SDP relaxation for a strategic pricing bilevel problem in electricity markets

Marcia Fampa
Instituto de Matemática-DCC and COPPE
Universidade Federal do Rio de Janeiro
C.P. 68530, 21945-970 Rio de Janeiro, Brasil
fampa@cos.ufrj.br

Wagner Pimentel
Centro Federal de Educação Tecnológica Celso Suckow da Fonseca
Unidade de Ensino Descentralizada
Estrada de Adrianpolis, 1.317, Santa Rita
26041-271 Nova Iguau, RJ, Brasil
wagner.pimentel@cefet-rj.br, pimentelwagner@cos.ufrj.br

Abstract

We consider a strategic bidding problem under uncertainty in a wholesale energy market, where the economic remuneration of each generator depends on the ability of its own management to submit price and quantity bids. We present a bilevel formulation for the problem and propose semidefinite programming (SDP) relaxations for it. The SDP relaxations are compared in order to measure the trade-off between their strength and the computational effort required to solve them. Numerical results are shown for case studies with configurations derived from the Brazilian system.

Keywords: SDP relaxation; bilevel problem; electricity pool market; strategic pricing.

1 Introduction

In the strategic pricing problem in electricity markets, generators compete for contracts for power sales to distribution companies. They make their price offers for energy production and then are loaded in order of increasing unit price until demand is met. All generators dispatched receive the most expensive unit price charged among them, which corresponds to the marginal cost of short-term or spot price of the system (Hunt(2003)).

The problem of determining the optimal price bids for a given company that owns one or more generators is a non-convex problem that may be modeled as a bilevel program, where the leader represents the company that aims to maximize its expected profit, while the follower represents the system operator, that aims to minimize the total cost of the energy production.

Mixed integer linear programming (MILP) reformulations for this strategic bidding problem were proposed in Fampa (2008) and Pereira (2005). The MILP formulation presented in Fampa (2008) was also applied in Fampa (2012) to obtain the optimal solutions of some instances derived from the Brazilian power system and then compare them to the solutions obtained by a genetic algorithm proposed in the paper.

In this paper, we consider the reformulation of the strategic pricing bilevel problem in electricity markets as a non-convex quadratically constrained quadratic program and investigate the application of semidefinite relaxations to obtain bounds for the problem. Semidefinite programming (SDP) relaxations of non-convex quadratically constrained quadratic programs (QCQPs) have been studied by a number of researchers, initially inspired by the seminal works of Lovász (1979), Lovász (1991) and Goemans (1995). The research in this field is still very active as shown, for example, on the recent works of Anstreicher (2009), Burer (2008), Fampa (2013), Rendl (2010), Saxena (2010), Saxena (2011) and on the survey paper of Bao (2011).

Although semidefinite relaxations have been very effective in generating strong bounds for QCQPs, it is well known that the required computation effort to solve the relaxations may be considerable, especially when the size of the relaxation becomes too big due to the inclusion of valid inequalities. The challenge is, therefore, to achieve a good trade-off between the size of the semidefinite programs and quality of the bounds obtained. Our goal with the research conducted in this paper is to verify the effect of the addition of valid inequalities to a basic SDP relaxation of the strategic pricing problem in order to achieve the best possible trade-off.

This paper is organized as follows: Section 2 presents the mathematical formulations of the strategic bidding problem under uncertainty as a bilevel program and as a QCQP. Section 3 presents the general QCQP and discuss semidefinite relaxations for the problem, starting from a basic and weaker relaxation and then proposing valid inequalities to strengthen it. Section 4 presents the numerical results comparing the different SDP relaxations for the strategic bidding problem. Section 5 concludes the paper.

Notation

In this paper, R^n refers to the n-dimensional Euclidean space, $e_i \in R^n$ represents the i-th unit vector, S^n_+ is the set of $n \times n$ positive semidefinite symmetric matrices, R^{1+n} and S^{1+n}_+ is used to denote the spaces R^n and S^n_+ with an additional 0-th entry or additional 0-th row and column prefixed. Given two symmetric $n \times n$ matrices X, Y, we let $X \bullet Y = \mathrm{trace}(X^T Y) = \sum_{i,j=1}^n X_{ij} Y_{ij}$ and we use $X \succeq 0$ to denote that the matrix X is positive semidefinite.

