MODELING INTERACTIONS BETWEEN CRITERIA IN MCDA: A COMPARATIVE ANALYSIS OF THE BIPOLAR CHOQUET INTEGRAL AND AN ELECTRE METHOD

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ABSTRACT

The focus of this paper is the use of a bi-capacity model for approaching the computation of interactions between criteria within the framework of Multiple Criteria Decision Aiding.. This model takes into consideration bipolar scales. The paper introduces the Choquet integral in the bipolar scale. The core of the paper is a comparative analysis of the uses of the bipolar Choquet integral and the ELECTRE IV method as decision aiding tools in a suppliers ranking problem. The matrix of performances is made explicit and the bipolar Choquet integral is used. The rankings of alternatives produced by the bipolar Choquet integral and the use of the ELECTRE IV method are then compared. The types of interaction are explained and the comparison is carried out with a sensitivity analysis over the ordering of criteria. The paper closes with conclusions within the context of the problem under analysis.

KEYWORDS. Choquet integral, ELECTRE IV method, Multicriteria Decision Aid.

RESUMO

O foco deste artigo é o uso de um modelo bi-capacidade para abordar-se o cálculo das interações entre critérios no Apoio Multicritério à Decisão. O modelo emprega escalas bipolares. O artigo introduz a integral de Choquet na escala bipolar. O cerne do artigo é uma análise comparativa entre os usos da integral de Choquet bipolar e o método ELECTRE IV como ferramentas de apoio à decisão em um problema de ordenação de fornecedores. Explicita-se a matriz de desempenhos e faz-se uso da integral de Choquet bipolar. Comparam-se em seguida as ordenações das alternativas produzidas por aquela integral e pelo emprego do método ELECTRE IV. Explicam-se os tipos de interações e efetua-se a comparação com uma análise de sensibilidade sobre as ordenações dos critérios. Conclui-se o artigo com considerações pertinentes ao contexto da análise realizada.

PALAVRAS CHAVE. Integral de Choquet, Método ELECTRE IV, Apoio Multicritério à Decisão.

Área principal: ADM - Multicriteria Decision Support

1. Introduction

One important problem in Multiple Criteria Decision Aiding (MCDA) is the existence of interactions between criteria. When using a multiple attribute utility (or value) theoretical model there are requirements that must be met if, for example, a linear additive function is to be used (Clemen & Reilly, 2001, pp. 647-654). An important mathematical model that has been used for modeling interactions between criteria is the Choquet integral (Choquet, 1953). In decision theory, the Choquet integral is a way to measure the expected utility of an uncertain event (Gilboa & Schmeidler, 1992). The Choquet integral is indeed a generalization of the weighted arithmetic mean and has been extensively used since the last decade in Multiple Criteria Decision Aiding in modeling interactions between criteria (Grabisch, 1996; Grabisch, 2006; Grabisch & Labreuche, 2005, 2010). Marichal (2000) also modeled interactions between criteria with the Choquet integral. A critical analysis of the use of the Choquet integral for modeling interactions between criteria was presented by Roy (2009) though. This author has pointed out that the generalization of the Choquet integral known as bipolar model (or model with bi-capacities) should be utilized in order to capture some particular aspects of interactions between criteria.

In this paper we show a comparison of the use of the bipolar Choquet integral against the use of the ELECTRE IV method. This is an outranking method that does not require knowledge of criteria weights (Roy & Bouyssou, 1993). The comparison is accomplished for a ranking problem under multicriteria. The paper starts by reviewing the Choquet integral in the unipolar scale as a multicriteria ranking model. Then it moves to presenting the Choquet integral in the bipolar scale as an equivalent model. Next the comparison between the use of the Choquet integral in the bipolar scale is compared against the results obtained from using ELECTRE IV for a suppliers ranking problem under multiple criteria. Our approach is therefore different from that of Figueira, Greco & Roy (2009), who extended the notion of concordance index that is not present in ELECTRE IV. These three authors have shown that the Choquet integral can be applied in order to build a transitive and complete pre-order relation of type existing in the ELECTRE methods. An extension of the concordance index of ELECTRE methods in order to deal with interactions between criteria was then proposed.

