THE SIDE-SENSITIVE SYNTHETIC MEDIAN CHART

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ABSTRACT

The synthetic chart signals when a second sample point falls beyond the control limits and it is not far from the first one, that is, the maximum sample points between them cannot exceed a threshold *L*. The side-sensitive synthetic (SSSyn) chart signals when these two points beyond the control limits are close from each other and on the same side of the centerline. The side-sensitive feature enhances the ability of the chart to signal. When the samples are small, two or three units per sample, the SSSyn Median chart competes with the standard $\overline{X}$ chart in terms of the speed they detect changes in the process mean. Devices, such as go-no-go gauges, allow monitoring processes without the need to measure any unit of the samples. With a go-no-go gauge device it is possible to know the position of the sample median; if above the upper control limit, if below the lower control limit, or if in the central region. The right value of the sample median is not necessary to to run the SSSyn Median chart. When the operator is in charge of the control, inspection by attribute, instead of by variable, is highly recommended. Stationary properties of the Markov chain were used to obtain the steady-state average run length (SSARL). The SSARL measures the speed with which the control chart signals processs changes under the assumption that the process starts adjusted and remains unaltereded until the occurrence of an unpredictable change in the process mean.

KEYWORDS. Median Chart; Side-sensitive Chart; Steady-state Average Run Length.

Main area (EST – Estatística - Statistics)

1. Introduction

The $\overline{X}$ charts detect mean changes faster than the median charts; because of that, the articles dealing with median charts are rare; see Park (2009) and Koo (2005). In this article, we consider the side sensitive synthetic signaling rule to improve the median chart performance.

The synthetic chart proposed by Wu and Spedding (2000) requires two points beyond the control limits to signal. The maximum sample points between them cannot exceed a threshold L, otherwise the synthetic chart doesn't signal. Wu *et al.* (2001), Huang and Chen (2005), and Chen and Huang (2005) studied the properties of the synthetic np, S and R charts, respectively. Costa and Rahim (2006) proposed a synthetic chart based on noncentral chi-square statistics. Costa *et al.* (2009) evaluated the properties of a synthetic chart with two-stage testing that is able to signal changes in the process mean and in the process variance. Machado *et al.* (2009) introduced a synthetic chart based on two sample variances for monitoring the covariance matrix of bivariate processes. Wu *et al.* (2010) proposed a $\overline{X}$ chart that signals when a sample point falls beyond control limits or when a second point, not far from the first, falls beyond warning limits. Khoo *et al.* (2010) proposed a synthetic double sampling chart. More recently, Zhang *et al.* (2011) evaluated the properties of the synthetic $\overline{X}$ chart when the process parameters are estimated. The growing interest in synthetic charts may be explained by the fact that many practitioners prefer waiting until the occurrence of a second point beyond the control limits before looking for an assignable cause.

Davis and Woodall (2002) introduced the side-sensitive version of the synthetic chart where the two points beyond the control limits should be not only close from each other but also on the same side of the centerline. Costa and Machado (2012) presented a Markov chain model of the side sensitive synthetic $\overline{X}$ chart and used it to evaluate the steady-stage average run length (ARL) performance. The side-sensitive feature enhances the ability of the synthetic chart to signal.

2. The Markov Chain Model of the Side-Sensitive Synthetic Chart

The following transition matrix of the Markov chain is used to obtain the steady-state ARLs of the side-sensitive synthetic median chart.

