughes (1968) in a strong theoretical basis.

If a closed-form for the error probability is not provided,one may seek either an approximate expression or an upper bound on the error probability. A closed form for the upper bound on the error probability is very useful for many reasons. Beyond of reducing the computational effort greatly, the evaluation of a simple formula may provide an insightful knowledge about the actual process state. Furthermore, the misclassification error increases significantly with the number of dimensions, reducing dramatically the standardconfidence levels that the process is actually in-control(Fukunaga, 1990).Due to this fact, the evaluation of a probabilitymeasure instead of raw distances givesmore valuable information about the price we have to pay for not knowing the alternative process state*a priori*.Focusing this objective, the monitoring of Gaussian mean vectors by means of a simple distance transformation that leads to a control chart directly based on probabilities is discussed in this work.

In the following sections, the main properties of the noncentrality parameter traditionally used to monitor the mean vector by the Hotelling's T and MEWMA control charts, as well the link with the upper bound on the error probability is described. Next, the experiments onthe Hotelling's T and the probability control chart performance with individual observations are comparedby the computation of the average run lengths (ARLs), or average time to signal (ATS) as the interval between observations is regular. Finally, some remarks and recommendations on the confidence control chart are made.

2. Methodology

It is known that the performance measured by the average run length (ARL) of traditional control charts like the Hotelling'sT and MEWMA depends only on the noncentrality parameter, not dependingon the shift's direction (Lowry et al., 1992). This distance is given by

$$\mathrm{d}_t^2 = (\mathbf{X}_t - \mathbf{M}_0)^{\mathrm{T}} \mathbf{\Sigma}_0^{-1} (\mathbf{X}_t - \mathbf{M}_0), \quad (1)$$

where $\mathbf{X}_t$, $\mathbf{M}_0$ and $\mathbf{\Sigma}_0$ are the observed vector, the in-control mean vector and the in-control covariance matrix, respectively. The decision rule gives an out-of-control signal as soon as $\mathrm{d}_t^2 > h_1$, where h_1 is a specified threshold that leads to a pre-specified false alarm rate, usually defined in terms of the ARL.

In his original paper in 1947, Hotelling suggested the utilization of d^2 instead of d to avoid the labor of extracting the square root, butas the computational power has massively increased in the last decades it is almost no matter anymore. Thus, to maintain clear the effect on the in-control limits,in this work d is used for experiment comparisons varying in the 0-4 range.

While the Hotelling's T considers the global process monitoring by outlying observations that are outside the in-control boundaries, the MEWMA statistic considers the entire process to be out-of-control as soon as

$$z_t^2 = (\mathbf{M}_t - \mathbf{M}_0)^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\mathbf{M}_t - \mathbf{M}_0) > h_2, \quad (2)$$

where$\mathbf{M}_t$ is the mean vector estimated with past and current information by a MEWMA scheme, such that

$$\mathbf{M}_t = (1 - \lambda)\mathbf{M}_{t-1} + \lambda \mathbf{X}_t. \quad (3)$$

and $0 < \lambda \leq 1$. Observe that when $\lambda = 1$, the MEWMA distance reduces to the Hotelling's distance.

The noncentrality parameteris very popular in the pattern recognition field (Therrien, 1989), also known as Mahalanobis distance, and with a straight connection with the Bhattacharyya distance, which is derived from the most general case, the Chernoffbounds (Fukunaga, 1990). That boundaries leads to a closed-form expression to computean upper limit on the Bayes error forthe case of normal distributed processes such as

$$\varepsilon = \sqrt{\mathrm{P}_1 * \mathrm{P}_2} \int \sqrt{\mathrm{p}_1(\mathrm{X}) * \mathrm{p}_2(\mathrm{X})}\, \mathrm{dX} = \sqrt{\mathrm{P}_1 * \mathrm{P}_2}\, \mathrm{e}^{-\mu(1/2)}, \quad (4)$$

where

$$\mu(1/2) = \frac{1}{8}(\mathbf{M}_2 - \mathbf{M}_1)^{\mathrm{T}} \left(\frac{\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2}{2}\right)^{-1} (\mathbf{M}_2 - \mathbf{M}_1) + \frac{1}{2} \ln \frac{\left|\frac{\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2}{2}\right|}{\sqrt{|\mathbf{\Sigma}_1||\mathbf{\Sigma}_2|}}. \quad (5)$$

