A TOPSIS based approach for evaluating alternatives in Fuzzy-Electre environments

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Abstract

This paper presents the formulation and the computational solution of a procedure for ranking alternatives in Fuzzy-Electre environments. It attempts to exploit the well-known advantages of the Fuzzy-TOPSIS method in order to define a new ranking procedure by combining the pure concordance and discordance index with the ideal positive and negative solutions. The final ranking of the alternatives is given by a modified closeness coefficient. A case study related to a purchasing situation in a hospital company is presented and discussed in details. Results obtained show that the approach developed in this work has potential as an alternative ranking procedure to Fuzzy-Electre method.

Key-words: Decision making, Multicriteria, Fuzzy sets, TOPSIS, Electre

1. Introduction

Making a selection among discrete decision alternatives in the presence of multiple and conflicting attributes is referred to as a multiple-attribute decision-making (MADM) problem (Belton and Stewart, 2002). MADM is a sub-discipline of the multiple-criteria decision-making class, consisting of developing models that sort or rank alternatives based on a set of attributes.

Roy (1990) points out that "solving" a MADM involves the following three fundamental problems concerning the assessment of a set of alternatives A: (1) Choice: choose the best alternative from A; (2) Sorting: sort the alternatives of A into clusters, which can then be arranged in preference order; and (3) Ranking: rank the alternatives of A from best to worst.

Basically, an MADM model should provide support to the decision makers in the process of structuring and solving decision problems involving multiple attributes. The multidimensionality and different perspectives involved in the evaluation of the alternatives based on a set of conflicting attributes generate a complex situation, in which there is no unique optimal solution. In many cases, it is necessary to establish trade-offs among attributes, incorporating the decision makers' (DM) information preferences. The situation is even more complex in group decision-making contexts where the intrinsic difficulties in trading off criteria should simultaneously incorporate different preferences. For instance, economic reasons may prevail in the views of some decision-makers in the group; while social and environmental motives may be prominent in the view of others.

In addition, decision makers face many problems with incomplete, unqualifiable, vague, and unquantifiable information in MADM (Kabak and Ruan, 2011). Traditional MADM methods, cannot effectively cope with several decision-making problems that very often require this kind of information (Cheng and Hwang, 1993). Fuzzy set theory (FST) was then introduced into MADM. Zimmerman (2000) defines FST as a very powerful modeling language that deals with a large portion of uncertainties in real-life situations, since much knowledge in the real-world is Fuzzy rather than precise. FST also provides a strict mathematical framework into which vague conceptual phenomena can be precisely and rigorously studied, the kind of situations commonly encountered in decision-making processes. There are successful applications and implementations of FST in MADM. FST has been primarily combined with traditional MADM, giving origin to Fuzzy-SAW (Yücel and Güneri, 2011), Fuzzy AHP/ANP (Van Laarhoven and Pedrycz, 1983; Wang, Luo and Hua, 2008), Fuzzy-Electre (Vahdani and Hadipour , 2011; Marbini and Tavana, 2011), Fuzzy-TOPSIS (Chen, 2000), and novelty methods (Wang and Parkan, 2005). Kahraman (2008), Wang and Parkan (2005) and Ribeiro (1996) present good reviews of Fuzzy-MADM.

The Electre method is a multiple-attribute model that uses outranking relations with the purpose of finding a set of alternatives dominating over other alternatives, while they cannot be dominated (Wang and Triantaphylloub, 2008). A special feature of this method is the non-compensatory effect, preventing alternatives to be ranked as the best when reaching an excellent score for one or more criteria and, simultaneously, a very low score for another criterion. This particularity avoids undesirable distortions in the final result, ensuring that the alternatives with the best position in the ranking outrank the others.

Nowadays, different versions of the Fuzzy-Electre method have been proposed, such as Marbini and Tavana (2011), Rouyendegh and Erol (2012) and Vahdani and Hadipour (2011), for example, and these papers did not present a ranking procedure. However, this paper addresses the multi-criteria ranking problem, presenting an alternative method for ranking in Fuzzy-Electre environment taking the TOPSIS' closeness coefficient into consideration. In summary, the major innovation and contribution of our model is calculating the positive and negative ideal solutions by using the fuzzy pure concordance and discordance indexes (Chaterjee *et al.*, 2003), instead of using the weighted normalized fuzzy-decision matrix as originally proposed by Chen (2000). After that, the alternatives can be ranked by closeness coefficient as in the original TOPSIS method.

The remainder of this paper is organized as follows: section 2 describe the fuzzy notation and definitions to be used in the paper, as well as summary descriptions of Fuzzy-TOPSIS and Fuzzy-Electre; section 3 describes the developed method with details; section 4 presents a numerical example, based on data collected from a real decision case; and, section 5 is reserved to the conclusions and final remarks.