2. The Choquet integral in the bipolar scale

Given a finite set $J=\{1,2,\dots,n\}$ a fuzzy measure μ is a function of the form: $\mu : 2^J \to [0,1]$ such that $\mu(\phi)=0$ and μ(J) = 1 (boundary conditions); μ(C) ≥ μ(D) if $D \subseteq C, \forall\, C,D \in J$ (monotonicity condition). Let P(J) be a set of pairs of subsets of J: $P(J)=\{(C,D), C,D \subseteq J, C \cap D = \phi\}$. A bi-capacity μ in J is a function $\mu : P(J) \to [0,1\}X[0,1]$ such that $\mu(C,\phi)=(c,0)$ and $\mu(\phi,D)=(0,D), c,d \in [0,1]$; $\mu(J,\phi)=(1,0)$ and $\mu(\phi,J)=(0,1)$ (boundary conditions); for each $(C,D),(E,F) \in P(J)$ such that $E \subseteq C, D \subseteq F$ we have $\mu(C,D)=(c,d)$ and $\mu(E,F)=(e,f)$, $c,d,e,f \in [0,1]$ with $c \geq e$ and $d \geq f$ (monotonicity condition). Here we use the following notation: $\mu^+(C,D)=c$, $\mu^-(C,D)=d$. A bi-capacity $\hat{\mu}$, on the set J, is a function $\hat{\mu} : P(J) \to [-1,1]$ such that $\hat{\mu}(\phi,\phi)=0$; $\hat{\mu}(J,\phi)=1$ and $\hat{\mu}(\phi,J)=-1$ (boundary conditions); if $E \subseteq C, D \subseteq F$, then $\hat{\mu}(C,D) \geq \hat{\mu}(E,F)$ (monotonicity condition). From each bi-polar capacity μ in J, we can obtain a bi-capacity $\hat{\mu}$ in J: $\hat{\mu}(C,D)=\mu^+(C,D)-\mu^-(C,D), \forall C,D \in P(J)$ (Greco & Figueira, 2003).

For each $x \in R^n : x^+ = \max\{x,0\}$ is the positive part of x, for each $x \in R$;

$x^- = \max\{-x, 0\}$ is the negative part of x, for each $x \in R$, $x^+ = \{x^+{}_1, x^+{}_2, x^+{}_3, ,..., x^+{}_n\}$ is the positive part of $x = \{x_1, x_2, x_3, ,..., x_n\} \in R^n$; $x^- = \{x^-{}_1, x^-{}_2, x^-{}_3, ,..., x^-{}_n\}$ is the negative part of $x(x_1, x_2, x_3, ,..., x_n) \in R^n$. Given $x \in R^n$ we consider a permutation (.) of the elements of J, such that $|x_{(1)}| \leq |x_{(2)}| \leq ... \leq |x_{(j)}| \leq |x_{(n)}|$. For each element $j \in J$ we have two subsets $C(j) = \{i \in J : x_i \geq |x_{(j)}|\}$ and $D(j) = \{i \in J : -x_i \geq |x_{(j)}|\}$. Considering a bi-capacity μ in J and a vector $x \in R^n$ we can define its bipolar Choquet integral of the positive part as follows: $Ch^+(x, \mu) = \sum_{j \in J^>} \left(|x_{(j)}| - |x_{(j-1)}|\right) \mu^+(C_{(j)}, D_{(j)})$, where $J^> = \{j \in J / |x_j| > 0\}$. In the same way we formulate the bipolar Choquet integral of the negative part as $Ch^-(x, \mu) = \sum_{j \in J^>} \left(|x_{(j)}| - |x_{(j-1)}|\right) \mu^-(C_{(j)}, D_{(j)})$. Therefore the bipolar Choquet integral is $Ch^B(x, \mu) = C^+(x, \mu) + Ch^-(x, \mu)$.

Other authors have used the bipolar Choquet integral for tackling different problems. For example, Greco & Rindone (2011) extended the bipolar Choquet integral representation towards bipolar Cumulative Prospect Theory. Greco & Rindone (2012) proposed the bipolar Choquet integral for the case in which the underlying scale is bipolar and provided a characterization of bipolar fuzzy integrals.

