A NEW IMPLEMENTATION TO THE VEHICLE TYPE SCHEDULING PROBLEM WITH TIME WINDOWS FOR SCHEDULED TRIPS

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ABSTRACT

Kliewer et al. (2006b, 2011) proposed a time window implementation to the multi-depot vehicle (and crew) scheduling problem using a time-space network (TSN). Based on this approach, we developed a new methodology to implement time windows to the vehicle-type scheduling problem (VTSP), which solves the vehicle scheduling problem considering heterogeneous fleet. Our method presents as main advantages the simplicity in the implementation, a smaller sized network, and facility to introduce new constraints closer to reality. In order to verify the effectiveness of our approach, experiments were carried out using real instances from a Brazilian city and large random instances. Analyzing the obtained results, it is possible to affirm that the developed method is able to present relevant savings in the daily operations of the public transportation service, reducing the required number of scheduled vehicles to satisfy the historic demand.

KEYWORDS. Vehicle type scheduling problem, heterogeneous fleet, time window.

L & T - Logistics and Transport

1. Introduction

The vehicle scheduling problem (VSP) has become an extensively studied research area in the last decades. The problem consists in the process of minimizing the assignment costs of vehicles to a given set of timetabled trips, satisfying two main constraints as follows: (i) each trip is assigned exactly once; and (ii) each vehicle performs a feasible sequence of trips. Each selected vehicle starts and ends a trip in the depot and the travel time and stations are fixed and previously defined. Different approaches to model the VSP have been developed, as well as solution methods and extensions for a better reality representation, e.g., the inclusion of multiple-depots and heterogeneous fleet (Bunte and Kliewer, 2009). The latter, however, has received little attention in the public transportation literature. Ceder (2011) stated that the literature of vehicle scheduling covers usually one type of vehicle; however in practice more than one type is used. In the context of heterogeneous fleet, the vehicle scheduling problem is known as the vehicle-type scheduling problem (VTSP). Compared to the VSP, VTSP increases the degrees of freedom for planning decisions, and therefore, the problem complexity.

Heterogeneous vehicle fleet is a common issue in public transportation and is present in most cities around the world. A METRO's survey (2012) with transit agencies from a hundred cities of U.S., Canada and Puerto Rico showed that all of them have heterogeneous fleet and at least half has articulated buses. New York City holds the top spot with 4.344 buses, with 3.704 buses over 35 feet in length and 640 articulated. In European Union, the International Association of Public Transport (2010) conducted a survey to assess the key characteristics of urban bus fleets in dozens of countries, and found that in Austria 52.50% are articulated buses, while in Belgium only 12,10% are articulated. In São Paulo, Brazil, the fleet is composed by five different types, characterized by different vehicles' capacities (Prefeitura de São Paulo, 2010).

By including time windows in the VTSP we allow to shift scheduled trips within defined interval, i.e., we introduce flexibility in the departure times of trips, increasing operational advantages on the number of required vehicles. When considering heterogeneous fleet, time windows becomes even more useful since the timetabling can be slightly redefined according to the demand and the bus type assigned to cover it.

Kliewer et al. (2006b, 2011) implemented time windows for homogeneous fleet in the context of multi-depot vehicle scheduling problem (MDVSP) and integrated MDVSP and crew scheduling problem, respectively. In both papers, with small time windows (few minutes), they made minor modifications in the timetable by shifting some trips. Based on a time-space network (TSN) structure, they insert time window arcs, which are multiplications of original service trip arcs, representing a trip displacement of a certain amount of time. The additional time window arcs leads to a more complex mathematical model, for which solution times can get very high. In order to solve the VSTP, a preprocessing routine to filter out useless arcs was implemented and two heuristic approaches to identify the critical set of trips and solve the problem faster were developed.

Based on Kliewer's approach, we developed a new time windows arcs implementation based on the space-time network. In order to qualify our approach, this paper aims to compare the characteristics of both time windows approach to VTSP, using real instances from a Brazilian city and large random instances. Comparing to Kliewer et al. (2006b, 2011) our approach is simpler to implement and results in a smaller size network, allowing its application to the VSTP. To the best of our knowledge the integrated formulation and solution comprising the VTSP and time windows based on a TSN network have not been previously published.