2. Preliminary Definitions

2.1. Fuzzy Set Theory

In this section, some basic definitions of the fuzzy set theory are reviewed from Zimmerman (2000), Chen (2000), Cheng (1998) and Zadeh (1965). All the definitions and the notation below are needed for understanding the proposed approach.

Definition 1: Let X be a set of objects represented by x, then a fuzzy set $\tilde{A}$ defined in X is a set of ordered pairs denoted by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

where $\mu_{\tilde{A}}(x)$ is the membership function associating each element x in X to a real number in the interval [0,1].

Definition 2: A trapezoidal fuzzy number $\tilde{n}$ is defined as $\tilde{n} = (n_1, n_2, n_3, n_4)$, where n_1, n_2, n_3 and n_4 are real numbers. The degree of membership of a trapezoidal Fuzzy number, denoted by $\mu_{\tilde{n}}(x)$, is calculated by

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1 \\ \dfrac{x - n_1}{n_2 - n_1}, & n_1 \le x \le n_2 \\ 1, n_2 \le x \le n_3 \\ \dfrac{x - n_4}{n_3 - n_4}, & n_3 \le x \le n_4 \\ 0, x > n_4 \end{cases}$$

If $n_2 = n_3$, $\tilde{n}$ is a triangular fuzzy number.

Definition 3: Given two positive trapezoidal fuzzy numbers, $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$, and a non-fuzzy number $r \ge 0$, where $0 \le a_1 \le a_2 \le a_3 \le a_4$, $0 \le b_1 \le b_2 \le b_3 \le b_4$), then the fuzzy operations of sum, subtraction and multiplication are respectively defined as follows:

$$\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$\tilde{a} \ominus \tilde{b} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

$$\tilde{a} \otimes \tilde{b} \cong (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4)$$

$$\tilde{a} \otimes r \cong (a_1 \times r, a_2 \times r, a_3 \times r, a_4 \times r)$$

Definition 4: A fuzzy number can be associated to a linguistic term. For example, we can use terms like *high* and *low* to express our satisfaction degree in relation to a purchased product.

Definition 5: A matrix $\tilde{Q}$ is a fuzzy matrix, if at least one of its elements is a fuzzy number.

Definition 6: The vertex distance $\delta(\tilde{a}, \tilde{b})$ between two trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ is defined as

$$\delta(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{4}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2]}$$

Definition 7: A trapezoidal fuzzy number $\tilde{k}$ is a convex and normal set defined as $\tilde{k} = (n1, n2, n3, n4, 1)$. The membership function, $\mu_{\tilde{k}}(x)$ is defined as follows:

$$\mu_{\tilde{k}}(x) = \begin{cases} f_{\tilde{k}}^L(x), a \le x \le b \\ 1, b \le x \ge c \\ f_{\tilde{k}}^R(x), c \le x \ge d \\ 0, otherwise \end{cases}$$

where $f_{\tilde{k}}^L(x): [a, b] \to [0,1]$ and $f_{\tilde{k}}^R(x): [c, d] \to [0,1]$. Since $f_{\tilde{k}}^L$ is continuous, strictly increasing and its inverse function exists, $f_{\tilde{k}}^R$ is continuous, strictly decreasing and its inverse function also exists. If $b = c$, then $\tilde{k}$ is called a triangular fuzzy number.

Definition 8: The centroid point $(\bar{x}_0, \bar{y}_0)$ of a fuzzy number $\tilde{k}$ is defined as

$$\bar{x}_o(\tilde{k}) = \frac{\int_a^b (x f_{\tilde{k}}^L) dx + \int_b^c x dx + \int_c^d (x f_{\tilde{k}}^R) dx}{\int_a^b (f_{\tilde{k}}^L) dx + \int_b^c dx + \int_c^d (f_{\tilde{k}}^R) dx}$$

$$\bar{y}_o(\tilde{k}) = \frac{\int_0^1 (y g_{\tilde{k}}^L) dy + \int_0^1 (y g_{\tilde{k}}^R) dy}{\int_0^1 (g_{\tilde{k}}^L) dy + \int_0^1 (g_{\tilde{k}}^R) dy}$$

Definition 9: The distance index between an original fuzzy number and its centroid is defined as

$$R(\tilde{A}) = \sqrt{(\bar{x}_0)^2 + (y_0)^2}$$

Thus, for any fuzzy numbers $\tilde{A}_i, \tilde{A}_j \in S$, where $S = \{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\}$ is a set of convex fuzzy numbers, the fuzzy number ranking has the following properties:

(1) If $R(\tilde{A}_i) < R(\tilde{A}_j)$, then $\tilde{A}_i < \tilde{A}_j$,
(2) If $R(\tilde{A}_i) = R(\tilde{A}_j)$, then $\tilde{A}_i = \tilde{A}_j$, and
(3) If $R(\tilde{A}_i) > R(\tilde{A}_j)$, then $\tilde{A}_i > \tilde{A}_j$;