The term $\mu(1/2)$ is called theBhattacharyya distance and is used as an important separabilitymeasure between two normal distributions, where$\mathbf{M}_i$and $\mathbf{\Sigma}_i$, $i = 1{,}2$, are the mean vector and covariance matrix of each class. This distance is composed of two terms, the first one carrying the information about the process difference in the mean vectors, and the second part corresponding to the difference in the covariance matrices.

Rao (1947) explained that this distance is an explicit function of the proportion of overlapping individuals in the two populations. Rao (1949) also commented that Bhattacharyya had developed a perfectly general measure defined by the distance between two populations based on a metric of Riemannian geometry, with the angular distance between points representing the populations in a unit sphere.

In the case of single-hypothesis tests, like in statistical process control (SPC) problems,the out-of-control state isgenerally undetermined. Then, instead of utilizing equation (4) which supposes two known processes, it is more interesting to evaluate only the upper bound for the Type I error, which refers only to the known process and isgiven by

$$\varepsilon_{\mathrm{I}} = \sqrt{\mathrm{P}_2/\mathrm{P}_1} \int \sqrt{\mathrm{p}_1(\mathrm{X}) * \mathrm{p}_2(\mathrm{X})}\, \mathrm{dX} = \sqrt{\mathrm{P}_2/\mathrm{P}_1}\, \mathrm{e}^{-\mu(1/2)}. \quad (6)$$

Also, as this work is focused in the monitoring of mean vectors only, the assumption of equal covariance matrices reduces the Bhattacharyya distance to the noncentrality parameter, exceptby a constant,assuming the form

$$\mu(1/2) = \frac{1}{8}(\mathbf{M}_t - \mathbf{M}_0)^{\mathrm{T}} \boldsymbol{\Sigma}_0{}^{-1} (\mathbf{M}_t - \mathbf{M}_0). \quad (7)$$

where$\mathbf{M}_t$ is the mean vector estimated at the instant *t*, $\mathbf{M}_0$ is the in-control mean vector and $\boldsymbol{\Sigma}_0$is the in-control covariance matrix.

This simplified form preserves all the known properties of the Hotelling's T and MEWMA control chart with respect to the performance measured by the average run length. To examine the mainproperties of this distance, let us consider the distribution of $\mathrm{d}^2$with expected vector $\mathbf{M}$ and the covariance matrix $\boldsymbol{\Sigma}$. Then, the standardized distance from individual observations to the process center is

$$\mathrm{d}^2 = (\mathbf{X} - \mathbf{M})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\mathbf{X} - \mathbf{M}) = \mathbf{Z}^{\mathrm{T}} \mathbf{Z} = \sum_{i=1}^{n} z_i^2, \quad (8)$$

where$\mathbf{Z} = \mathbf{A}^{\mathrm{T}}(\mathbf{X} - \mathbf{M})$ and $\mathbf{A}$ is the whitening transformation. Since the expected vector and covariance matrix of $\mathbf{Z}$ are $\mathbf{0}$ and $\boldsymbol{I}$ respectively, the $\mathrm{z_i}$'s are uncorrelated, and $\mathrm{E}(\mathrm{z_i}) = 0$ and $\mathrm{Var}(\mathrm{z_i}) = 1$. Thus, the expected value and variance of $\mathrm{d}^2$for the in-control process (IC) are

$$\mathrm{E}(\mathrm{d}^2|\mathrm{IC}) = n\, \mathrm{E}\left(\mathrm{z_i^2}\right) = n \quad (9)$$

$$\mathrm{Var}(\mathrm{d}^2|\mathrm{IC}) = \mathrm{E}((\mathrm{d}^2)^2) - \mathrm{E}^2(\mathrm{d}^2)$$

$$\mathrm{Var}(\mathrm{d}^2|\mathrm{IC}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{E}\left(\mathrm{z_i^4}\right) + \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1_{\mathrm{i}\neq\mathrm{j}}}^{\mathrm{n}} \mathrm{E}\left(\mathrm{z_i^2 z_i^2}\right) - n^2\, \mathrm{E}^2\left(\mathrm{z_i^2}\right). \quad (10)$$