The paper is organized as follows: In Section 2 we firstly describe the time-space network and subsequently we formulate mathematically the VTSP. In Section 3, the time windows implementation presented by Kliewer (2006b, 2011) is described in detail. Following, Section 4 contains a comprehensive description of our proposed approach to time windows implementation. In Section 5, we present computational results comparing Kliewer's and our approach to VTSP and time windows considering real and random instances. Finally, in Section 6 conclusions and outlook to future research are drawn.

2. Modeling the problem

We adopt the time–space network as the basis for the mathematical formulation presented in this paper. Section 2.1 describes the TSN and section 2.2 presents the mathematical formulation for the VTSP.

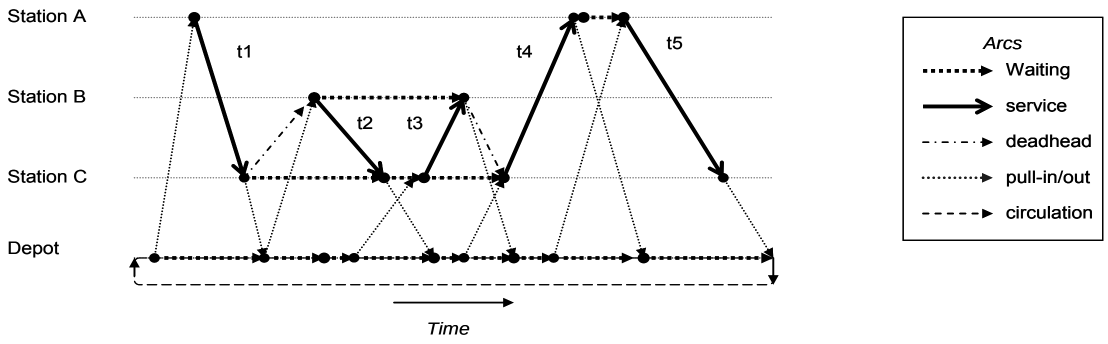
2.1. Time-space network

The VSP has been traditionally formulated using a connection network (Carpaneto et al., 1989), in which nodes represent trips and the arcs, deadheading trips (empty movement). The TSN has been introduced by Kliewer et al. (2002), based on underlying networks developed in the airline context. In their paper, Kliewer et al. argue that the TSN constitutes a better representation both in size (mainly, in the number of arcs) and in the informational level to solve the problem. In subsequent papers (Kliewer et al., 2006a,b, 2011; Steinzen et al., 2010), the TSN was validated for a proper solution of the VSP and its variations This network is composed by an acyclic directed graph $G=(N,A)$ with N as the set of nodes, which represent a specific location at time, and A as the set of arcs, which corresponds to a transition in time and, possibly, space. The set A is divided into five subsets:

- A^s is the set of service arcs, used to connect the corresponding departure and arrival nodes at the start and end locations of a trip with passengers
- A^{wait} is the set of waiting arcs, representing transitions in time-space network where the vehicle are waiting at a station.
- A^{dh} is the set of deadhead arcs, where the vehicles move without passengers between two compatible trips from the end location of the first trip to the start location of the second one.
- A^{pin} is the set of pull-in arcs, expressing the arcs from the depot to a station to starts a trip.
- A^{pout} is the set of pull-out arcs and represents the arcs from a station toward the depot, when a vehicle returns to the depot, for each trip.

Fig. 1 exemplifies a network with one depot and three stations. The service, deadhead and pull-in/out arcs denote the bus in movement while waiting arcs represent the bus stopped at a station. This example represents the TSN considering one vehicle type (for heterogeneous fleet, the network has a multi-layer structure, with one layer for each vehicle-type). It is interesting to note that this example introduces a set of circulation arcs, A^c, connecting the last node (in time) of the depot to its first node, representing a daily schedule for each vehicle type (Steinzen et al., 2010). This small change in the network was introduced later to explicitly minimize the number of scheduled vehicles, easily done by inserting in the objective function the costs of each circulation arc, expressing the fixed costs of using a certain vehicle.

