

## AN OPTIMIZATION MILP MODEL TO INTEGRATE TANK USAGE IN REFINERY AND PIPELINE NETWORK SCHEDULING

**Guilherme Alceu Schneider**

Federal University of Technology – Parana  
Av. Sete de Setembro, 3165 - Rebouças, Curitiba - PR – Brasil  
guilherme@utfpr.edu.br

**Flávio Neves Junior**

Federal University of Technology – Parana  
Av. Sete de Setembro, 3165 - Rebouças, Curitiba - PR – Brasil  
neves@utfpr.edu.br

**Lúcia Valéria Ramos de Arruda**

Federal University of Technology – Parana  
Av. Sete de Setembro, 3165 - Rebouças, Curitiba - PR – Brasil  
lvrarruda@utfpr.edu.br

### ABSTRACT

Nowadays many works have been published aiming solutions to optimize planning and scheduling problems in oil industry. Mixed Integer Linear Programming (MILP) is the technique especially used for modeling and solving these kinds of problems that are characterized by a combinatorial complexity. This paper's proposes a MILP model with time continuous representation for optimizing the tanks' usage in a specific refinery. The model catches information from the external pipeline network's schedule. It has as input variables: tank initial inventory, pipelines scheduling, refinery internal production and local demand market. It considers a refinery scenario with: 9 tanks, 1 kind of product, 2 inputs, 3 outputs, and a programming horizon during one month (H). The MILP model's objective is to ensure feasibility and better usage of the refinery's tank farm while respecting tank's limits. The results show that these objectives can be achieved with a low computational charge.

**KEYWORDS. MILP. Scheduling. Refinery.**

**Main area: P&G – OR in Oil & Gas. IND - OR in Industry. PM - Mathematical Programming**

## 1. Introduction

Refineries, harbors and terminals are components of the oil industry multi-modal supply chain. In this industry, products transportation is carried out by means of pipelines, railways, roads and waterway. This transport logistic requires detailed planning and scheduling tasks in order to assure a good performance of product transfers among the several chain entities.

Planning and scheduling problems present combinatorial complexity, due to the great number of parameters, variables and constraints need to completely characterize problems. As a result, problems solution is cumbersome. In general, planning and scheduling tasks are manually carried out by schedulers, but when complexity grows, the manually execution of these tasks becomes difficult or even impossible.

However this complexity can be circumvented by conservative operational politics (Magatão et al., 2008). In this case, the goal is to assure a feasible solution to problems despite of all system capacities not being fully exploited. On the other hand, idle equipment and resource misusing (including natural resources) can lead to profit loss and also environmental damage. In this context, several researchers are looking for optimized solutions to oil industry supply chain related problems.

Some research works focus on product distribution problem through a multimodal network considering pipes, trains, ships and trucks that transport light oil products to final client (Banaszewski et al., 2010), Banaszewski et al. 2011 e Banaszewski et al., 2012). The authors propose a multi agent heuristic model based on auctions to decide about volume and products to be moved and also about routes and timetable to satisfactorily attain client demands. The FIPA protocol CONTRACT-NET is adopted during simultaneous auctions to negotiate with refineries, depots, pipes and other supply chain agents (manager, auctioneer and participant) in order to attain the problem equilibrium point. The main advantage of this approach is that a feasible solution (equilibrium point) is always reached with a low computational charge.

Product blending problem are modeled with mathematical programming tools by Singh et-al (2000), Li and Karimi (2011), Zhang et-al (2012) and Liang et-al (2012). Blending solutions compute optimal mixtures assuring product quality standard without discharge or product loss. Optimal flow rates of incoming products, blended final specifications (for example octane number of produced gasoline) and other specifications are also involved at blending models. However, in order to assure quality specification ranges, incoming product flow rates must vary during product blending. As a result optimization blending problems are non linear ones.

Magatão et-al (2004), Magatão et-al (2008), Felizari (2009), Magatão et-al (2010) and Boschetto (2011) use MILP models for solving scheduling problems of a pipeline network connecting depots, harbors and refineries. In these works, pipeline operations consider operational and physical limits at refineries, harbors and depots. Features like pump restrictions, pipeline flow rates, flow reversion, product compatibility, and other constraints are also considered and the model goal is to attain a feasible operational schedule to the complete pipeline network. Good solutions at a low computational cost are assured by use of a decomposition approach (the problem is divided into small problem that are individually solved) and hybrid models combining different solution methods. These MILP models compute the sequencing of product batches (time and volume) that arrive and leave refineries by pipelines.

More recently Tong et-al (2012) developed a integrated refinery and pipeline model to scheduling problems. This model comprises production scheduling, pipeline scheduling, product blending and inventory management by means of a monolithic model. The developed model reached global solution in a good computational time for small instances, for example, only one pipeline.

Many other works such as Pinto et-al (2000), Glismann and Gruhn (2001), Jia and Ierapetritou (2003), Barboza (2005), Stebel (2006) present mathematical models developed to assure a better usage of refinery tank farm. In general, all these scheduling models respect tank capabilities and constraints such as mass balance, tank capability and campaign production during programming horizon. Some models also consider blending problems which occur at

intermediary tanks. Other models consider the mixer as a dedicated tank. However, all models goals are to compute inventory time evolution to a given production campaign and satisfy client demand while respect tank operational capacity limits.