2.2. The Fuzzy-TOPSIS Method

This section presents a succinct description of the Fuzzy-TOPSIS method as described in Chen (2000) and Chen, Lin and Huang (2006). The seminal TOPSIS method has the following principle: the alternative, simultaneously closest to a positive ideal solution (A^*) and farthest from a negative ideal solution (A^-), is the best choice in relation to a set of evaluation criteria (Hwang and Yoon, 1981). Chen (2000) extended this method for solving the group decision-making problem under fuzzy environments. Thus, the Fuzzy-TOPSIS method assumes that a decision group has K decision-makers; a decision matrix $\tilde{Q} = [\tilde{q}_{ijk}]_{m \times n \times k}$, to represent the fuzzy rating of each alternative i in relation to each criterion j given by decision-maker k; and a matrix $\tilde{Z} = [\tilde{z}_{jk}]_{n \times k}$ to express the importance weight of each criterion j given by decision-maker k.

Initially, a procedure to aggregate the fuzzy ratings of the alternatives and the weight of the criteria is required. Chen, Lin and Huang (2006) derived a matrix $\widetilde{W} = [\widetilde{w}_j]_n$, where $\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$, aggregating the weight of each criterion j as follows

$$w_{j1} = \min_k \{z_{jk1}\} \qquad w_{j2} = \frac{1}{K}\sum_{k=1}^{K} z_{jk2} \qquad w_{j3} = \frac{1}{K}\sum_{k=1}^{K} z_{jk3} \qquad w_{j4} = \max_k \{z_{jk4}\}$$

and a matrix $\widetilde{D} = [\tilde{x}_{ij}]_{m \times n}$, where $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4})$, that aggregates the ratings of each alternative i in relation to each evaluation criterion j as follows:

$$x_{ij1} = \min_k \{q_{ijk1}\} \qquad x_{ij2} = \frac{1}{K}\sum_{k=1}^{K} q_{ijk2} \qquad x_{ij3} = \frac{1}{K}\sum_{k=1}^{K} q_{ijk3} \qquad x_{ij4} = \max_k \{q_{ijk4}\}$$

The next step consists of normalizing the matrix $\widetilde{D}$ in order to transform all the criteria into a comparable scale. A normalized fuzzy-decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m \times n}$ is produced as follows

$$\tilde{r}_{ij} = \left(\frac{x_{ij1}}{d_j^*}, \frac{x_{ij2}}{d_j^*}, \frac{x_{ij3}}{d_j^*}, \frac{x_{ij4}}{d_j^*}\right), \forall\, j \in B \qquad \tilde{r}_{ij} = \left(\frac{a_j^-}{x_{ij4}}, \frac{a_j^-}{x_{ij3}}, \frac{a_j^-}{x_{ij2}}, \frac{a_j^-}{x_{ij1}}\right), \forall\, j \in C$$

where B is the set of benefit criteria and C is the set of cost criteria. The values of d_j^* and a_j^- for each criterion j are calculated as follows

$$d_j^+ = \max_i x_{ij4}, \forall\, j \in B \qquad a_j^- = \min_i x_{ij1}, \forall\, j \in C$$

After calculating the matrix $\tilde{R}$, the weighted normalized fuzzy-decision matrix is computed as follows

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n}, \text{where } [\tilde{v}_{ij}] = \tilde{r}_{ij} \otimes \tilde{w}_j$$

Next, the method obtains the fuzzy ideal positive solution (FPIS, $\tilde{A}^*$) and the fuzzy negative ideal solution (FNIS, $\tilde{A}^-$) according to the following equations

$$\tilde{A}^* = \{\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*\} \qquad \tilde{A}^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\}$$

where

$$\tilde{v}_j^* = \max_i\{v_{ij4}\} \qquad \tilde{v}_j^- = \min_i\{v_{ij1}\}$$

The distances of each alternative i in relation to the ideal solutions previously defined are calculated as follows

$$d_i^* = \sum_{i=1}^{n} \delta(\tilde{v}_{ij}, \tilde{v}_i^*) \qquad d_i^- = \sum_{i=1}^{n} \delta(\tilde{v}_{ij}, \tilde{v}_i^-)$$

Finally, the closeness coefficient CC_i for each alternative is obtained through the following equation

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}$$

The highest CC_i value indicates the best performance in relation to evaluation criteria, i.e., the alternative which is simultaneously closest to $\tilde{A}^*$ and farthest from $\tilde{A}^-$.