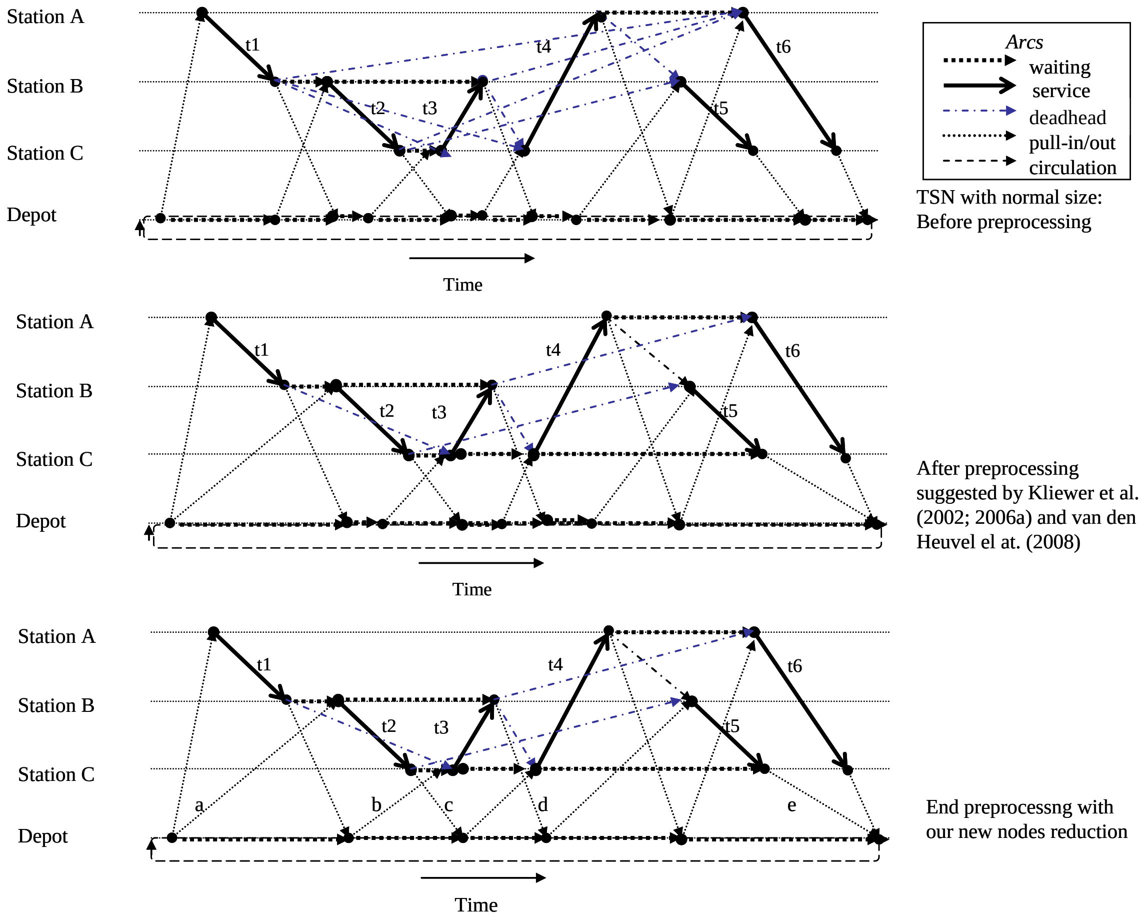
Figure 1. TSN with a depot, five trips and three stations and heterogeneous fleet



The arcs in the TSN represent for the vehicle performing trip 1 (t1) that it can wait for the next trip at the same station (t3), or go until station B by a deadhead arc to perform trip 2 (t2), or also return to depot. The decision is based on the company's policy and in the times and costs involved. Considering that deadhead and waiting arcs means additional costs factor, the minimization of these costs are relevant optimization goals.

However, in some situations the empty movements between two stations (deadhead arcs) is less expensive than pull-out costs, for example, since there is no need for the vehicle to go back to the depot or select an additional vehicle to perform a trip. Though, it is not practical to model all possible deadhead arcs in the TSN, because of the high combinatorial complexity. Thus, we implement the preprocessing suggested by Kliewer et al. (2002; 2006a), so-called "latest-first-matches approach". This routine carries out a nodes aggregation procedure to reduce the total number of deadhead arcs, minimizing the TSN size into a fraction of the original size, still implicitly considering all possible empty movements. Further, we applied a complementary preprocessing step on the nodes that originate or receive pull-in/out arcs, based on van den Heuvel el at. (2008). Concerning pull-in arcs, in complementary preprocessing, if two subsequent nodes at a depot just have pull-in arcs, they can be merged into the first node, by including a waiting arc between the two consecutive nodes at the depot. In the same way, if there are two subsequent pull-out arcs we can merge them into the last node. Besides these preprocessing previously suggested in the literature, we merged pull-in and pull-out arcs on the same node. When pull-in node is subsequent to pull-out node, they may be grouped to the latter. To applying theses preprocessing we obtain a reduced number of arcs and nodes without losing the characteristics of the network representation. Fig. 2 represents the performed preprocessing on the TSN represented in Fig. 1.