3. Comparing the use of the bipolar Choquet integral against ELECTRE IV

To compare the Choquet integral using bipolar scale against ELECTRE IV we make use of the decision matrix taken from the doctoral thesis of Alencar (2006). This last author analyzed the problem of ranking suppliers within the context of group decision making by applying ELECTRE IV. Table 1 displays the judgments by experts on the relative importance of each supplying firm with respect to every evaluation criterion. Those criteria are the following: $C_1 =$ Cost, $C_2 =$ Culture, $C_3 =$ Design, $C_4 =$ Quality, $C_5 =$ Time, $C_6 =$ Experience. The criteria order is: $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$.

Criteria ↓	Firm # 1	Firm # 2	Firm # 3	Firm # 4	Firm # 5	Firm # 6
C_1	0.1	0.15	0.15	0.03	-0.03	0.05
C_2	4	2	1	1	4	3
C_3	5	2	2	1	4	5
C_4	4	4	3	4	3	3
C_5	0.1	0.05	-0.05	0.15	0.2	0.1
C_6	8	20	15	6	10	5

Table 1 - Decision matrix of Criteria versus Alternative supplying firms

In order to run a sensitive analysis over the computation of the bipolar Choquet integral we adopt two hypothesis: (A) hypothesis # 1 (H$_1$): $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$ with the following interactions between criteria: $\mu_1 = 0.35$, $\mu_2 = 0.5\mu_1$, $\mu_2 = \mu_3 = \mu_4$, $\mu_5 = 0.6\mu_1$, $\mu_5 = 0.84\mu_1$, given that $\sum_{i=1}^{3} \mu_i = 1$; (B) hypothesis # 2 (H$_2$): $C_6 > C_1 = C_5 > C_2 = C_3 = C_4$ with the following interactions between criteria: $\mu_6 = 0.32$, $\mu_1 = 0.32\mu_6$, $\mu_5 = \mu_1$,

$\mu_2 = 0.6\mu_5$, $\mu_2 = \mu_3 = \mu_4$, given that $\sum_{i=1}^{3} \mu_i = 1$. Next, we show the steps for computing the bipolar Choquet integral.

Step 1 – Determination of fuzzy measures

In Table 2 we present the fuzzy measures for each hypothesis and for every criterion.

Individual fuzzy measures	Fuzzy measures H_1/H_2	Criteria
μ_1	0.35/ 0.32	C_1
μ_2	0.09/ 0.18	C_2
μ_3	0.09/ 0.18	C_3
μ_4	0.09/ 0.11	C_4
μ_5	0.21/ 0.11	C_5
μ_6	0.18/ 0.11	C_6

Table 2- Fuzzy measures for each hypothesis and for every criterion

Step 2 – Determination of the fuzzified decision matrix

The fuzzified decision matrix is obtained by multiplying each element of decision matrix by its fuzzy measure. In Table 3 we present the results for hypothesis H_1. An equivalent table has been determined for hypothesis H_2.

Criteria ↓	Firm # 1	Firm # 2	Firm # 3	Firm # 4	Firm # 5	Firm # 6
C_1	0.035	0.0525	0.0525	0.0105	-0.0105	0.0175
C_2	0.3528	0.1764	0.0882	0.0882	0.3528	0.2646
C_3	0.441	0.1764	0.1764	0.0882	0.3528	0.441
C_4	0.3528	0.3528	0.2646	0.3528	0.2646	0.2646
C_5	0.021	0.0105	-0.0105	0.0315	0.042	0.021
C_6	1.4112	3.528	2.646	1.0584	1.764	0.882

Table 3 - Fuzzified decision matrix for hypothesis H_1

Step 3: Computation of the bipolar Choquet integral

In Table 4 we display all the computational results for hypothesis H_1. Equivalent results are obtained for hypothesis H_2.

Criteria ↓	Firm # 1	Firm # 2	Firm # 3	Firm # 4	Firm # 5	Firm # 6
C_1	0.035	0.0525	0.0525	0.0105	-0.0105	0.0175
C_2	0.3528	0.1764	0.0882	0.0882	0.3528	0.2646
C_3	0.441	0.1764	0.1764	0.0882	0.3528	0.441
C_4	0.3528	0.3528	0.2646	0.3528	0.2646	0.2646
C_5	0.021	0.0105	-0.0105	0.0315	0.042	0.021
C_6	1.4112	3.528	2.646	1.0584	1.764	0.882

Min operator	5.6934 E-05	20.13502 E-5	-6.00476 E-06	9.60761 E-07	-2.56203 E-05	1.00079 E-05
Max operator	2.6138	4.2966	3.2172	1.6296	2.7657	1.8907
Values of the bipolar Choquet integral	2.61	4.30	3.22	1.63	2.77	1.89
Ranking of alternatives	4	1	2	6	3	5

Table 4 - Computation of the bipolar Choquet integral

In table 5 we show the comparisons between the ELECTRE IV and the bipolar Choquet rankings.