2.3. The Fuzzy-Electre method

Electre is a family of multi-attribute methods introduced by Roy (1996), and is currently composed by several extensions, such as Electre I, Electre II, Electre III, Electre IV, Electre IS and Electre TRI. Nevertheless, all Electre-based methods aim at: (i) the construction of outranking relations, and (ii) the exploitation of the outranking relations to classify the alternatives (Wang and Triantaphylloub, 2008). The basic difference among the several Electre-based methods is related to how the outranking relations are defined and how they are applied in order to classify the alternatives. Nowadays, different versions of the Fuzzy-Electre method have been proposed, such as Marbini and Tavana (2011), Rouyendegh and Erol (2012) and Vahdani and Hadipour (2011).

According to the classical Electre definition, an outranking relation represents a binary relation over a set of alternatives. Let A_a and A_b be two alternatives for a decision problem, A_a outranks A_b ($A_a S A_b$) if A_a is, at least as good as A_b, based on arguments derived from the set of evaluation criteria. The main objective is to find the alternatives that dominate over other alternatives without being dominated.

To define an outranking relation it is necessary to calculate two main measures called concordance index $[c(a,b)]$ and discordance index $[d(a,b)]$. In fuzzy environments, the fuzzy concordance index (Marbini and Tavana, 2011) can be calculated as follows

$$\tilde{c}(a,b) = \sum_{j=1}^{n} g_j^*(a,b)\tilde{w}_j, \forall a,b = 1,2,\dots,m, a \neq b$$

where $g_j(a,b)$ is a function that compares the performance rating of alternatives a and b in relation to the jth evaluation criterion, whose definition is given by

$$g_j(a,b) = \begin{cases} 1.0, \text{if } \tilde{r}_{aj} > \tilde{r}_{bj} \\ 0.5, \text{if } \tilde{r}_{aj} = \tilde{r}_{bj} \\ 0.0, \text{otherwise} \end{cases}$$

The concordance index measures the strength of the hypothesis of a given alternative A_a, that is at least as good as alternative A_b. On the other hand, the discordance index measures the strength of the evidence against this first hypothesis (Wang and Triantaphylloub, 2008). The fuzzy discordance index (Marbini and Tavana, 2011) can be calculated as follows

$$\tilde{d}(a,b) = \begin{cases} 0, \text{if } \tilde{r}_{aj} \geq \tilde{r}_{bj}, j = 1 \dots m \\ \dfrac{\max_{j^*}(\tilde{r}_{bj} \ominus \tilde{r}_{aj})}{\max_{j=1\dots m} \delta(\tilde{r}_{aj}, \tilde{r}_{bj})}, \text{otherwise} \end{cases}$$

where $j^* = \{j |\, r_{bj} \geq r_{aj}\}$ contains the index of all criteria against the assertion "A_a is at least as good as A_b". Once these two indices were define, a binary outranking relation S can be defined as

$$A_a S A_b \; iff \; c(a,b) \geq \bar{C} \; and \; d(a,b) \leq \bar{D}$$

where $\bar{C}$ and $\bar{D}$ are fuzzy thresholds defined by the decision makers.

After calculating the concordance and discordance matrices, a directed graph, called outranking graph, can be drawn with purpose of representing the outranking relations between all alternatives. Thus, Electre I provide a partial ranking and a set of promising alternatives. When a full ranking of the alternatives is required, different extension of the ELECTRE may be used.

3. Algorithm of the proposed method

The ranking procedure presented in this section is based on Fuzzy-TOPSIS concepts presented in section 2.2. First of all, we must assume that all the assessments were made in relation to the level of importance of the criteria and performance ratings of alternatives, and that the fuzzy concordance $(\tilde{C})$ and discordance $(\tilde{D})$ matrices were calculated, as the first four steps of the Electre method presented in section 2.3. Next, the ranking of the alternatives is carried out though the following steps:

Step 1: Normalize the fuzzy concordance $(\tilde{C} = [\tilde{c}_{ab}]_{m\times m})$, where $\tilde{c}_{ab} = (c_{ab1}, c_{ab2}, c_{ab3}, c_{ab4})$, with a procedure similar to those utilized by Chen, Lin and Huang (2006) as

$$\tilde{c}_{ab} = \left(\frac{c_{ab1}}{c^+}, \frac{c_{ab2}}{c^+}, \frac{c_{ab3}}{c^+}, \frac{c_{ab4}}{c^+}\right)$$

where, $c^+$is the maximum value of c_{ab4} in the concordance matrix $\tilde{C}$.

Step 2: Normalize the fuzzy discordance matrix $\tilde{D} = [\tilde{d}_{ab}]_{m\times m}$, where $\tilde{d}_{ab} = (d_{ab1}, d_{ab2}, d_{ab3}, d_{ab4})$, as

$$\tilde{d}_{ab} = \left(\frac{[d_{ab1} + d^-]}{[d^- + d^+]}, \frac{[d_{ab2} + d^-]}{[d^- + d^+]}, \frac{[d_{ab3} + d^-]}{[d^- + d^+]}, \frac{[d_{ab4} + d^-]}{[d^- + d^+]}\right)$$

where d^+ is the maximum value of d_{ab4} and d^- is the minimum values of d_{ab1} in the discordance matrix $\tilde{D}$. d^- is a necessary parameter to normalize the discordance matrix, given that not all the values of $\tilde{D}$ are positive fuzzy numbers.