When the z_i^2's are uncorrelated (this is satisfied when the $\mathrm{z_i}$'s are independent), and $\mathrm{E}\left(z_i^4\right)$ is independent of i, the variance of d^2can be further simplified to

$$\mathrm{Var}(\mathrm{d}^2|\mathrm{IC}) = n\gamma. \quad (11)$$

$$\gamma = \mathrm{E}\left(\mathrm{z_i^4}\right) - \mathrm{E}^2\left(\mathrm{z_i^2}\right) = \mathrm{E}\left(\mathrm{z_i^4}\right) - 1. \quad (12)$$

For normal distributions, when the $\mathrm{z_i}$'s are uncorrelated, they are also independent, therefore, (10) can be used to compute $\mathrm{Var}(\mathrm{d}^2|\mathrm{IC})$, and $\gamma = 2$. Note that in (9) and (10), only the first and second order moments of d^2 are given. However, if the $\mathrm{z_i}$'s are normal, the density function of d^2 is the gamma density with $\beta = n/2 - 1$ and $\alpha = 1/2$.Since the $\mathrm{z_i}$'s are obtained by a linear transformation of X, the $\mathrm{z_i}$'s are normal if X is normal. Note that the gamma distribution becomes an exponential distribution for$n = 2$. Indeed,the distribution of d^2 with the mean n and standart deviation $\sqrt{n\gamma}$ approximates the normal when n large.

Considering the out-of-control state (OC) with mean vector $\mathbf{M}_1$, the expected value of d^2 under assumption of equal covariance matrices *I-I* is given as

$$\mathrm{E}(\mathrm{d}^2|\mathrm{OC}) = n + \mathbf{M}_1{}^{\mathbf{T}} \mathbf{M}_1, \quad (11)$$

and variance as

$$\mathrm{Var}(\mathrm{d}^2|\mathrm{OC}) = 2n + 4\mathbf{M}_1{}^{\mathrm{T}}\mathbf{M}_1. \quad (12)$$

These results may be extended to the case where the sample mean and sample covariance matrix are used in place of known parameters, as

$$\zeta = \frac{1}{N-1}\left(\mathbf{X} - \widehat{\mathbf{M}}\right)^{\mathrm{T}}\widehat{\boldsymbol{\Sigma}}^{-1}\left(\mathbf{X} - \widehat{\mathbf{M}}\right). \quad (13)$$

When X is normal, ζ has the beta distribution with $\mathrm{E}(\zeta|\mathrm{IC}) = n/(N-1)$and $\mathrm{Var}(\zeta|\mathrm{IC}) = 2n/(N-1)^2$.

The simulated experiments presented in the following section agree with the presented theoretical values for the first and second moments of the Bhattacharyya distance as well for the Hotelling's T with high precision.

2.1. Confidencecontrol charts

The theoretical resultsearlypresented provide a different look in the process monitoring by transforming the statistical raw distances and theirs respective in-control boundaries into probability values as standard patterns. First, if there is no special reason to weight the in- and out-of-control process differently, the processes are equally weighted in equation (6), thus reducing the upper bound on the Type I error to $\exp(-\mu(1/2))$. Different weights for the processes will result in a scale modification, but still preserving the 0-1 domain.

Observe that when the process is actually in-control, the estimatedmean vector, or individual observations, must show no significant difference from the in-control standard error levels. This leads to an upper bound ofε_{I} that is near to one because the in-control and current processesare completely overlapped. When the mean vector shiftsto the out-of-control state, the upper bound onε_{I} decreases indicating less overlappingamong the processes. By other hand, if the complementary probability is taken, it indicates an upper bound on the confidencelevel which is closer to zero, meaning that the current process is not being apart from the in-control state.