Figure 2. Illustration of the TSN preprocessing



TSN with normal size:
Before preprocessing

After preprocessing
suggested by Kliewer et al.
(2002; 2006a) and van den
Heuvel el at. (2008)

End preprocessng with
our new nodes reduction

The main advantage of the TSN structure is the reduced number of variables and constraints, compared with the traditional connection-based network. If the problem contains *m* stations and *n* trips, then the number of deadhead arcs in a TSN is O(*mn)* in opposite to O(n^2) of the connection-based network, with *n*>>*m*. Steinzen et al. (2010) showed a comparison between the number of deadhead arcs plotted on a TSN and on a connection-based network. The TSN is especially relevant when the number of stations involved in the problem is low compared to the number of trips.

2.2. Mathematical formulation

Before defining formally the VSTP, we need to introduce some notation. F is the set of vehicle types (heterogeneous fleet). Let be p_f the capacity of vehicle type $f \in F$, and P_{ij} the expected number of passenger on service trips $(i, j) \in A$. Finally, c_{ij} is the cost of vehicle using the arc $(i, j) \in A$, which is a travel and waiting time function. Proportionately to the size or capacity of the bus, a relative weight α_f is multiplied by the total vehicle cost. Binary variable x_{ijf} define if arc $(i, j) \in A$ is on the route of vehicle $f \in F$. Based on the TSN, the VTSP can be formulated as follows:

$$\min \sum_{(i,j)\in A} \sum_{f\in F} c_{ij} x_{ijf} \qquad (1)$$

s.t.

$$\sum_{(i,j)\in A} x_{ijf} - \sum_{(j.l)\in A} x_{jlf} = 0 \qquad \forall j \in N, \forall f \in F \qquad (2)$$

$$\sum_{f\in F} p_f \cdot x_{ijf} \geq P_{ij} \qquad \forall (i, j) \in A^S \qquad (3)$$

$$\sum_{f\in F} x_{ijf} \leq 1 \qquad \forall (i, j) \in A^S \qquad (4)$$

$$x_{ijf} \in \{0,1\} \qquad \forall (i, j) \in A^S, \forall f \in F \qquad (5)$$

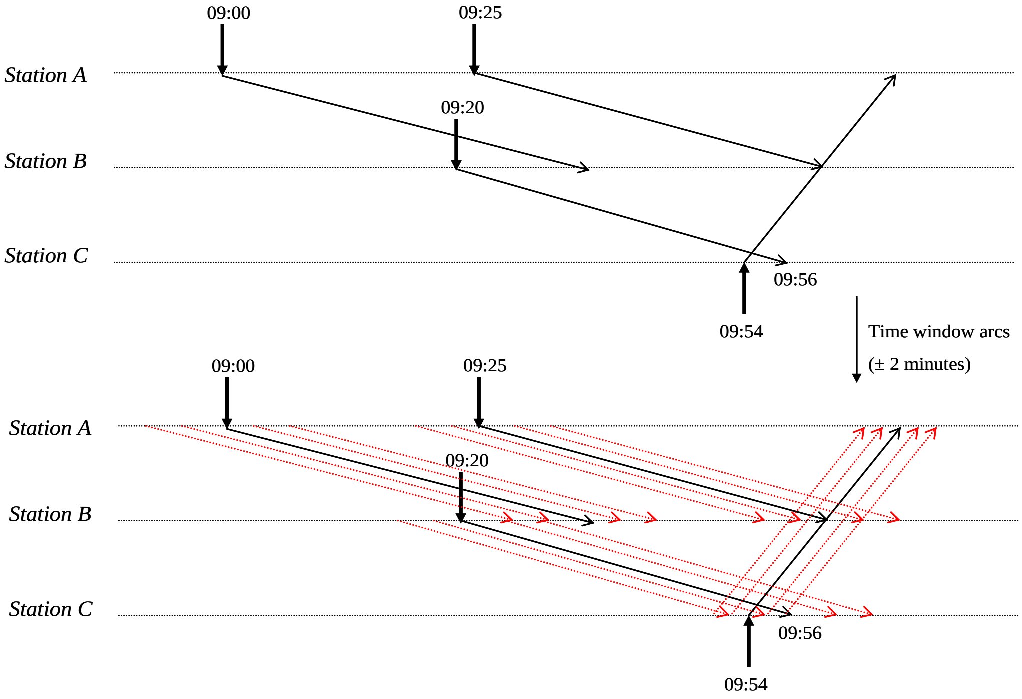
$$x_{ijf} \quad integer \qquad \forall (i, j) \in A \setminus A^S, \forall f \in F \quad (6)$$