Step 3: Calculate the fuzzy pure concordance matrix $\widetilde{PC} = [\widetilde{pc}_a]_m$ as

$$\widetilde{pc}_a = \sum_{b=1}^{m} \tilde{c}(a,b) \ominus \sum_{b=1}^{m} \tilde{c}(b,a), \forall a = 1,2,\dots,m, a \neq b,$$

where $\widetilde{pc}_a = (c_{a1}, c_{a2}, c_{a3}, c_{a4})$.

Step 4: Calculate the fuzzy pure discordance matrix $\widetilde{PD} = [\widetilde{pd}_a]_m$ as

$$\widetilde{pd}_a = \sum_{b=1}^{m} \tilde{d}(a,b) \ominus \sum_{b=1}^{m} \tilde{d}(b,a), \forall a = 1,2,\dots,m, a \neq b,$$

where, $\widetilde{pd}_a = (d_{a1}, d_{a2}, d_{a3}, d_{a4})$.

Step 5: Normalize the matrices calculated in the previous steps as

$$\widetilde{pc}_a = \left(\frac{c_{a1}}{cc^+}, \frac{c_{a2}}{cc^+}, \frac{c_{a3}}{cc^+}, \frac{c_{a4}}{cc^+}\right)$$

where the value of $cc^+$is the maximum value of c_{a4} in $\widetilde{PC}$, while $\widetilde{PD}$ is normalized as

$$\widetilde{pd}_a = \left(\frac{[d_{a1} + dd^-]}{[dd^- + dd^+]}, \frac{[d_{a2} + dd^-]}{[dd^- + dd^+]}, \frac{[d_{a3} + dd^-]}{[dd^- + dd^+]}, \frac{[d_{a4} + dd^-]}{[dd^- + dd^+]}\right)$$

where dd^+ is the maximum value of d_{a4} and $dd^-$is the minimum values of d_{a1} in $\widetilde{PD}$.

Step 6: Calculate the fuzzy positive ideal solution $(\tilde{A}^*)$ based on normalized fuzzy pure concordance and discordance matrices as

$$\tilde{A}^* = \{\tilde{v}_c^*, \tilde{v}_d^*\}$$

where, $\tilde{v}_{\tilde{c}}^* = \max_a \{nc_{a4}\}$ and $\tilde{v}_{\tilde{d}}^* = \min_a \{nd_{a1}\}$.

Step 7: Calculate the fuzzy negative ideal solution $(\tilde{A}^-)$ based on normalized fuzzy pure concordance and discordance matrices as

$$\tilde{A}^- = \{\tilde{v}_c^-, \tilde{v}_d^-\}$$

where, $\tilde{v}_{\tilde{c}}^- = \min_a \{nc_{a1}\}$ and $\tilde{v}_{\tilde{d}}^- = \max_a \{nd_{a4}\}$.

Step 8: Calculate the distance of each alternative (Δ_a^*) in relation to the positive ideal solution as

$$\Delta_a^* = \delta(\widetilde{nc}_a, \tilde{v}_{\tilde{c}}^*) + \delta(\widetilde{nd}_a, \tilde{v}_{\tilde{d}}^*)$$

Step 9: Calculate the distance of each alternative (Δ_a^-) in relation to the negative ideal solution as

$$\Delta_a^- = \delta(\widetilde{nc}_a, \tilde{v}_{\tilde{c}}^-) + \delta(\widetilde{nd}_a, \tilde{v}_{\tilde{d}}^-)$$

Step 10: Calculate the closeness coefficient (CC_a) of each alternative as $CC_a = \frac{\Delta_a^-}{\Delta_a^* + \Delta_a^-}$.

4. Numerical Example

In this section, we describe in details the application of the algorithm presented in section 3, using a case study conducted by the authors to select, among five candidates, an appropriate supplier of a frequently used item in a hospital. The example involved a sole decision maker (DM), the responsible for selecting suppliers. To facilitate the computations and use by the decision maker, the designed method was computationally implemented using Java™ language.

Initially, the Electre method were applied and seven criteria were defined by the decision maker, based on his experience, as follows: price (C_1), conditions of payment (C_2), communication capability (C_3), quality certification (C_4), location (C_5), delivery conditions and/or shipment quality (C_6) and compliance with due date (C_7). Next, the linguistic variables for the importance weight of the criteria and the ratings of the alternatives are presented in Tables 1 and 2, respectively, while the ratings and weight of the criteria are presented in Table 3.