In this context, the present work proposes a mathematical model with time continuous representation to optimize tank usage at a refinery. Products arrive at tank farm by one pipeline but they can be delivered by two pipelines. Input variables are tank initial inventory, pipelines scheduling, refinery internal production and local demand market. Production, demand, tank limits and pipeline scheduling restrictions are respected and the computed model output is the tank farm management.

The paper is organized as follows: in section 2, tank farm management problem is described and the considered scenarios are presented, the proposed MILP model with time continuous representation is presented in section 3 and, in section 4, several results are discussed. Finally some conclusion and future works are addressed in section 5.

## 2. Problem Definition

Figure 1 shows a pipeline network connecting, refineries, harbors, distribution points or final clients represented by labeled squares. These nodes are numbered from one to N and all pipes (arcs) can transport product in both direction. Figure 1 also highlights products movements at each node, in special to a refinery node.

A MILP model was developed by Boschetto (2011) to support decisions about network planning and scheduling. The considered network is composed by 30 pipes linking 4 refineries, 2 harbors, 2 distribution point and 2 final clients and the model goal is to optimize product transportation during a schedule horizon. Products are moved through different routes over the network. The model also considers flow direction reversion, seasonal cost constraints, surge operation, human labors constraints, initial inventories and local constraints at each node.

In this model, internal production and local market demand are considered as input parameters. Aggregated inventory is computed as the sum of all tanks inventories, that is, the product quantity at each tank is unknown. Network planning and scheduling results are computed to a time horizon equal to one month. As specific result, Boschetto (2011) model presents a table with all products movements, product volume, including fragmentations, coming in and out all nodes are also reported.

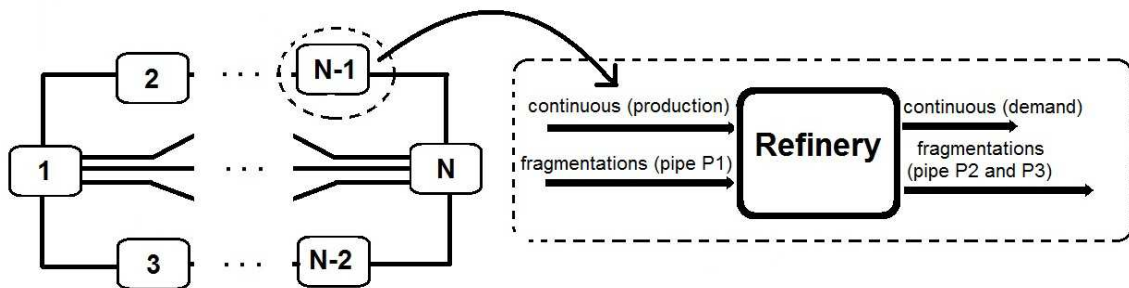


Figure 1. Pipeline network

Figure 2 shows the scenario studied this paper: a refinery tank farm used to move product into refinery. This farm is composed by 9 tanks with initial inventory, a material continuous input flow from refinery production, a refinery input pipeline to incoming products (pipeline P1), an output gate to supply local demand, two output pipelines to flow outgoing products (pipelines P2 and P3). This paper considers mensal handling of one product (p) as presented in Boschetto (2011).

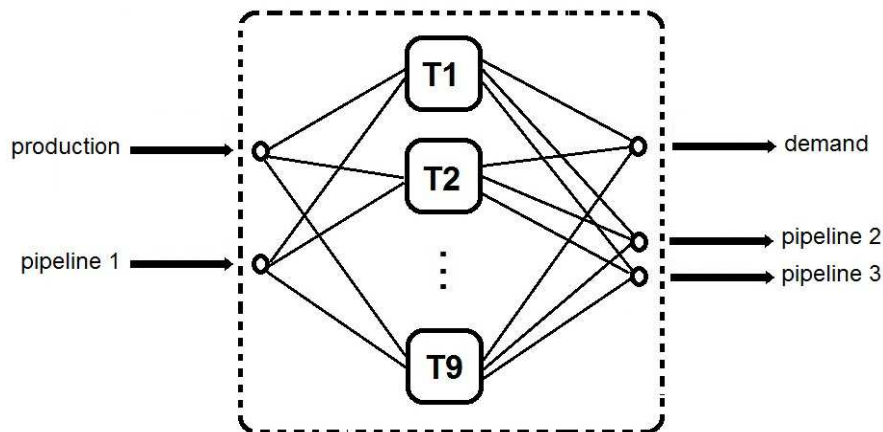


Figure 2. Refinery tank farm

Table 1, 2 and 3 show input data from Boschetto (2011). These data are related to volumes and flow of internal production, local demand, incoming products and outgoing products. The considered programming time horizon is 720 hours.

Table 1 presents continuous intake (internal production) and local departure (demand) of one product. After the first scheduled day, internal production generates flow lines which fill tanks able to receive a priori amount of product (volume) at a constant flow rate. The amount of product (volume) which must be send in order to supply client demands also must be at a constant flow.