The objective function (1) minimizes the total vehicle costs. Constraint (2) ensures the conservative flow properties of the network, while constraint (3) guarantees that the vehicle capacities scheduled to perform the trips is greater than the demand. Constraint (4) ensures that all trips are operated exactly once for a single vehicle type. Constraint (5) and (6) are domain constraints. If the vehicle fixed costs is high, the VTSP corresponds to find the minimum number of vehicles required to perform all trips, corresponding to the service with minimum operating cost. The objective function can be easily modified to minimize the total number of vehicles, considering the costs associated with each circulation arc. As a consequence, only the really needed vehicles will be selected, ensuring a better use of the heterogeneous fleet to attend the daily demand.

3. Time windows implementation: Kliewer et al. (2006b, 2011)

The time window arcs implementation applied by Kliewer et al. (2006b, 2011) consists in multiplying the service arcs of the TSN. For each service trips, time window arcs are inserted, each of them represents a trip displacement of a certain amount of time. They assumed discrete time window values, usually few minutes, to shift some trips of a given timetable and modify possible departure and arrival times. Fig. 3 shows an example with four service trips and a time window for each trip of ±2 minutes. By anticipating a trip that starts at 9:20 at Station B for one or two minutes this trip would be compatible with the trip that starts at 9:54 at Station C.

Figure 3. Arcs Multiplication to consider time window of ± 2 minutes (Kliewer et al., 2006b; 2011)



The insertion of time window arcs leads to a larger mathematical model, generating very high computational solution times. Because of this, a preprocessing technique was developed to avoid insertion of a time window arc that does not enable new trip compatibilities compared to the original service arc. Such unused arcs can be identified from the TSN structure, since only one arc between all available arcs (service arcs and time window arcs) can be selected. In Fig. 3, we represent all inserted time window arcs within a defined time interval. In the preprocessing step, all time window arcs are checked if they enable a new connection from their departure to their arrival station. If it occurs, the arc is kept on the network, otherwise it is deleted. These aspects make the computational implementation of time window arcs quite time consuming.

Due to the increasing model complexity, two heuristics were tested to solve the problem considering the case with multiple depots. The first one, called “trip shortening heuristic” uses a type of what-if analysis, solving the bus scheduling for the original timetable with equal time windows for all trips. The solution provides the trips whose shifting lead to additional connections and in turn reduces the number of necessary vehicles. The resolution for the problem is obtained from these shifting trips. The second one, “cutting-heuristic”, apply the time-windows in specific trips, in general, the ones that occur in peaks hours. The second one, “cutting-heuristic”, is faster than the trip shortening because applies the time-windows just in specific trips. Compared to the global time windows for all trips of a timetable, which provide the largest savings but with a long solution time, these heuristics provided compatibles results in a much shorter time. Concerning the mathematical model each new time window arc requires an additional flow variable. However, additional constraints are not needed, since the existing cover constraints are enhanced by variables corresponding to new arcs. To get a better understanding about this model, we suggest reading of Kliewer et al. (2006b, 2011).

4. The new time window approach

This section describes in details the time window approach developed towards enhancing the efficiency of the method, making it possible to apply for a heterogeneous fleet context. The basic idea of our approach is to add time window arcs linking two trips from the same station, since they are within the defined time window (usually 1 or 2 minutes). The time window arcs are expressed in the network like "reversed waiting arcs", i. e., waiting arcs implemented with an inverse direction.