Table 1 – Linguistic terms for the importance weight of the criteria

Linguistic term	Abbreviation	Fuzzy number
Not important	NI	(0.0, 0.10, 0.20, 0.3)
Moderately important	MI	(0.2, 0.30, 0.40, 0.5)
Important	I	(0.4, 0.50, 0.60, 0.7)
Very important	VI	(0.6, 0.70, 0.80, 0.9)
Extremely important	EI	(0.8, 0.85, 0.95, 1.0)

Table 2 – Linguistic terms for the rating of the alternatives

Linguistic term	Abbreviation	Fuzzy number
Extremely undesirable	EU	(0.0, 0.0, 0.1, 0.2)
Undesirable	U	(0.1, 0.2, 0.3, 0.4)
Acceptable	A	(0.3, 0.4, 0.5, 0.6)
Desirable	D	(0.5, 0.6,0.7, 0.8)
Extremely desirable	ED	(0.7, 0.9, 1.0, 1.0)

Table 3 – Rating of the alternatives and weight of the criteria

A_i / C_j	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	ED	A	ED	EU	ED	EU	ED
A_2	ED	D	A	ED	ED	ED	EU
A_3	ED	A	D	A	ED	A	U
A_4	A	U	ED	A	D	ED	ED
A_5	ED	ED	D	ED	ED	ED	D
W_j	MI	MI	EI	MI	VI	VI	EI

The next step of the Electre method is calculating the concordance and discordance indexes. Table 4 and 5 presents the results obtained.

Table 4 – Fuzzy concordance matrix

	A1	A2	A3	A4	A5
A1	(0.0, 0.0, 0.0, 0.0)	(1.3, 1.5, 1.75, 1.95)	(1.5, 1.8, 2.15, 2.45)	(1.6, 2.02, 2.48, 2.9)	(1.3, 1.5, 1.75, 1.95)
A2	(2.1, 2.5, 2.95, 3.35)	(0.0, 0.0, 0.0, 0.0)	(1.4, 1.8, 2.2, 2.6)	(1.5, 1.95, 2.4, 2.85)	(0.8, 1.0, 1.2, 1.4)
A3	(1.9, 2.2, 2.55, 2.85)	(2.0, 2.2, 2.5, 2.7)	(0.0, 0.0, 0.0, 0.0)	(1.1, 1.45, 1.8, 2.15)	(0.8, 0.92, 1.08, 1.2)
A4	(1.8, 1.97, 2.22, 2.4)	(1.9, 2.05, 2.3, 2.45)	(2.3, 2.55, 2.9, 3.15)	(0.0, 0.0, 0.0, 0.0)	(1.9, 2.05, 2.3, 2.45)
A5	(2.1, 2.5, 2.95, 3.35)	(2.6, 3.0, 3.5, 3.9)	(2.6, 3.07, 3.62, 4.1)	(1.5, 1.95, 2.4, 2.85)	(0.0, 0.0, 0.0, 0.0)

Table 5 – Fuzzy discordance matrix
Table 8 – Fuzzy discordance matrix

	A1	A2	A3	A4	A5
A1	(0.0, 0.0, 0.0, 0.0)	(0.6, 0.96, 1.21, 1.21)	(0.46, 0.77, 1.07, 1.23)	(0.6, 0.96, 1.21, 1.21)	(0.6, 0.96, 1.21, 1.21)

A2	(0.6, 0.96, 1.21, 1.21)	(0.0, 0.0, 0.0, 0.0)	(-0.22, 0.22, 0.66, 0.88)	(0.6, 0.96, 1.21, 1.21)	(0.52, 0.87, 1.21, 1.39)
A3	(0.46, 0.92, 1.23, 1.38)	(0.22, 0.88, 1.33, 1.55)	(0.0, 0.0, 0.0, 0.0)	(0.46, 0.92, 1.23, 1.38)	(0.22, 0.88, 1.33, 1.55)
A4	(0.12, 0.48, 0.72, 0.84)	(0.12, 0.48, 0.72, 0.84)	(0.15, 0.61, 0.92, 1.07)	(0.0, 0.0, 0.0, 0.0)	(0.46, 0.92, 1.23, 1.38)
A5	(-0.12, 0.24, 0.48, 0.6)	(0.0, 0.0, 0.0, 0.0)	(0.0, 0.0, 0.0, 0.0)	(-0.15, 0.31, 0.61, 0.77)	(0.0, 0.0, 0.0, 0.0)

Once the fuzzy concordance and discordance index were estimated with the Fuzzy-Electre method, the algorithm present in section 3 can be applied through the following steps:

(1 and 2) The normalized fuzzy concordance and discordance matrices are constructed as showed in Tables 6 and 7.