Table 1. Continuous intake and local departure (one kind of product)

Production (Input 1)			Demand (Output 1)		
Time (h)	Volume (m <sup>3</sup> )	Flow (m <sup>3</sup> /h)	Time (h)	Volume (m <sup>3</sup> )	Flow (m <sup>3</sup> /h)
0 - 24	0	0	0 - 168	29232	174
24 - 48	12684	528.5	168 - 336	29232	174
48 - 72	12684	528.5	336 - 504	29232	174
...	...	...	504 - 672	29232	174
696 - 720	12684	528.5	672 - 720	8352	174

Tables 2 and 3 present information about fragmentations that are related to pipeline streams. Batches arriving at tank farm are driven from pipeline P1 (input 2) and batches leaving tank farm are driven to pipelines P2 and P3 (outputs 2 and 3).

Table 2 shows batch fragmentation occurring to match pipe P1 schedule to available tanks. In this table, one batch arrives by pipe P1 during programming time horizon. The volume of incoming products and the flow rate by pipeline P1 are considered.

Table 2. Fragmented intake (one kind of product)

Incoming products			
Pipe	Streams	Volume (m <sup>3</sup> )	Flow (m <sup>3</sup> /h)
P1 (Input 2)	Single	4600	450

Table 3 describes volume moved from (outgoing product) tank farm through pipes P2 and P3. The pipelines scheduling is computed as proposed by Boschetto (2011) model. Flow rates are maintained constant, thus batches volumes are time varying in order to match both pipeline and tank scheduling. In table 3, there are several batches (streams) which are pumped to both pipes (P2 and P3). The amount of product at each stream can vary into a volume range and the number of streams is strictly defined, 14 batches for movements in P2 and 13 batches to movements in P3.

Table 3. Fragmented departure (one kind of product)

Outgoing products							
Pipe	Streams	Streams Range (m <sup>3</sup> )	Flow (m <sup>3</sup> /h)	Pipe	Streams	Streams Range (m <sup>3</sup> )	Flow (m <sup>3</sup> /h)
P2 (Output 2)	S1	550	600	P3 (Output 3)	S1	18000	1000
	S2				S2		
	S3				S3		
	S4				S4		
	S5				S5		
	S6	S6					
	S7	S7					
	S8	S8					
	S9	S9					
	S10	S10					
	S11	S11					
	S12	S12					
	S13	S13					
	S14	S14					

Boschetto (2011) considers aggregated inventory like as the sum of the inventory at all tanks, in order to solve planning and scheduling in all level of oil supply chain. However from Boschetto (2011) the quantity of product at each tank is unknown. To fill this gap, we propose the model described below.

### 3. Mathematical Formulation

This section presents the MILP model developed to manage inventories and to schedule tanks at a refinery tank farm. The model uses a time continuous representation and it is looking for to optimize tank usage. Other works such as Barboza (2005) and Stebel (2006) also proposed MILP models to solve problems related to optimal usage of a tank farm. However the herein proposed model is able to solve refinery scheduling by considering fragmented batches. That is not addressed in above mentioned works. These fragmentations occur due to arriving and leaving batches related to pipeline streams (external network pipeline).

Next sections describe model characteristics, its objective function and constraints and also the adopted denomination of indices, parameters and variables.

#### 3.1 Model Definition

The considered scenario is composed by 9 tanks which can receive or send product batches using 5 refinery outer points. Received batches come from refinery internal production (input 1) and from pipeline P1 (input 2). Send batches are delivered by demand (output 1), pipelines P2 (output 2) and P3 (output 3).

The developed MILP model addresses following situations: only one product is moved, refinery production and local demand are a priori known, all fragmented batches are also a priori known, initial tank inventories are a priori known, programming horizon is 720 hours corresponding to one month.

#### 3.2 Nomenclature

*Indices:*

*t* Tanks  
*o* Outer  
*b* Batches

*Sets:*

*T* Set of tanks

$O_I$	Set of inputs where $o \in O_I$ and $I = 1, 2$ (production and P1)
$O_O$	Set of outputs where $o \in O_O$ and $O = 1, 2, 3$ (demand, P2 and P3)
$O$	$O_I \cup O_O$
$B$	Set of batches

*Binary MILP model variables:*

$MOVING_{t,o,b}$	1, if tank $t$ is moving product during batch $b$
$FREE_{t,b}$	1, if tank $t$ is free during batch $b$

*Continuous MILP model variables:*

$STREAM_{t,o,b}$	Product arriving to tank $t$ or leaving from tank $t$ during batch $b$
$STOCK_{t,b}$	Inventory of tank $t$ during batch $b$
$TLVIOMIN_{t,b}$	Lower storage violation of product on tank $t$ during batch $b$
$TLVIOMAX_{t,b}$	Upper storage violation of product on tank $t$ during batch $b$
$TOTAL_o$	Total volume of product received or delivered at each outer $o$
$TS_{o,b}$	Stream start time of outer $o$ at each batch $b$
$TF_{o,b}$	Stream end time of outer $o$ at each batch $b$
$TSO_{t,b}$	Operation start time of tank $t$ at each batch $b$
$TFO_{t,b}$	Operation end time of tank $t$ at each batch $b$