The procedure to add time window arcs is defined as follows:

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for each station find $t_0, t_1 \in A^s$, $t_0 < t_1$ (where $t_0$ is an arrival and $t_1$ is a departure)
    if $(t_1 - t_0) <$ time window limit
        then add to the network a TW arc which starts in $t_1$ and ends in $t_0$.
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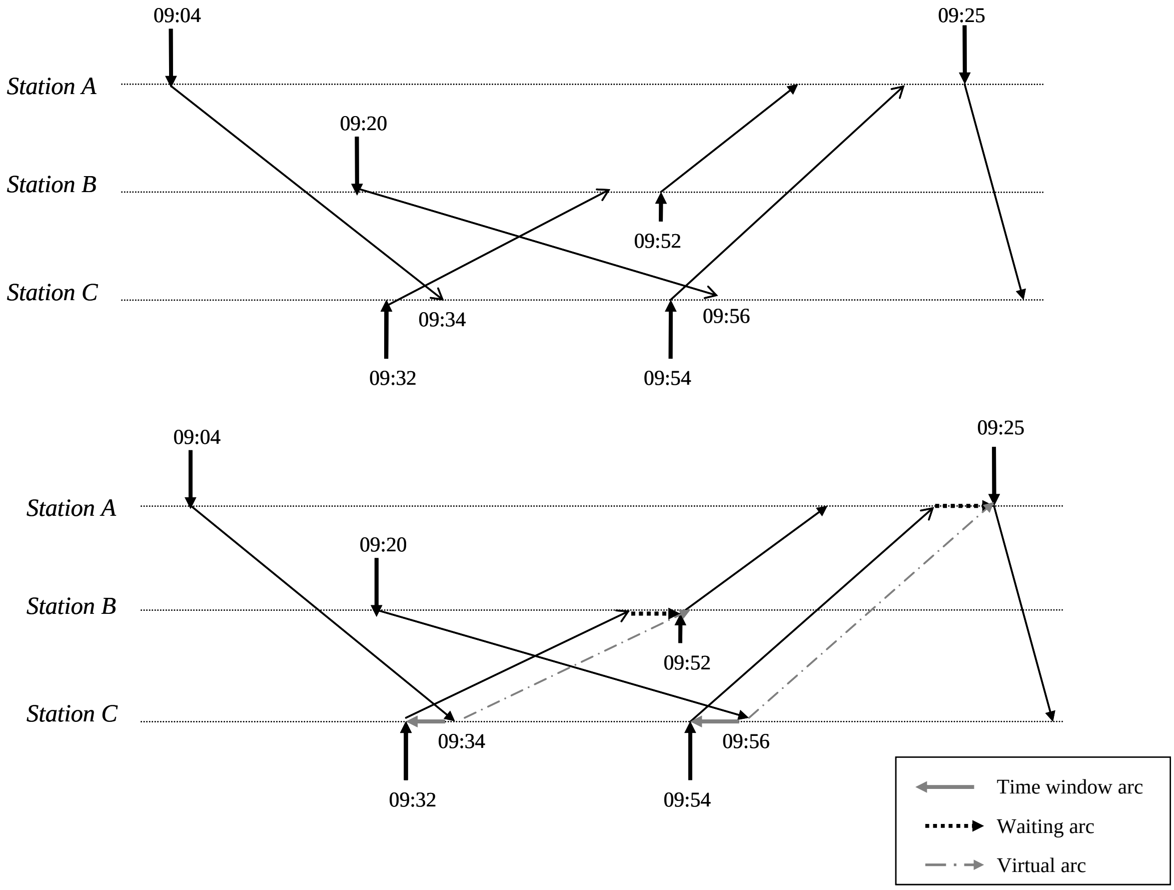
Fig. 4 illustrates two situations in which it is possible to use the time windows arcs and save vehicles, for a two minutes interval.