Based on those appointments, the probability control chart when individual observations are compared to the in-control mean vector is taken as the standard level for the different waysof estimating the mean vector. This approach can be viewed as the MEWMA chart with$\lambda = 1$, which is a simple scale transformation of the Hotelling's T by the use of Bhattacharyya distance, triggering a signal as soon as

$$p_t = 1 - \exp\left[-\frac{1}{8}(\mathbf{X}_t - \mathbf{M}_0)^{\mathrm{T}}\boldsymbol{\Sigma}_0^{-1}(\mathbf{X}_t - \mathbf{M}_0)\right] > h_1^*, \quad (14)$$

whereh_1^* is the in-control upper limit to achieve a desired ARL_0.

By reducing the λvalue, the individual observed vector is changed by a mean vector, performing like a transformedMEWMA control chart, where Equation (3) is utilized to estimate the current mean vector, $\mathbf{M}_t$. In this case, the confidence control chart triggers an out-of-control signal as soon as

$$p_t = 1 - \exp\left[-\frac{1}{8}(\mathbf{M}_t - \mathbf{M}_0)^{\mathrm{T}}\boldsymbol{\Sigma}_0^{-1}(\mathbf{M}_t - \mathbf{M}_0)\right] > h^*. \quad (15)$$

whereh^* is chosen to achieve a desired ARL_0.

The control chart calibration procedure was carried out in two steps to achieve an ARL_0 = 200 for all control charts. The first step adjusts a linear regression models in the form $\mathrm{d}^2 = a + b * \ln(ARL)$. This procedure gives an approximate first estimative of in-control thresholds for each chart. The second step in the calibration procedure iteratively adjusts the threshold by interpolation.Next section illustrates the functionality of the proposed control chart and analyses the comparative experiments.

3. Results and discussion

The first part of the experimentscompares the Hotelling's T and the confidencecontrol chart for individual observation vectors($\lambda = 1$), which performs a scale transformation of the Hotelling'sdistance. Figure 1 (a) shows the signal pattern for the case of no change in the mean vector, d = 0, while Figure 1 (b) and (c) shifts the mean vector process at time t = 201 to the distances d = 3 and 6, respectively.In the scatter-plot below the control chart, the out-of-control observation vectors are markedwithred dots in the scatter-plot,while the in-control dots are black. The vertical line in the middle of the chart delimits the change point.The horizontal dashed linesare the in-control thresholds for the pre-defined ARL_0 = 200. Given in probability value, the in-control upper limit for the confidence chart is h^* = 0.7362 (73,62%). The correspondent in-control noncentral distance that holds for an ARL_0 = 200 in the Hotelling's T control chart is h = 3.265, which is just a scale transformation of h^*.