2 Strategic Pricing in Electricity Markets

In deregulated electricity markets, generators submit a set of hourly generation prices and available capacities for the following day. Based on these data and on an hourly load forecast, the system operator carries out the following economic dispatch at each time step (Fampa (2008)):

$$
\begin{array}{llll}
 & & \text{dual variable} & \\
\text{Minimize}_{g_j} & \sum_{j\in J} \lambda_j g_j, & & \\
\text{subject to} & \sum_{j\in J} g_j = d, & \pi_d & \\
 & g_j \leq \bar{g}_j, & \pi_{g_j} & j \in J, \\
 & g_j \geq 0, & & j \in J,
\end{array}
\tag{2.1}
$$

where the input data d, λ_j and $\bar{g}_j$ represent, respectively, load (MWh), price bid ($\$/MWh$) and generation capacity bid (MWh) of generator j and the variable g_j represents the energy production of generator j (MWh). The optimal value of the dual variable π_d is considered as the system spot price. The profit of each generator $j \in J$, in each time step, corresponds to $(\pi_d - c_j)g_j$, where c_j represents its unit operating cost. Note that c_j may be different from λ_j, its price bid.

The net profit of a generation company E, which may be a utility or an independent power producer that owns several different generation units, is given by:

$$\sum_{j\in E} (\pi_d - c_j) g_j,$$

where E is also used to denote the set of indexes associated to the plants belonging to the company E ($E \subset J$).

In the optimal price bidding problem, company E aims to determine a set of price bids $\lambda_E = \{\lambda_j, j \in E\}$ that maximize its total net profit, considering the quantity bid of each generator of the company fixed as its maximum generation capacity, denoted by $\bar{g}_j^*$.

The complexity of this problem is increased by the fact that the calculation of π_d and g_j in the dispatch problem (2.1) depends on the knowledge of price vectors for all companies, as well as their generation availability and system load values. However, this information is not available to any single company at the time of its bid. Therefore, the bidding strategy has to take into account the uncertainty around these values. An approach used to deal with the uncertainty on the data of the problem is to define a set of scenarios for the remaining agent's behavior and maximize the profit of the company over all scenarios, in a classical strategic bidding under uncertainty problem. In this case, the bids from generators not belonging to company E and the load are considered uncertain, and represented by a set of scenarios indexed by s, which occur with exogenous probabilities $\{p_s$, s=1,...,S$\}$. The bilevel formulation for the problem is given by

$$
\begin{array}{lllll}
\text{Maximize}_{\lambda_E} & \multicolumn{4}{l}{\sum_{s\in S} p_s \sum_{j\in E} [\pi_d^s - c_j] g_j^s,} \\
\text{subject to} & & & & \\
 & \text{Minimize}_{g_j^s} & \sum_{s\in S}\sum_{j\in E} \lambda_j g_j^s + \sum_{j\notin E} \lambda_j^{*s} g_j^s, & & \\
 & \text{subject to} & \sum_{j\in J} g_j^s = d^s, & & s \in S, \\
 & & 0 \leq g_j^s \leq \bar{g}_j^*, & j \in E, & s \in S, \\
 & & 0 \leq g_j^s \leq \bar{g}_j^{*s}, & j \notin E, & s \in S.
\end{array}
\tag{2.2}
$$

The first level of problem (2.2) represents the interest of company E, maximize expected profits), while the second level represents the interest of the system operator (minimize operational costs). The company is classified as leader of the bilevel program and controls the variables λ_j, for $j \in E$, while the system operator is classified as follower and controls the variables g_j^s for $j \in J$, $s \in S$.