*Parameters*

$STORED_t$	Initial volume at each tank $t$
$MINCAP_t$	Tank $t$ minimum capacity
$MAXCAP_t$	Tank $t$ maximum capacity
$OUTSIDE_o$	Monthly volume of outer $o$
$FLOW_o$	Flow rate at each outer $o$
$MAXIMUM_o$	Greater amount of product moved at each outer $o$
$MINIMUM_o$	Lower amount of product moved at each outer $o$
$NumberSHOTS_o$	Number of fragmentations at each outer $o$
$H$	Programming time horizon (720 hours or 1 month)
$SETUP$	Idle time to product certification (4 hours)
$\Psi$	Volume minimum reference
$\Phi$	Volume maximum reference

### 3.3 Objective Function (OF)

The model goal is to manage product inventory among all tanks. Moreover tank capacity must be exploited without exceed tank operational and physical limits. That is a good practical to respect mass balances at each tank during  $H$ . These limits are modeled by variables  $TLVIOMIN$  and  $TLVIOMAX$ . Thus the model objective function is looking for minimize these variables as given below.

$$\min(Z) = \sum_{t \in T} \sum_{b \in B} (TLVIOMIN_{t,b} + TLVIOMAX_{t,b}) \quad (1)$$

### 3.4 Tank Operation

A tank can assume one of three states: receiving, sending or free ( $MOVING$  or  $FREE$ ) of batches. The tanks also have an idle time that corresponds a time to product certification ( $SETUP$ ). This wait condition must be respected, only after this interval, a tank can delivery again. These conditions are modeled by constraints 2 and 3.

$$\sum_{o \in O} MOVING_{t,o,b} + FREE_{t,b} = 1 \quad \forall t \in T, b \in B \quad (2)$$

$$TSO_{t,b} \geq TFO_{t,b-1} + SETUP \cdot \sum_{o \in O_I} MOVING_{t,o,b-1} \quad \forall t \in T, b \in B \mid b \geq 2 \quad (3)$$

Constraint 4 states that only one tank can be receiving from production or from P1 for each batch. Similarly only one tank can be sending a batch to demand, pipe P2 or pipe P3.

$$\sum_{t \in T} MOVING_{t,o,b} \leq 1 \quad \forall o \in O, b \in B \quad (4)$$

### 3.5 Timing Constraints

Constraints 5 and 6 state that streams occurs until the end of programming timing horizon. Constraint 7 states that flow for each refinery outer point defines stream timing duration. If there is no stream, the start time and end time are equals. The product volume of each batch arriving or leaving each tank must respect input and output flow rates.

$$0 \leq TS_{o,b} \leq H \quad \forall o \in O, b \in B \quad (5)$$

$$0 \leq TF_{o,b} \leq H \quad \forall o \in O, b \in B \quad (6)$$

$$TF_{o,b} = TS_{o,b} + \sum_{t \in T} STREAM_{t,o,b} / FLOW_o \quad \forall o \in O, b \in B \quad (7)$$

Barboza (2005) also had results to scheduling for each tank individually. The author used a mathematical formulation to link streams time with operating times in the tanks. This formulation was adapted to the problem in this paper (constraints 8 to 13). Constraints 8 to 11 impose that for a stream occurrence, there is a synchronism among the stream timing and the operation tank. When there is no stream the condition is relaxed. Constraints 12 and 13 imposes that the operation has null time duration when the tank is free, otherwise the condition is relaxed.

$$TSO_{t,b} \leq TS_{o,b} + H.(1 - MOVING_{t,o,b}) \quad \forall t \in T, o \in O, b \in B \quad (8)$$

$$TSO_{t,b} \geq TS_{o,b} - H.(1 - MOVING_{t,o,b}) \quad \forall t \in T, o \in O, b \in B \quad (9)$$

$$TFO_{t,b} \leq TF_{o,b} + H.(1 - MOVING_{t,o,b}) \quad \forall t \in T, o \in O, b \in B \quad (10)$$

$$TFO_{t,b} \geq TF_{o,b} - H.(1 - MOVING_{t,o,b}) \quad \forall t \in T, o \in O, b \in B \quad (11)$$

$$TSO_{t,b} \leq TFO_{t,b} + H.(1 - FREE_{t,b}) \quad \forall t \in T, b \in B \quad (12)$$

$$TSO_{t,b} \geq TFO_{t,b} - H.(1 - FREE_{t,b}) \quad \forall t \in T, b \in B \quad (13)$$

### 3.6 Mass Balance at Tank

Tank current inventory are computed by mass balance at each batch. The value is computed by summing previous inventory to received volume minus delivered volume. When the first inventory is computed the previous inventory are initial ones. This mass balance is modeled by constraints 14 and 15.

$$STOCK_{t,b} = STOCK_{t,b-1} + \sum_{o \in O_I} STREAM_{t,o,b} - \sum_{o \in O_O} STREAM_{t,o,b} \quad \forall t \in T, b \in B | b \geq 2 \quad (14)$$

$$STOCK_{t,b} = STORED_t + \sum_{o \in O_I} STREAM_{t,o,b} - \sum_{o \in O_O} STREAM_{t,o,b} \quad \forall t \in T, b \in B | b = 1 \quad (15)$$