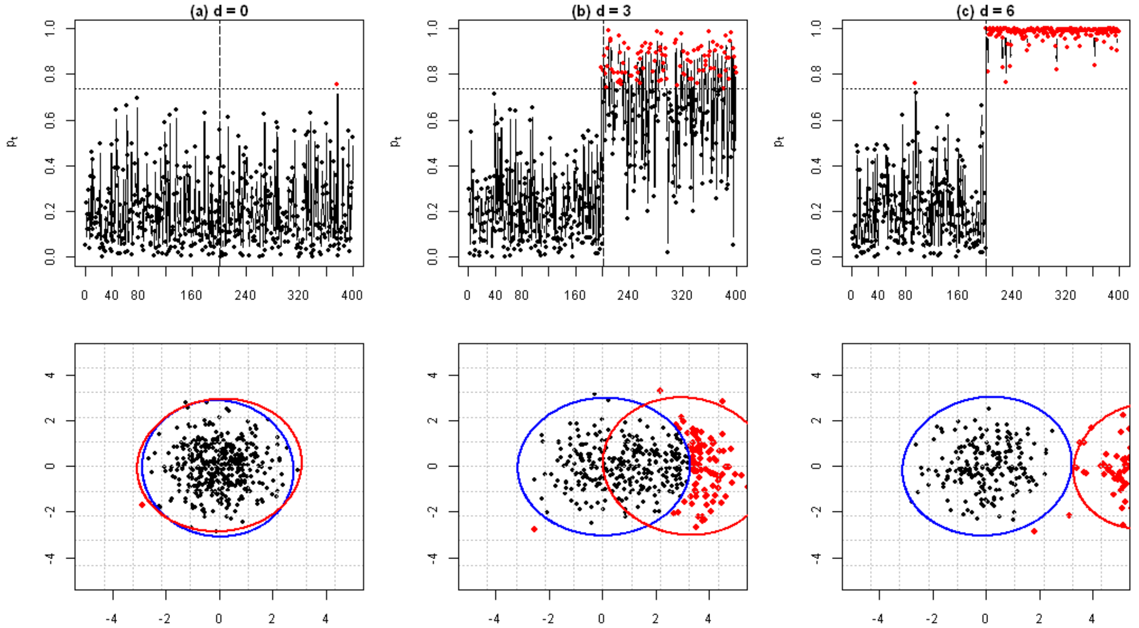


Figure 1:Confidence MEWMA control chart for individual vectors(λ = 1)with scatter plots

Observing Figure 1 (c), notice that most of the out-of-control observation vectors do not overlaps the in-control region, resulting in probability values closer to 1. This indicates that the confidence level converge to 1 when the processes are not overlapped. This characteristic pattern does not happen with the Hotelling's T statistic because it has no bound for maximum values, making the interpretation of out-of-control signals difficult to evaluate.

A more detailed summary of the raw distances and theirs equivalent confidence levels are given in Table 1, where $\overline{d^2}$ and $\bar{p}$ are average values and Sd(*) is the standard deviation from 100.000 sample replications of size 10. Note that the simulated experiments confirm with high precision the expected parameters of the Hotelling's T statistic (d^2). Also as expected, the ARL for both charts are exactly equal, demonstrating that the transformation of the Hotelling's T by the Bhattacharyya distance and into probabilities does not modify the control chart performance with respect to the ARL.

Figure 2 is composed of three sets of confidence control charts and theirs respective two-dimensional scatter plots below the control chart. The first 200 observations are generated from the in-control process with d = 0, while the second group of 200 observations are generated from the out-of-control process with d = 3. The confidence MEWMA control chart utilisingλ = 1 performs exactly as the transformed Hotelling's T^2 and can be viewed in Figure 2 (a). This control chart serves as a standard confidence level to protect the global in-control process region.

As well known in the MEWMA procedure, the reduction on the λ parameter turns the control chart more sensitive to small shifts in the mean vector. Figure 2 (b) shows how the proposed control chart performs when utilising λ = 0.7. This reduced value for λ acts by reducing the standard confidence level for the in-control process. In this case, the in-control confidence upper limit to achieve an ARL_0 = 200 is h^* = 0.5086 (50.86%). When the smoothing factor λ is reduced for 0.4 as shown in Figure 2 (c), the standard confidence level for the in-control process is reduced to a lower baseline than the previous values, with $h^* = 0.2747$ (27.47%), indicating that the control chart will be more rigorous concerning to the variation of the current mean vector.

Table 1: Summary of the Hotelling'sT^2 and Confidence CCwith ARL comparison

d	$\overline{d^2}$	$Sd(d^2)$	ARL	$\bar{p}(\%)$	$Sd(p)(\%)$	ARL
0.0	2.000	1.850	200.6	20.00	15.74	200.6
	0.006	0.006	0.634	0.001	0.000	0.634
0.5	2.251	2.070	118.8	21.97	16.93	117.7
	0.007	0.007	0.376	0.001	0.001	0.372
1.0	3.001	2.642	43.1	27.62	19.57	43.1
	0.009	0.008	0.136	0.001	0.001	0.136
1.5	4.252	3.407	16.0	36.13	21.96	16.0
	0.013	0.011	0.051	0.001	0.001	0.051
2.0	6.003	4.263	7.0	46.39	22.98	7.0
	0.019	0.013	0.022	0.001	0.001	0.022
2.5	8.253	5.163	3.6	57.19	22.30	3.6
	0.026	0.016	0.011	0.002	0.001	0.011
3.0	11.004	6.086	2.2	67.49	20.13	2.2
	0.035	0.019	0.007	0.002	0.001	0.007
3.5	14.270	7.039	1.5	76.51	16.97	1.5
	0.143	0.070	0.005	0.002	0.001	0.005
4.0	18.021	7.987	1.2	83.86	13.39	1.2
	0.180	0.080	0.004	0.003	0.000	0.004
h(ARL_0=200)	10.66			73.62		