$$
\begin{array}{c|cccccccccccc}
 & \underline{1}00..0 & 0\underline{1}0..0 & \ldots & 0..0\underline{1}0 & 0..00\underline{1} & 00..00 & 0..001 & 0..010 & 0..100 & \ldots & 100..0 & Signal \\
\underline{1}00..0 & 0 & 0 & \ldots & 0 & 0 & A & B^{+} & 0 & 0 & \ldots & 0 & B^{-} \\
0\underline{1}0..0 & A & 0 & \ldots & 0 & 0 & 0 & B^{+} & 0 & 0 & \ldots & 0 & B^{-} \\
00\underline{1}..0 & 0 & A & \ldots & 0 & 0 & 0 & B^{+} & 0 & 0 & \ldots & & B^{-} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0..00\underline{1} & 0 & 0 & \ldots & A & 0 & 0 & B^{+} & 0 & 0 & \ldots & 0 & B^{-} \\
00..00 & 0 & 0 & \ldots & 0 & B^{-} & A & B^{+} & 0 & 0 & \ldots & 0 & 0 \\
0..001 & 0 & 0 & \ldots & 0 & B^{-} & 0 & 0 & A & 0 & \ldots & 0 & B^{+} \\
0..010 & 0 & 0 & \ldots & 0 & B^{-} & 0 & 0 & 0 & A & \ldots & 0 & B^{+} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
010..0 & 0 & 0 & \ldots & 0 & B^{-} & 0 & 0 & 0 & 0 & \ldots & A & B^{+} \\
100..0 & 0 & 0 & \ldots & 0 & B^{-} & A & 0 & 0 & 0 & \ldots & 0 & B^{+} \\
Signal & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1
\end{array}
\qquad (1)
$$

The transient states describe the position of the last L sample points; "$\underline{1}$" means that the sample point fell below the lower control limit, "0" means that the sample point fell in the central region, and "1" means that the sample point fell above the upper control limit. For instance, the transient state (0$\underline{1}$0..0) is reached when the second of the last L points falls below the lower control limit and all others points fall in the central region. If the current state is state (00...01), (0…010)…(001...0), (010…0), or (100…0) and the next sample point falls below the lower control limit, the Markov chain moves to state 00...0$\underline{1}$. Similarly, if the current state is state (00...0$\underline{1}$), (0…0$\underline{1}$0)…(00$\underline{1}$...0), (0$\underline{1}$0…0), or ($\underline{1}$00…0) and the next sample point falls above the upper control limit, the Markov chain moves to state 00...01. The events "$\underline{1}$", "0", and "1" occur with probabilities B^{-}, A, and B^{+}, respectively. If $X_i \sim N(\mu, 1)$, with i=1, 2… n, are the sample observations, Z_m the median of the sample observations, UCL the upper control limit of the median chart, and LCL the lower control limit of the median chart, follows that: $B^{-} = \Pr[Z_m < LCL]$, $B^{+} = \Pr[Z_m > UCL]$ and $A = 1 - B^{-} - B^{+}$.

The steady-state ARL is given by $\mathbf{S}'\,\mathbf{ARL}$, where $\mathbf{S}$ is the vector with the stationary probabilities of being in each nonabsorbing state and **ARL** is the vector of ARLs taking each nonabsorbing state as the initial state. The $\mathbf{ARL} = (\mathbf{I} - \mathbf{R})^{-1}\mathbf{1}$, where $\mathbf{I}$ is an (2L+1) by (2L+1) identity matrix, $\mathbf{R}$ is the transition matrix given in (2) with the last row and column removed, and **1** is an (2L+1) by one vector of ones.

$$\begin{pmatrix} A & B & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & 0 & 0 & \ldots & 0 & 0 \end{pmatrix} \qquad (2)$$

The **S** vector, with the stationary probabilities of being in each nonabsorbing state, is given by $D^{-1}\mathbf{S}_{\mathbf{C}}$. $\mathbf{S}_{\mathbf{C}}^{'} = (C^{L-1}, C^{L-2}, \ldots, C^{0}, 2C^{L}(1-A)^{-1}, \; C^{0}, \ldots, C^{L-2}, C^{L-1})$, $D=2+2C+2C^2+\ldots+2C^{L-1}+2C^{L}(1-A)^{-1}$, and $C=2A/(1+A)$, with $A = \Pr[|Z| < k | Z \sim N(0;1)]$. It was obtained with the process mean adjusted (μ=0) by solving the system of linear equations $\mathbf{S}^{'}\mathbf{R}_{\mathbf{adj}} = \mathbf{S}$, constrained to $\mathbf{S}^{'}\mathbf{1} = 1$. The matrix $\mathbf{R}_{\mathbf{adj}}$ is an adjusted version of **R**, where all nonzero cells are divided by (1-B), with $B = B^{+} = B^{-}$, except the nonzero cells in the “00...00” row that are kept unaltered. The matrix $\mathbf{R}_{\mathbf{adj}}$ is as follows:

$$\frac{1}{1-B}\begin{pmatrix} 0 & 0 & \ldots & 0 & 0 & A & B & 0 & 0 & \ldots & 0 \\ A & 0 & \ldots & 0 & 0 & 0 & B & 0 & 0 & \ldots & 0 \\ 0 & A & \ldots & 0 & 0 & 0 & B & 0 & 0 & \ldots & 0 \\ .. & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & A & 0 & 0 & B & 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & B-B^2 & A(1-B) & B-B^2 & 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & B & 0 & 0 & A & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & B & 0 & 0 & 0 & A & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & B & 0 & 0 & 0 & 0 & \ldots & A \\ 0 & 0 & \ldots & 0 & B & A & 0 & 0 & 0 & \ldots & 0 \end{pmatrix} \qquad (3)$$

Figures 1 through 4 show the SSARLs for the $\overline{X}$ charts and the side-sensitive synthetic median chart considering different shifts in the process mean, expressed in units of σ, the standard deviation of the observations. The control charts were designed fixing $SSARL_0$=370.4 or 1000. The $SSARL_0$ measures the incidence of false alarms; it is the SSARL computed with the process mean adjusted (μ=0). The SSSyn median chart signals larger shifts faster the $\overline{X}$ chart. In terms of the overall performance - considering different mean shifts, the SSSyn median chart is better than the $\overline{X}$ chart, especially when the $SSARL_0$ is large (=1000). When n is even, the median is the mean of the two middle X values. To avoid the mean computation, we might run the SSSyn chart considering the position of the two middle values. For instance, when n=2, a sample point is plotted below the lower control limit (above the upper control limit) if the diameter of both units are smaller than the LCL (bigger than the UCL). Otherwise, the sample point is plotted in the central region.

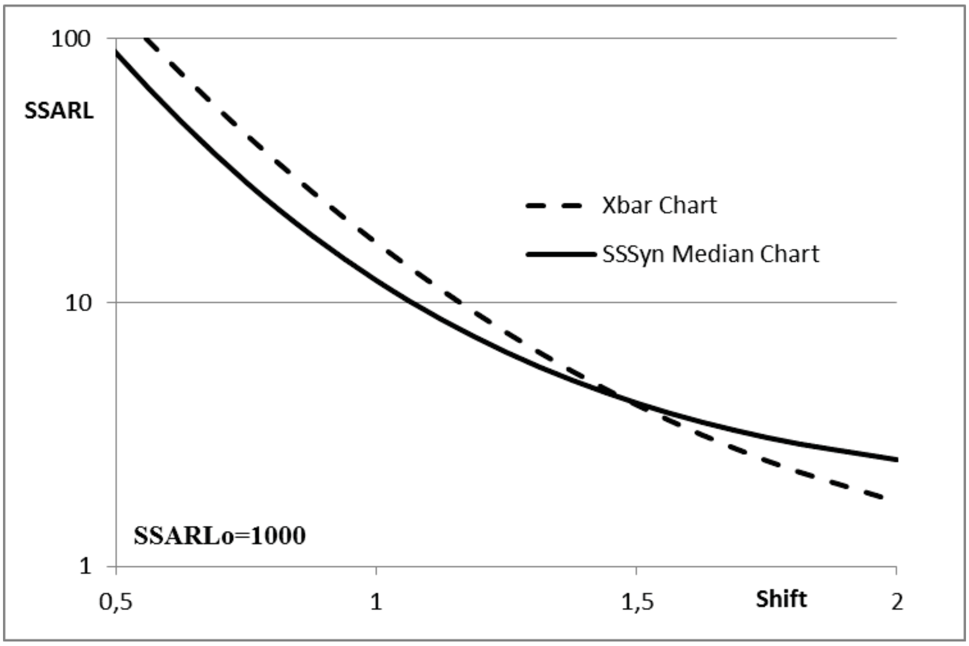

Figure 1: SSARLs for the SSSyn Median chart and $\overline{X}$ chart (L=3, n=3)