### 3.7 Capacity Constraints at Tank

Inventory violations are computed based on maximum and minimum capacity limits by constraints 16 and 17. These violations must be minimized or nulled by model objective function optimization (OF).

$$STOCK_{t,b} - TLVIOMAX_{t,b} \leq MAXCAP_t \quad \forall t \in T, b \in B \quad (16)$$

$$STOCK_{t,b} + TLVIOMIN_{t,b} \geq MINCAP_t \quad \forall t \in T, b \in B \quad (17)$$

### 3.8 Mass Balance at Refinery Tank Farm

The sum of all received and sent volumes (STREAM) allows computing total product amounts at refinery tank farm during one month (TOTAL). The amount of intake product and of delivered product during one month must be respected, surplus factors  $\Psi$  and  $\Phi$  allows respecting volume ranges. This condition is modeled by constraints 18 and 19.

$$\sum_{t \in T} \sum_{b \in B} STREAM_{t,o,b} = TOTAL_o \quad \forall o \in O \quad (18)$$

$$\Psi * OUTSIDE_o \leq TOTAL_o \leq \Phi * OUTSIDE_o \quad \forall o \in O \quad (19)$$

### 3.9 Production Constraints and Demand Constraints

Constraints 20 to 22 consider streams come from production and streams delivered to demand. Constraint 20 imposes that if the product is moved, the volume of stream must not exceed the maximum capacity of the tank and must be greater than a minimum. Equation 21 ensures continuous carrying of products from production to demand, without interrupts. Equation 22 treats a specific production condition, the carrying of products starts only after the first day of the month.

$$(-MAXCAP_t - MINIMUM_o).(1 - MOVING_{t,o,b}) + MINIMUM_o \leq STREAM_{t,o,b} \leq MAXCAP_t.MOVING_{t,o,b} \quad \forall t \in T, b \in B, o \in O \mid o = 1,3 \quad (20)$$

$$TS_{o,b} = TF_{o,b-1} \quad \forall b \in B, |b| \geq 2, o \in O \mid o = 1,3 \quad (21)$$

$$TS_{o,b} \geq 24 \quad \forall b \in B, o \in O \mid o = 1 \quad (22)$$

### 3.10 Fragmentation Constraints

Constraints 23 to 25 consider product flows comes from P1 and product flows are moved through P2 and P3. Constraint 23 imposes that if the product is moved, the stream amount must be among the greater and the lower value allowed for each outer  $o$ . The fragmentations must occur at time slots that can be discontinued, therefore the constraint 24 allows a timing break. At each outer point of refinery (P1, P2 and P3) a specific number of batches for moving products (NumberSHOTS) occurs, this condition is modeled by constraint 25.

$$(-MAXIMUM_o - MINIMUM_o).(1 - MOVING_{t,o,b}) + MINIMUM_o \leq STREAM_{t,o,b} \leq MAXIMUM_o.MOVING_{t,o,b} \quad \forall t \in T, b \in B, o \in O \mid o = 2,4,5 \quad (23)$$

$$TS_{o,b} \geq TF_{o,b-1} \quad \forall b \in B, |b| \geq 2, o \in O \mid o = 2,4,5 \quad (24)$$

$$\sum_{t \in T} \sum_{b \in B} MOVING_{t,o,b} = NumberSHOTS_o \quad \forall o \in O \mid o = 2,4,5 \quad (25)$$

## 4. Results

Figure 3a to 3j show inventory evolution to all nine tanks during time horizon (0 to 720 hours). It is worthwhile to note that all tanks begin with an initial inventory, receive and deliver products during 720h without violate maximum or minimum tank capacity limits. A few tanks (3,4,6,9) have operated near limits during most of the time, however it is possible to note that the mass balance at refinery tank farm was executed without high overload on others tanks.

Idle time to product certification is respected by tanks at all figures. That is, after



receiving operation, tanks must wait 4 hours or more to send product. Figure 3f highlights the idle time to product certification on graphic of tank 5 (figure 3e), where the waiting time (4 hours) was accurately respected. All results validate the proposed model to inventory management at each tank.