Table 6 – Normalized fuzzy concordance matrix

	A1	A2	A3	A4	A5
A1	(0.0, 0.0, 0.0, 0.0)	(0.32, 0.37, 0.43, 0.48)	(0.37, 0.44, 0.52, 0.6)	(0.39, 0.49, 0.6, 0.71)	(0.32, 0.37, 0.43, 0.48)
A2	(0.51, 0.61, 0.72, 0.82)	(0.0, 0.0, 0.0, 0.0)	(0.34, 0.44, 0.54, 0.63)	(0.37, 0.48, 0.59, 0.7)	(0.2, 0.24, 0.29, 0.34)
A3	(0.46, 0.54, 0.62, 0.7)	(0.49, 0.54, 0.61, 0.66)	(0.0, 0.0, 0.0, 0.0)	(0.27, 0.35, 0.44, 0.52)	(0.2, 0.23, 0.26, 0.29)
A4	(0.44, 0.48, 0.54, 0.59)	(0.46, 0.5, 0.56, 0.6)	(0.56, 0.62, 0.71, 0.77)	(0.0, 0.0, 0.0, 0.0)	(0.46, 0.5, 0.56, 0.6)
A5	(0.51, 0.61, 0.72, 0.82)	(0.63, 0.73, 0.85, 0.95)	(0.63, 0.75, 0.88, 1)	(0.37, 0.48, 0.59, 0.7)	(0.0, 0.0, 0.0, 0.0)

Table 7 – Normalized fuzzy discordance matrix

	A_1	A_2	A_3	A_4	A_5
A_1	(0.13, 0.13, 0.13, 0.13)	(0.47, 0.67, 0.81, 0.81)	(0.39, 0.56, 0.73, 0.82)	(0.47, 0.67, 0.81, 0.81)	(0.47, 0.67, 0.81, 0.81)
A_2	(0.47, 0.67, 0.81, 0.81)	(0.13, 0.13, 0.13, 0.13)	(0.0, 0.25, 0.5, 0.63)	(0.47, 0.67, 0.81, 0.81)	(0.42, 0.62, 0.81, 0.91)
A_3	(0.39, 0.65, 0.82, 0.91)	(0.25, 0.63, 0.88, 1)	(0.13, 0.13, 0.13, 0.13)	(0.39, 0.65, 0.82, 0.91)	(0.25, 0.63, 0.88, 1)
A_4	(0.19, 0.4, 0.53, 0.6)	(0.19, 0.4, 0.53, 0.6)	(0.21, 0.47, 0.65, 0.73)	(0.13, 0.13, 0.13, 0.13)	(0.39, 0.65, 0.82, 0.91)
A_5	(0.06, 0.26, 0.4, 0.47)	(0.13, 0.13, 0.13, 0.13)	(0.13, 0.13, 0.13, 0.13)	(0.04, 0.3, 0.47, 0.56)	(0.13, 0.13, 0.13, 0.13)

(3 and 4) The Fuzzy pure concordance matrix ($\widetilde{PC}_a$) and the Fuzzy pure discordance ($\widetilde{PD}_a$) matrices are constructed and presented in Table 8.

Table 8 – Fuzzy pure concordance and discordance matrices

	$\widetilde{PC}_a$	$\widetilde{PD}_a$	$\widetilde{NC}_a$	$\widetilde{ND}_a$
A_1	(-1.52, -0.94, -0.26, 0.33)	(-1.0, 0.01, 1.18, 2.14)	(0.02, 0.17, 0.34, 0.49)	(0.36, 0.52, 0.7, 0.85)
A_2	(-1.27, -0.68, 0, 0.59)	(-1.18, -0.13, 1.11, 2.12)	(0.08, 0.23, 0.41, 0.56)	(0.33, 0.49, 0.69, 0.85)
A_3	(-1.59, -1, -0.32, 0.27)	(-1.03, 0.54, 1.98, 3.09)	(0.0, 0.15, 0.33, 0.48)	(0.35, 0.6, 0.83, 1.0)
A_4	(-0.7, -0.11, 0.57, 1.16)	(-2.1, -0.99, 0.25, 1.49)	(0.23, 0.38, 0.56, 0.71)	(0.19, 0.36, 0.55, 0.75)
A_5	(0.44, 1.02, 1.71, 2.29)	(-3.28, -2.5, -1.44, -0.25)	(0.52, 0.67, 0.85, 1.0)	(0.0, 0.12, 0.29, 0.48)

(5) The normalized Fuzzy pure concordance matrix ($\widetilde{NC}_a$) and the normalized Fuzzy pure discordance matrix ($\widetilde{ND}_a$) are constructed. See the last two columns of Table 8.

(6) The fuzzy positive ideal solution is derived as $\tilde{A}^* = \{(1;1;1;1).(0;0;0;0\}$.