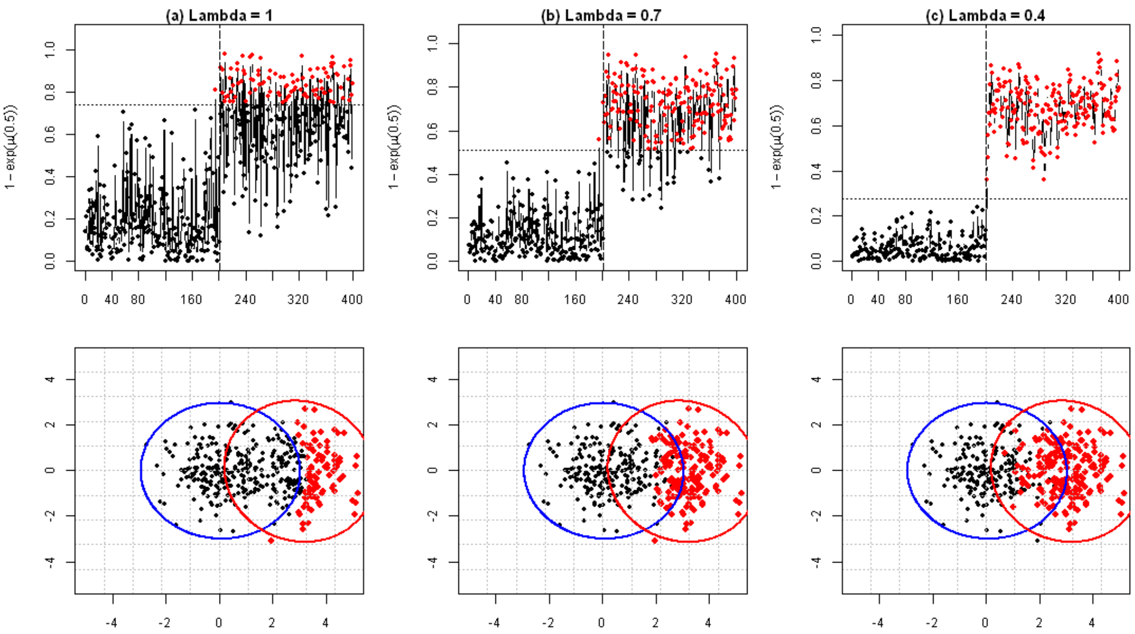


Figure 2:Confidence MEWMA control chart for individual vectors (a) and mean vectors(b, c)with scatter plots

A detailed comparison between the confidence control charts baselines (mean values) and standard deviations for the in-control process with λ varying from 1 to 0.1 by 0.1 units is given in Table 2. Figure 3 (a) and (b) shows how the mean confidence value and its respective standard deviation estimated in Table 2 behave by reducing the smooth factor.

To analyse the out-of-control behaviour of the proposed statistic, the mean vector shifts with d varying in the 0-4 range by 0.5 units. This information is also valid to inspect in order to have a support for the decision maker. The results are presented in Table 3 and Figure 4, from which is easy to notice the interesting patterns of the out-of-control process for the proposed statistic.

Table 2:Summary of the Confidence MEWMACC statistics for the in-control process

λ	$\bar{p}(\%)$	$\mathrm{Sd}(p)(\%)$
1.0	0.20013	0.15740
	0.00090	0.00070
0.9	0.16995	0.13706
	0.00076	0.00061
0.8	0.14299	0.11748
	0.00064	0.00053
0.7	0.11870	0.09872
	0.00053	0.00044
0.6	0.09664	0.08078
	0.00043	0.00036
0.5	0.07644	0.06367
	0.00034	0.00028
0.4	0.05778	0.04740
	0.00026	0.00021
0.3	0.04039	0.03211
	0.00018	0.00014
0.2	0.02407	0.01811
	0.00011	0.00008
0.1	0.00908	0.00634
	0.00004	0.00003