Figure 4. Network representation for two minutes time windows

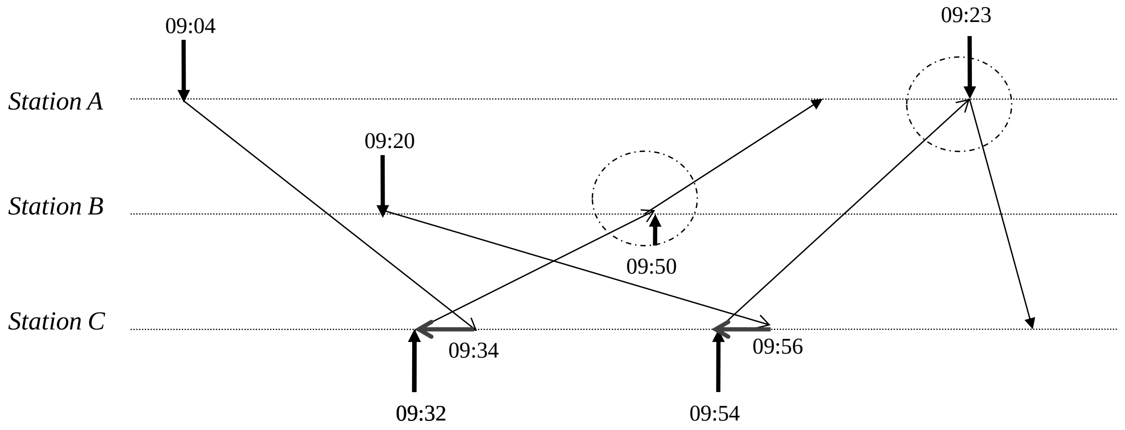


To allow a consistent and adjusted timetable and avoid the accumulation of successive delays on the optimal solution, we added to the model of Section 2.2 a constraint (7), which requires that whenever there is flow in a time window arc, there must be flow in a waiting arc that immediately precedes the service trip performed. The size, in time, of each respective waiting arc should be at least equal to the time window arc. Thus, the new time window application becomes very similar to Kliewer's, which may be represented by the "virtual arc" in Fig. 4. Let TW the set of time window arcs and WA the set of waiting arcs, the constraint (7) can be represented by:

$$x_{ijf} - x_{lkf} = 0 \qquad \forall(i,j) \in TW,\ \forall(l,k) \in WA: (l,k)\ succeed\ (j,l) \in A^S,\ f \in F \qquad (7)$$

Fig. 5 shows a particular case when two consecutive trips are performed and do not has a waiting arc immediately after the service arc from the time window arc. The proposed implementation does not consider the flow represented in the time window arcs because if there is no waiting arc (l,k) successor of a service arc (j,l), the constraint (7) is reduced to $x_{ijf} = 0$. However, in practice, the waiting time between two trips is necessary for the entrance and exit of passengers at stations and/or for the crew to have a rest, which minimizes this drawback.

Figure 5. Network representation without waiting arcs



We applied a penalty costs to each time window arc to ensure that they only take place if savings are obtained, as well as for minimizing changes to the original timetabling. From the solution obtained by the time window model, the timetable can be readjusted. Small time windows intervals enable few changes at the original timetabling, ensuring the service level and the passengers' satisfaction.

5. Computational results

In this section, we describe the results of the carried out experiments to compare the two different time windows approaches. For those tests, we used real-world instances from a public transit companies located in the south of Brazil, and large random instances generated based on real instances with 1000 and 1500 trips. In all instances, the heterogeneous fleet is composed by three different vehicle types as follows: (i) type A, corresponding to an articulated bus, with capacity for 141 passengers; (ii) type B, the most commonly used type, with capacity for 100 passengers; and type C, a smaller vehicle able to carry up to 83 passengers. We overestimated the circulation arcs to find the minimal number of vehicles and define the time window arcs cost twice more expensive than waiting arcs in our approach and twice more expensive than service arcs in Kliewer's approach. We can measure different costs for these arcs because they not interfere directly in the total number of vehicles (which is subject to the circulation arcs), but may impact the distribution of vehicle by types. Time windows are considered with ranges of 2 minutes.

Our main objective is comparing the characteristics of both approaches minimizing the total number of vehicles. All tests were performed on an Intel Core i7-3612QM 2.10GHz and 8 GB RAM running Ubuntu 12.04.2 LTS. We use ILOG CPLEX 12.4 for computing the solutions.

Table 1 shows the characteristics for real and random instances with its respective origin, the number of trips (#Trips) and the number of stations (#Stations).