(7) The fuzzy negative ideal solution is derived as $\tilde{A}^- = \{(0;0;0;0).(1;1;1;1)\}$.

(8) The distance of each alternative from the positive ideal solution (Δ_a^*) is computed (see Table 9).

(9) The distance of each alternative from the negative ideal solution (Δ_a^-) is computed and showed in Table 9.

(10) The closeness coefficient (CC_a) is calculated and presented in Table 9.

Table 9 – Positive and negative ideal solution

	$\delta(\widetilde{nc}_a.\tilde{v}_c^*)$	$\delta(\widetilde{nd}_a.\tilde{v}_d^*)$	Δ_a^*	$\delta(\widetilde{nc}_a.\tilde{v}_c^-)$	$\delta(\widetilde{nd}_a.\tilde{v}_d^-)$	Δ_a^-	CC_a
A_1	0.766	0.634	1.400	0.311	0.435	0.747	0.347
A_2	0.702	0.621	1.323	0.367	0.454	0.822	0.383
A_3	0.782	0.735	1.517	0.299	0.390	0.689	0.312
A_4	0.561	0.507	1.068	0.501	0.578	1.080	0.502
A_5	0.299	0.285	0.584	0.782	0.798	1.580	0.730

Finally, the Fuzzy-TOPSIS (Chen, 2000; Chen, Ling and Huang, 2006) and the Chaterjee's method (Chaterjee *et al*., 2010) were applied to the same problem with purpose of comparing the final ranking of the alternatives. Based on the results presented in Table 10, A_5 is clearly the best alternative based on the set of criteria previously defined. However, the proposed method presented some advantages in relation to the Fuzzy-TOPSIS and the Chaterjee's method, given that it provided better rankings than both methods. Table 10 also shows that the difference between the two best alternatives increased from 0.04 (a quite small value given the vagueness involved in the process) to 0.28, and the range of the closeness coefficient increased from 0.1827 to 0.418, when the proposed method is compared with Fuzzy-TOPSIS method. However, as these methods are different in relation to decision making preference structure and the weight of the criteria have different meaning in compensatory (Fuzzy-TOPSIS) and non-compensatory methods (Fuzzy-Electre), this comparison must be relativized. On the other hand, when we compare the proposed method with the Chaterjee's method, the first one differentiated better the final ranking of the alternatives than the last one, given that the Chaterjee's method provided a ranking with two indifferences.

Table 10 – Chaterjee's method x Fuzzy-TOPSIS x proposed method

	Chaterjee et al. (2010)				Fuzzy-TOPSIS				Proposed method			
A_a	$\widetilde{NC}_a$	$\widetilde{ND}_a$	Partial position	Ranking	d_i^*	d_i^-	CC_a	Ranking	Δ_a^*	Δ_a^-	CC_a	Ranking
A_1	(0.36, 0.51, 0.69, 0.85)	(0.32, 0.48, 0.65, 0.8)	2 / 3	2.5	2.4962	2.6476	0.4852	3	1.400	0.747	0.347	4
A_2	(0.06, 0.22, 0.4, 0.55)	(0.32, 0.48, 0.67, 0.82)	5 / 3	4	2.3720	2.8134	0.4574	4	1.323	0.822	0.383	3
A_3	(0, 0.15, 0.33, 0.49)	(0.37, 0.61, 0.83, 1)	4 / 4	4	2.1181	2.9588	0.4172	5	1.517	0.689	0.312	5
A_4	(0.29, 0.45, 0.63, 0.78)	(0.18, 0.35, 0.54, 0.73)	3 / 2	2.5	2.8456	2.3006	0.5529	2	1.068	1.080	0.502	2
A_5	(0.51, 0.67, 0.85, 1)	(0, 0.12, 0.28, 0.46)	1 / 1	1	3.1275	2.0852	0.5999	1	0.584	1.580	0.730	1

5. Conclusions

In this paper, we proposed an alternative procedure for ranking alternatives in Fuzzy-Electre environments. This approach combines the pure concordance and discordance indexes with the TOPSIS' closeness coefficient. Although we cannot affirm that our approach obtains the most appropriate rankings for all MADM problems, the experiments presented in section 4 demonstrated that the rankings provided are consistent and more informative than the ones obtained by Fuzzy-TOPSIS and Chaterjee' method.

Future work will be directed towards a more general comparison between Fuzzy-Electre and our method through random generated scenarios, with different combinations in number of alternatives, criteria and decision makers. Additionally, we can verify the ability of the method to avoid ranking irregularities, by considering different analysis criteria such as: (i) the effect of the inclusion of a clear non-optimal alternative in the analysis; (ii) the transitivity property; and (iii) the impact on the rankings when the MADM problem is decomposed. Based on the results obtained in both tests, we can better guarantee the reliability and effectiveness of our method.

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