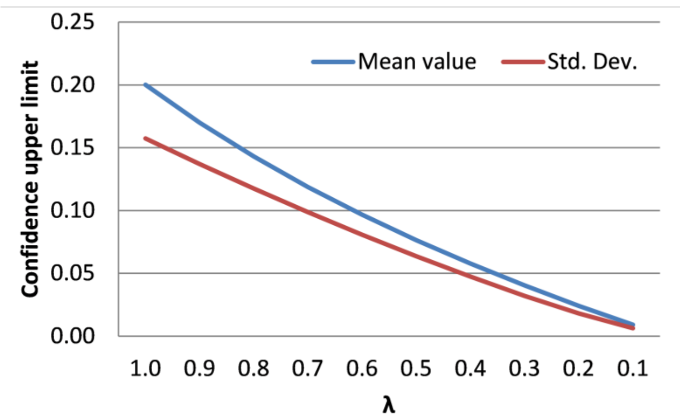


Figure 3:Mean value and standard deviation of the Confidence MEWMA CC for the in-control process with various λ's

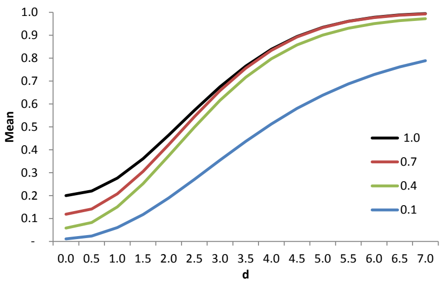
As observed previously, the proposed control chart does not bring improvements with respect to the control chart performance as measured by the ARL. A simulated comparison with the selected values for the smooth factor λ is presented in Table 4 and Figure 5. These results show the same known pattern of the Hotelling's T and the traditional MEWMA control charts, where the ARL is reduced with a reduction in the smoothing factor. Table 4 presents the average run lengths and standard errors of the estimate, while in Figure 5 is presented the ARL in a natural logarithm scale to emphasize the performance differences. The main result of Figure 5 is to notice that when λ = 0.4, the control chart starts to suffer the inertial effect to change detection as from d = 3.5.

Table 3:Summary of the Confidence MEWMACC statistics for out-of-control processes and λ = 1.0, 0.7, 0.4 and 0.1

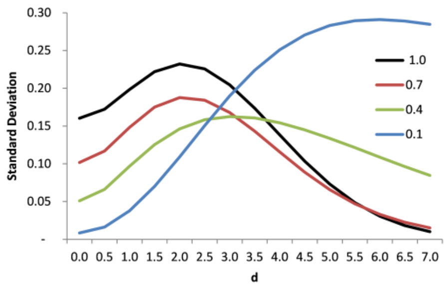
λ	1		0.7		0.4		0.1	
d	$\bar{p}$(%)	Sd(p)(%)	$\bar{p}$(%)	Sd(p)(%)	$\bar{p}$(%)	Sd(p)(%)	$\bar{p}$(%)	Sd(p)(%)
0.0	19.99	16.03	11.86	10.17	5.83	5.09	1.08	0.83
	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000
0.5	21.97	17.22	14.16	11.69	8.23	6.62	2.35	1.61
	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
1.0	27.61	19.83	20.72	14.82	15.04	9.67	6.03	3.77
	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000
1.5	36.12	22.19	30.53	17.50	25.22	12.53	11.75	7.00
	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.000
2.0	46.37	23.21	42.24	18.76	37.33	14.62	18.97	10.92
	0.002	0.001	0.002	0.001	0.002	0.001	0.001	0.000
2.5	57.17	22.57	54.40	18.42	49.87	15.83	27.08	15.05
	0.003	0.001	0.002	0.001	0.002	0.001	0.001	0.001
3.0	67.47	20.46	65.79	16.78	61.58	16.26	35.49	18.98
	0.003	0.001	0.003	0.001	0.003	0.001	0.002	0.001
3.5	76.49	17.36	75.59	14.33	71.64	16.07	43.67	22.39
	0.003	0.001	0.003	0.001	0.003	0.001	0.002	0.001
4.0	83.84	13.82	83.40	11.56	79.69	15.43	51.25	25.11
	0.004	0.001	0.004	0.001	0.004	0.001	0.002	0.001
4.5	89.43	10.35	89.22	8.89	85.74	14.49	58.00	27.06
	0.004	0.000	0.004	0.000	0.004	0.001	0.003	0.001
5.0	93.43	7.30	93.29	6.58	90.06	13.36	63.82	28.31
	0.004	0.000	0.004	0.000	0.004	0.001	0.003	0.001
5.5	96.11	4.86	95.98	4.71	93.03	12.14	68.73	28.94
	0.004	0.000	0.004	0.000	0.004	0.001	0.003	0.001
6.0	97.81	3.05	97.67	3.28	95.01	10.88	72.79	29.10
	0.004	0.000	0.004	0.000	0.004	0.000	0.003	0.001
6.5	98.83	1.81	98.68	2.24	96.33	9.63	76.13	28.91
	0.004	0.000	0.004	0.000	0.004	0.000	0.003	0.001
7.0	99.40	1.02	99.27	1.50	97.21	8.45	78.86	28.47
	0.004	0.000	0.004	0.000	0.004	0.000	0.004	0.001