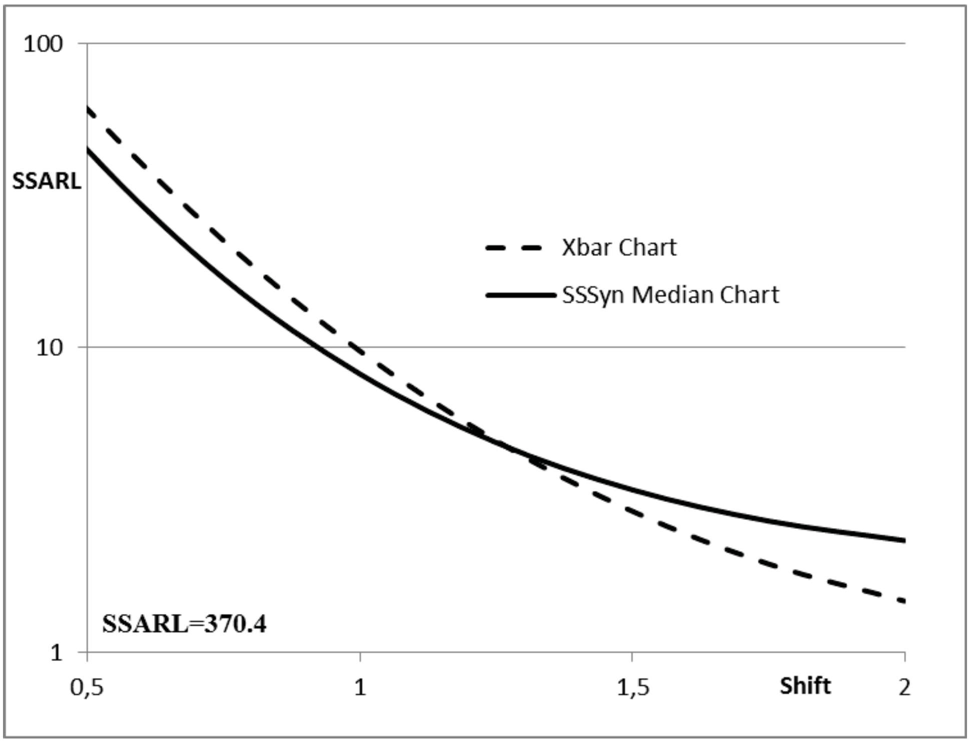

Figure 2: SSARLs for the SSSyn Median chart and $\overline{X}$ chart (L=3, n=3)