Ranking from ELECTRE IV assuming that $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$	Ranking from bipolar Choquet for H_1, $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$	Ranking from bipolar Choquet for H_2, $C_6 > C_1 = C_5 > C_2 = C_3 = C_4$
Firm # 6	Firm # 2	Firm # 2
Firm # 3	Firm # 3	Firm # 3
Firms # 1 and 5	Firm # 5	Firm # 5
Firm # 4	Firm # 1	Firm # 1
Firm # 2	Firm # 6	Firm # 6
- - - -	Firm # 4	Firm # 4

Table 5- comparison of results from ELECTRE IV against bipolar Choquet

4. Results

It can be seen that by using bipolar Choquet integral a complete pre-order is produced and ranking is different from that obtained by ELECTRE IV. In both hypotheses, when $C_1 > C_5 > C_6 > C_2 = C_3 = C_4$ (H_1) and when $C_6 > C_1 = C_5 > C_2 = C_3 = C_4$ (H_2), bipolar Choquet integral leads to the same ranking. According to both approaches, Firm # 3 ranks as second alternative. Firm # 3 has low cost and low delay as compared against the other alternatives. Using ELECTRE IV leads to the conclusion that Firm # 6 ranks as first alternative. Firm # 6 has lowest cost and cost (C_1) is the most important. Considering time, the second most important criterion (C_5), Firm # 6 has the lowest value. Firm # 6 ranks in the fifth position by using bipolar Choquet. By using bipolar Choquet Firm # 2 is the best alternative, although its cost is three times higher than Firm # 6. On the other hand, Firm # 2 has four times more experience (C_6) than Firm # 6, and this is indeed the third most important criterion. Firm # 2 is also one and a half times lower in time (C_5), that is the second more important criterion; it also has more quality (C_4), that is the less important criterion. Firm # 2 has lower culture (C_2), that is the fourth important criterion, and design (C_3), that has relatively low importance as compared against other criteria. In the third position we have, from using ELECTRE IV, a tie: Firms # 1 and 5. From using bipolar Choquet we obtain Firm # 5 in that third position. It would be fair to say that bipolar Choquet leads to Choquet bipolar is more realistic when we consider that Firm # 5 has lower price and higher experience. ELECTRE IV classifies Firm # 4 in the fourth position and Choquet bipolar classifies Firms # 1 in that same position. This firm has higher cost but has more experience, better time, design and culture. Finally, Firm # 2 is classified in the fifth position according to ELECTRE IV and bipolar Choquet classifies Firm # 2 in the first position as commented before.

5. Conclusions

We have shown in this paper that applications of the bipolar Choquet integral and ELECTRE IV are comparable, with an advantage to that first model when a bipolar scale is used.

The tradeoffs between criteria were explained in a realist way and measures of interactions were associated to the fuzzy measures related to two different hypotheses.

For this particular numerical example we saw that interactions between criteria were indeed quite relevant. Another important dimension of the advantage of using bipolar Choquet instead of ELECTRE IV was that this method has distinctly grouped all firms. The key conclusions from this case study are listed below: (i) the use of the Choquet integral minimizes the calculations by TODIM since it is unnecessary to normalize the raw data; (ii) not only precise values can be used but also interval data; this second situation would lead to using a fuzzy triangular number; (iii) by using the Choquet integral more complex additive models can be used that allow for taking dependencies between criteria into consideration.

More research along the line pursued in this paper will likely proceed along two major lines:

(i) carrying out similar comparisons for larger problems; and
(ii) focusing the comparison on multiple criteria decision aiding models that rely on bipolar Cumulative Prospect Theory (Greco & Rindone, 2012). In particular, new concepts such as generalizations of Choquet integral as well as the bipolar Cumulative Prospect Theory should be considered.

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