Table 1. Instances characteristics

Instances	Origin	#Trips	#Stations
A_r	Real	97	21
B_r	Real	499	21
C_r	Real	532	9
D_r	Real	651	17
A_rd	Random	1000	21
B_rd	Random	1500	21

Table 2 compares the results for these instances without time windows and with time windows applying the new (N) and Kliewer’s (K) approaches. We registered the number of vehicles by type, the total number of vehicles and the solution times in seconds.

Table 2. Comparison of solution approaches to two minutes time windows

Instance	Model type		#vehicles		#total	#Solution
		type A	type B	type C	vehicles	time (sec.)
A_r	without time window	4	10	35	49	0,02
	N	4	10	34	48	0,02
	K	4	9	35	48	0,02
B_r	without time window	4	10	35	49	0,15
	N	4	10	34	48	0,16
	K	4	9	35	48	0,19
C_r	without time window	1	1	44	46	0,15
	N	1	1	43	45	0,18
	K	1	1	43	45	0,19
D_r	without time window	4	10	46	60	0,19
	N	4	10	45	59	0,47
	K	4	10	45	59	0,33
A_rd	without time window	48	19	127	194	0,77
	N	46	19	129	194	0,82
	K	47	20	127	194	0,77
B_rd	without time window	50	37	183	270	0,84
	N	50	34	184	268	0,90
	K	50	35	183	268	2,20

Performing a comprehensive analysis, it is clear that the use of TW arcs lead to savings in the number of scheduled vehicles. Since a very short time window interval was used, the current timetable was slightly modified, minimally changing the passenger’s routines. Concerning the objective of this paper, the results indicate that both time window approaches (N and K) are equivalents, changing (for some instances) only the distribution of vehicles by type.

We obtained the same vehicle savings for all instances, with low computational cost for both approaches. These results confirm the practical applicability of the new approach.

Table 3 analyzes the number of service arcs generated in the TSN to solve the model of the problem, the total number of nodes and arcs generated in both approaches. Table 4 shows the proportional network reduction in service arcs, nodes and total arcs for the new approach if compared with Kliewer's (K / N).

Table 3. Comparison of network size for two minutes time windows

Instance	Model type	#Service arcs	#Nodes	#Arcs
A_r	without time window	97	178	596
	N	97	178	646
	K	145	227	775
B_r	without time window	499	1063	3501
	N	499	1063	3809
	K	762	1361	4637
C_r	without time window	532	1133	3726
	N	532	1133	4044
	K	813	1456	4944
D_r	without time window	651	1360	5002
	N	651	1360	5401
	K	989	1730	6506
A_rd	without time window	1000	2281	12925
	N	1000	2281	13197
	K	1127	2429	13561
B_rd	without time window	1500	3385	19517
	N	1500	3385	20140
	K	1862	3786	21527

Table 4. Network reduction with the new approach

Instance	Reduction		
	Service arcs	Nodes	Arcs
A_r	49,5%	27,5%	20,0%
B_r	52,7%	28,0%	21,7%
C_r	52,8%	28,5%	22,3%
D_r	51,9%	27,2%	20,5%
A_rd	12,7%	6,5%	2,8%
B_rd	24,1%	11,8%	6,9%

The network of the developed approach (N) is smaller than the original approach (K), configured as an alternative proposal to solve problems with a large number of variables and constraints. With the new approach we can reduce up to 52% the number of service arcs for some instances, since we do not need perform the multiplication of service arcs to generate the time window arcs.

6. Conclusions

In this paper, we consider a new mathematical formulation to insert time window constraints to the vehicle-type scheduling problem. The approach is based on a time-space network representation and considers the passengers demand for the fleet scheduling. This is another differential of the new model, using the demand as a parameter to solve the problem, a factor often overlooked in the public transportation optimization literature.

The new time window approach is based on the implementation suggested by Kliewer et al. (2006b, 2011), with some differences: our computational implementation do not need preprocessing to delete unused time-window arcs and time window arcs are generated in a similar way to the waiting arcs, instead of multiplying the original service arcs. As a result, a smaller size network is generated. As shown in Table 2, both approaches are equivalent in terms of computation time and solution. However, by having a smaller number of nodes and arcs, the new approach facilitates problem solutions with a large number of variables and constraints.

We intend to apply this new time window approach in order to solve multiple-depot problems and the integrated vehicle and crew scheduling, considering heterogeneous fleet.

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