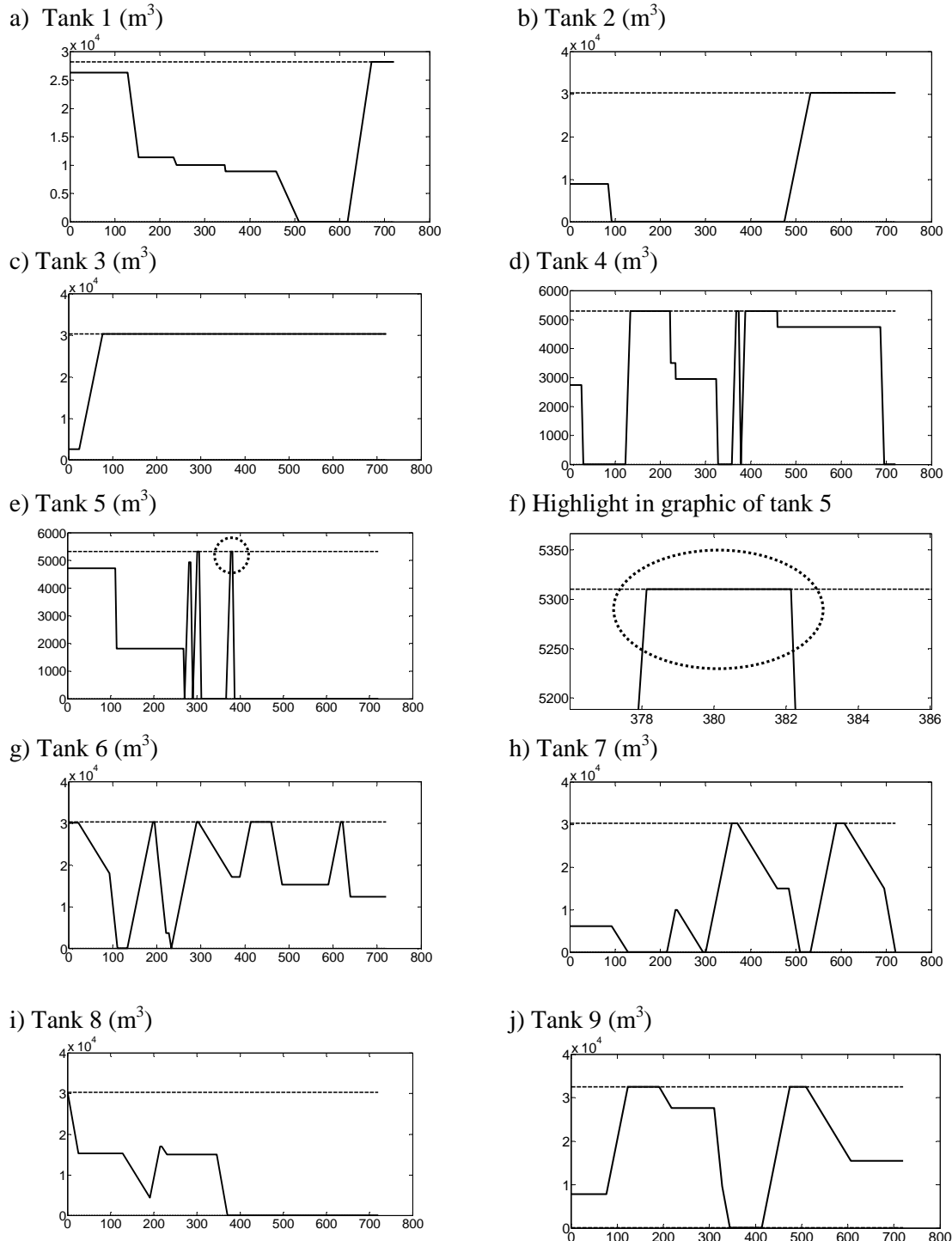


Figure 3. Inventories of all tanks

Figures 4a to 4d show respectively batches receiving from production and batches leaving to attain monthly demand (Table 1). Figures 4a and 4b show streams from production and streams forwarded to demand. Continuous flow batch was maintained: the refinery tank farm

receives from production and sends to demand without interruption. It should also be noted that the first production was only considered after the first day of programming horizon (criteria defined by scheduling of the external pipeline network).

For proposed mathematical model, flow rates are parameters, therefore these values must be respected. To prove this, flow rates presented in Figures 4c and 4d are computed conversely, from model variables values (TS, TF and STREAM). Flow rates were accurately respected, with decimal disagreements. Production and demand scheduling were finished before the end of the programming horizon, due to the use of surplus factors  $\Psi$  and  $\Phi$ . These factors allow respecting monthly volume of production and demand with flexibility. This volume flexibility affects time variables allowing programming horizon decrease.