Conclusion

In this work we propose a new manner of monitoring Gaussian mean vectors by the use of an upper bound on the confidence that the process is in-control. Instead the monitoring of the traditional noncentrality parameter, we suggest the use of the Bhattacharyya distance due to its relationship with the upper bound on the misclassification error. While the Hotelling's traditional distance has no maximum values, the proposed confidence control chart based on probabilities for individual observation vectors manifest a useful distinction between processes in the 0-1 range. In this case, when the out-of-control process became completely separable (not overlapped) from the in-control process, the proposed statistic converge to 1, not going to infinity.



(a)



(b)

Figure 4:Mean value (a) and standard deviation (b) of the Confidence MEWMA CC for the out-of-control process with various d's

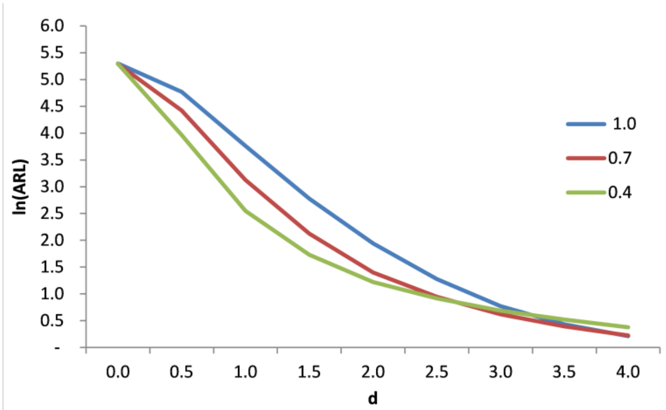


Figure 5:Confidence MEWMA CC performance comparison for the out-of-control process with various d's

Additionally, we show that the probability control chart for individual observation vectors is a particular case that can be extended to a more general optics, the mean vector process monitoring by the use of a MEWMA based control chart. Despite the fact that the performance measured by the ARL is the same of the traditional approach, the interesting particularity of the proposed statistic is its limited behaviour in the 0-1 range.

Future work on this topic includes the monitoring of the covariance matrix of a Gaussian process by the use of probability based control charts, as well the global process monitoring, i.e., the jointly monitoring of the mean vector and covariance matrix of a multivariate Gaussian process.

Table 4:Average run length comparison for λ = 1.0, 0.7 and 0.4

d \ λ	1.0	0.7	0.4

0.0	200.6	198.9	199.3
	0.634	4.45	4.46
0.5	117.7	83.2	52.6
	0.372	0.83	0.53
1.0	43.1	22.7	12.8
	0.136	0.23	0.13
1.5	16.0	8.4	5.6
	0.051	0.08	0.06
2.0	7.0	4.1	3.4
	0.022	0.04	0.03
2.5	3.6	2.6	2.5
	0.011	0.03	0.03
3.0	2.2	1.9	2.0
	0.007	0.02	0.02
3.5	1.5	1.5	1.7
	0.005	0.01	0.02
4.0	1.2	1.3	1.5
	0.004	0.01	0.01

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