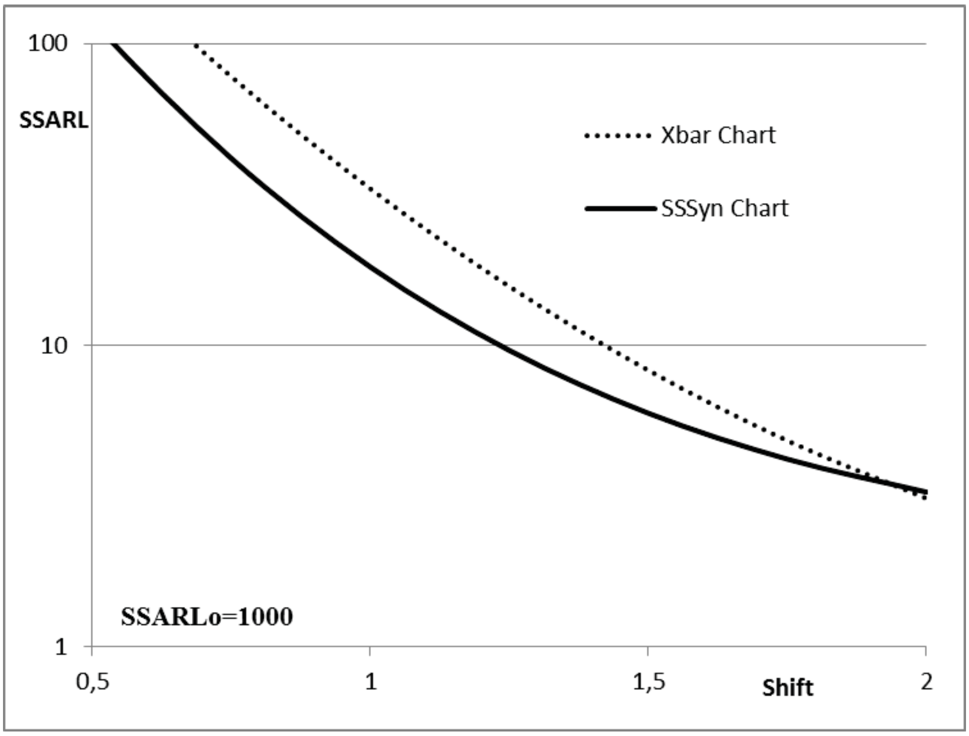

Figure 3: SSARLs for the SSSyn Median chart and $\overline{X}$ chart (L=3, n=2)

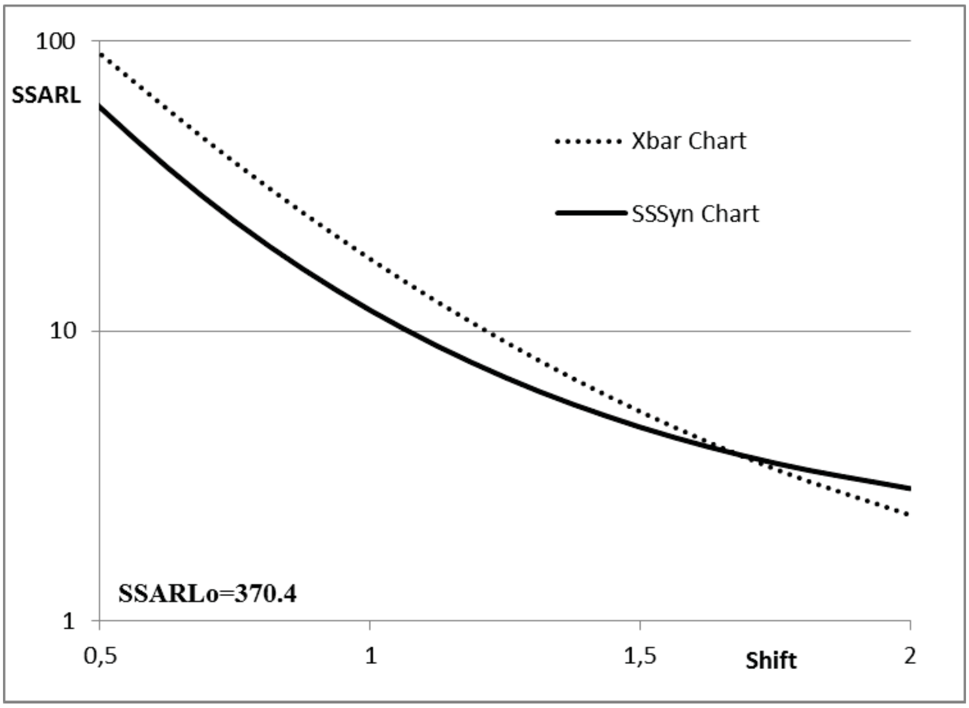

Figure 4: SSARLs for the SSSyn Median chart and $\overline{X}$ chart (L=3, n=2)

4. Conclusions

In this article, we considered the side-sensitive synthetic signaling rule to improve the median chart performance. When the samples are small, two or three units per sample, the SSSyn Median chart has a better overall performance than the standard $\overline{X}$ chart in terms of the speed they detect changes in the process mean. When the operator is in charge of the control, inspection by attribute, instead of by variable, is highly recommended.

Acknowledgements

This work was supported by CNPq – National Council for Scientific and Technological Development, Projects 307744/2006-0 and 306189/2011-9.

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