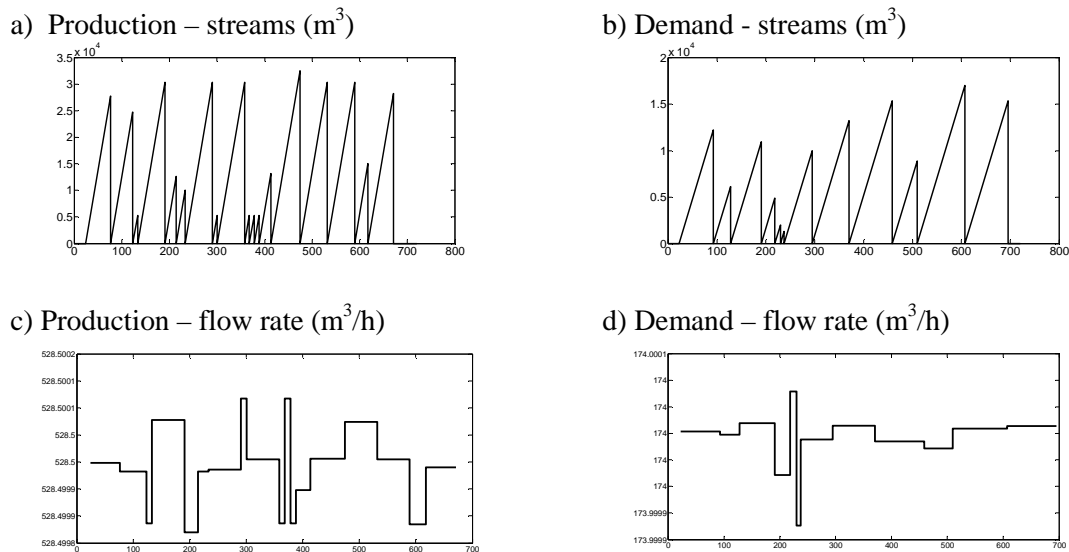


Figure 4. Continuous batches

Figures 5a to 5d show receiving and delivering operations. These figures are representatives of fragmentation batch occurrences. Pipelines have a constant flow rate, but at each pipe the amount of product forming a batch can vary. Fragmentation operations, beyond the monthly amount, must also respect upper and lower limits of each batch and the number of batches to be pumped per pipeline (NumberSHOTS).

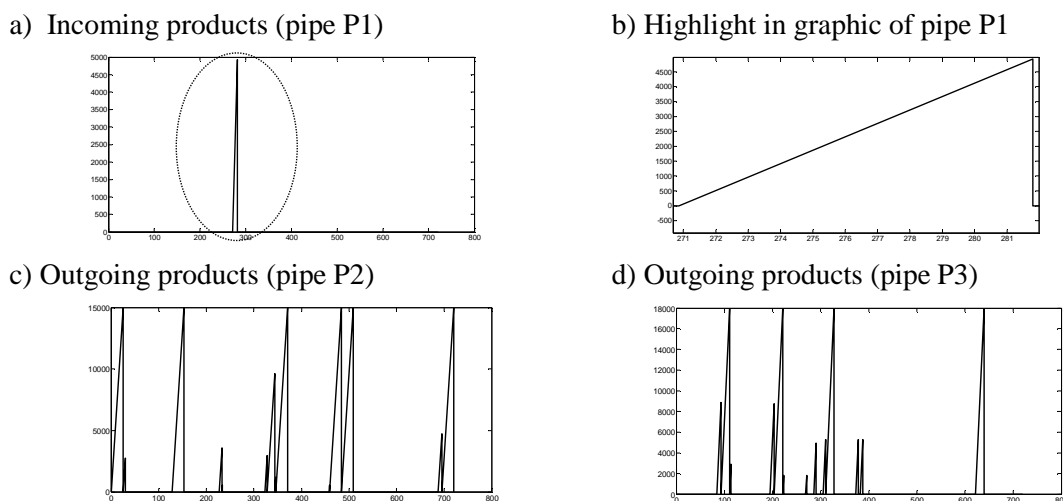


Figure 5. Fragmented batches

Figures 5a, 5c and 5d show that the model was able to schedule all batches within the required volume range. There is a single batch arriving by pipe P1, there are 14 batches delivered

by pipe P2 and 13 batches delivered by pipe P3 (according Table 3). All these batches occur according to respective flow rates: 450m<sup>3</sup>/h, 600 m<sup>3</sup>/h and 1000 m<sup>3</sup>/h. Figure 5b highlights the single batch arriving by pipe P1. We can see that this batch drives 4500 m<sup>3</sup> during about 10 hours, considering flow rate.

The developed MILP model to inventory management has 7705 variables being 2700 integer variables and 18455 constraints. This model is solved in the commercial solver Lingo 8.0, running at a personal computer with a 2,5GHz processor and 4GB ram memory. The model takes 1812976 iterations to attain global optimal point presenting computational time of 35min32seg.

## 5. Conclusions

This paper has presented a MILP model with time continuous representation to optimize refinery tank farm usage. The considered time horizon was one month (720h) and only one product has been managed. The studied tank farm was composed by 9 tanks and 3 pipelines and local production and demand. Presented results validate the model. The computed tank schedule takes account all restrictions: any tank capacity limits were violated, local demand was supplied, production flow was constantly received (without interruptions) and pipeline restrictions were respected by batch fragmentation operations. Time and volume specifications are attained by the computed scheduling.

With the use of time continuous representation is not possible to assure synchronism among movements of streams and tank operations. To circumvent this problem different timing variables were created for streams and for operations. These variables were linked per batch. Thus it was possible to compute the mass balance at the tanks in each batch and therefore became feasible to manage tank inventories individually.

Boschetto's model (2011) is a time continuous model that allocates fragmentations in fixed time windows, resulting in an external pipeline network scheduling, where the aggregated inventory at all tanks is considered. But when we looking for individually optimize tank usage, fixed time windows can generate infeasible solutions. To circumvent this problem, fragmentations are fixed into volume range and the total number of batches is also fixed, but they are free in the timeline. Thus the relaxed model can be able to find optimal global solution and also meets scenario requirements.

Surplus factors  $\Psi$  and  $\Phi$  were included in the model to meet the monthly volume range and make the model less strict. However, this flexibility has a direct impact on the horizon time programming of production and demand (continuous movement). Smaller range generates ideal solution but requires much computational time to find global optimum. A future improvement to this model considers exploring better these factors.

The computational time (35min32seg) was satisfactory for a problem that involves programming time horizon of 1 month. Other future work includes applying the proposed model in scenarios with two or more products.

## 6. Acknowledgements

The authors acknowledge financial support from ANP and FINEP (PRH-ANP / MCT PRH10 UTFPR), Brazilian Oil Company PETROBRAS (grants 6000.0062055.10.4 and 6000.0067933.11.4), CENPES-PETROBRAS (TC 0050.0077136.12.2) e CNPq (grants 304037/2010-9 e 311877/2009-5).

## Referências

- Banaszewski, R. F., Pereira, F. R., Tacla, C. A., Simão, J. M., Arruda, L. V. R., Ribas, P. C.** Simultaneous Auctions in Transport planning of Multiple Derivatives of Petroleum in Multi-modals Networks, *Brazilian Symposium of Operational Research – XLIISBPO*, Bento Gonçalves – RS, Brazil, 2010.
- Banaszewski, R. F., Nogueira, K. E., Simão, J. M., Arruda, L. V. R., Tacla, C.A.** Protocols of Simultaneous Auctions: Application to the Problem of Planning Movements of Oil, *Journal of Computation Theoretical and Applied - RITA*, n. 2, v. 18, 2011.
- Banaszewski, R. F., Nogueira, K. E., Arruda, L. V. R., Simão, J. M., Tacla, C.A., Relvas, S.,**

- Povoa, A. P. B.** Transportation planning of Oil Products: An Application of Multi-agents Auction-based Protocol with Improvements in the Bidding Strategy. In: *Simultaneous Auctions: Application to the Problem of Planning Movements of Oil*, *Symposium on Computer Aided Process Engineering*, London, 2012.
- Barboza, A. O.** Simulation and Techniques of Computation Evolutionary applied to Problems of Mixed Integer Linear Programming, *PhD Thesis*, Federal University of Technology – Parana, CPGEI, Brazil, 2005.
- Boschetto, S. N.** Optimization of Operations Transfer and Storage in a External Pipeline Network, *PhD Thesis*, Federal University of Technology – Parana, CPGEI. Brazil, 2011.
- Felizari, L. C.** Scheduling Transport Operations of Oil in Pipeline Networks, *PhD Thesis*, Federal University of Technology – Parana, CPGEI, Brazil, 2009.
- Glismann, K., Gruhn, G.** Short-term Scheduling and Recipe Optimization of Blending Processes, *Computers and Chemical Engineering*, v. 25, p. 627-634, 2001.
- Jia, Z., Ierapetritou, M.** Mixed-Integer Linear Programming Model for Gasoline Blending and Distribution Scheduling. *Industrial & Engineering Chemical Research*, v. 42, p. 825-835, 2003.
- Li, J., Karimi, A.** Scheduling Gasoline Blending Operations from Recipe Determination to Shipping Using unit Slots. *Industrial & Engineering Chemical Research*, v. 50, p. 9156-9174, 2011.
- Liang, B., Vongheng, J., Dexian, H.** A Novel Two-Level Optimization framework Based on Constrained Ordinal Optimization and Evolutionary Algorithms for Scheduling of Multipipeline Crude Oil Blending. *Industrial & Engineering Chemical Research*, v. 51, p. 9078-9093, 2012.
- Magatao, L. Arruda, L. V. R. Neves-Jr, F.** A Mixed Integer Programming Approach for Scheduling Commodities in a Pipeline. *Computer & Chemical Engineering*, v. 28, p. 171-185, 2004.
- Magatao, L., Arruda, L. V. R., Neves-Jr, F.** A Hybrid Model (CLP-MILP) for Operations of Scheduling in Pipelines. *Operational Research*, n. 3, v. 28, p. 511 – 543, 2008.
- Magatao, L. Arruda, L. V. R. Neves-Jr, F.** A Combined CLP-MILP Approach for Scheduling Commodities in a Pipeline. *Journal of Scheduling, Springer Netherlands*, p. 1-31, 2010.
- Pinto, J. M., Joly, M., Moro, L. F. L.** Planning and Scheduling Models for Refinery Operations. *Computers and Chemical Engineering*, v. 24, p. 2259-2276, 2000.
- Singh, A., Forbes, J. F., Vermeer, P. J., Woo, S. S.** Model-based Real-time Optimization of Automotive Gasoline Blending Operations. *Journal of Process Control*, v. 10, p. 43-58, 2000.
- Stebel, S. L.** Optimization Techniques applied to Scheduling Problems of Storage Resources. *PhD Thesis*, Federal University of Technology – Parana, CPGEI. Brazil. 2006.
- Tong, K., Feng, Y., Rong, G.** Integrated Model for Refinery Production and Pipeline System Scheduling. . In: *Symposium on Computer Aided Process Engineering*. London. 2012.
- Zhang, J., Wen, Y., Xu, Q.** Simultaneous Optimization of Crude Oil Blending and Purchase Planning with Delivery Uncertainty Consideration. *Industrial & Engineering Chemical Research*, v. 51, p. 8453-8464, 2012.