Finally replacing the follower linear program by its optimality conditions we derive the following non-convex quadratically constrained quadratic program, with a bilinear objective function and one bilinear constraint.

$$
\begin{array}{lll}
\text{Maximize}_{\lambda_j, g_j^s, \pi_d^s, \pi_{g_j}^s} & \sum_{s \in S} p_s \sum_{j \in E} [\pi_d^s - c_j] g_j^s & \\
\text{subject to} & & \\
\sum_{j \in J} g_j^s = d^s, & & s \in S, \\
0 \leq g_j^s \leq \bar{g}_j^*, & j \in E, & s \in S, \\
0 \leq g_j^s \leq \bar{g}_j^{*s}, & j \notin E, & s \in S, \\
\pi_d^s - \pi_{g_j}^s - \lambda_j \leq 0, & j \in E, & s \in S, \\
\pi_d^s - \pi_{g_j}^s \leq \lambda_j^{*s}, & j \notin E, & s \in S, \\
\pi_{g_j}^s \geq 0, & j \in J, & s \in S, \\
\multicolumn{3}{l}{\sum_{s \in S} (\sum_{j \in E} \lambda_j g_j^s + \sum_{j \notin E} \lambda_j^{*s} g_j^s - d^s \pi_d^s + \sum_{j \in E} \bar{g}_j^* \pi_{g_j}^s + \sum_{j \notin E} \bar{g}_j^{*s} \pi_{g_j}^s) = 0.}
\end{array}
\tag{2.3}
$$

3 SDP relaxations of quadratically constrained quadratic programs

A general non-convex Quadratically Constrained Quadratic Program (QCQP) may be formulated as:

$$
(\text{QCQP}) \left\{
\begin{array}{lll}
\text{maximize} & x^T Q_0 x + 2 q_0^T x + r_0 & \\
\text{subject to} & x^T Q_j x + 2 q_j^T x + r_j \leq 0, & j = 1, \ldots, m_q \\
& p_j^T x = v_j, & j = 1 \ldots, m_{l_e} \\
& b_j \leq a_j^T x \leq c_j, & j = 1 \ldots, m_{l_1} \\
& \alpha_j^T x \leq \gamma_j, & j = 1 \ldots, m_{l_2} \\
& \beta_j \leq \delta_j^T x, & j = 1 \ldots, m_{l_3}
\end{array}
\right.
$$

where $Q_j \in S^n$, $q_j \in R^n$, $r_j \in R$, for $j = 0, \ldots, m_q$, $p_j \in R^n$, $v_j \in R$, for $j = 1, \ldots, m_{l_e}$, $a_j \in R^n$, $b_j, c_j \in R$, for $j = 1, \ldots, m_{l_1}$, $\alpha_j \in R^n$, $\gamma_j \in R$, for $j = 1, \ldots, m_{l_2}$, $\delta_j \in R^n$, $\beta_j \in R$, for $j = 1, \ldots, m_{l_3}$.

A standard approach to derive a convex relaxation of QCQP is to introduce the variable $Y \in S_+^{1+n}$ in the formulation, obtaining the following lifted reformulation of the problem.

$$
(\text{QCQP'}) \left\{
\begin{array}{lll}
\text{maximize} & S_0 \bullet Y & \\
\text{subject to} & S_j \bullet Y \leq 0, & j = 1, \ldots, m_q \\
& p_j^T x = v_j, & j = 1 \ldots, m_{l_e} \\
& b_j \leq a_j^T x \leq c_j, & j = 1, \ldots, m_{l_1} \\
& \alpha_j^T x \leq \gamma_j, & j = 1 \ldots, m_{l_2} \\
& \beta_j \leq \delta_j^T x, & j = 1 \ldots, m_{l_3} \\
& Y = \begin{pmatrix} 1 & x^T \\ x & x x^T \end{pmatrix}, &
\end{array}
\right.
$$

where $S_j = \begin{pmatrix} r_j & q_j^T \\ q_j & Q_j \end{pmatrix}$, $j = 0, \ldots, m_q$.

The only non-convex constraint in QCQP' is the last one, which imposes Y to be a positive semidefinite rank-1 matrix with $Y_{00} = 1$. A convex relaxation of QCQP is then given by the following SDP problem obtained by relaxing the rank-1 constraint.

$$
(\text{SDP}) \begin{cases} \text{maximize} & S_0 \bullet Y \\ \text{subject to} & S_j \bullet Y \leq 0, \qquad j = 1, \ldots, m_q \\ & p_j^T x = v_j, \qquad j = 1 \ldots, m_{l_e} \\ & b_j \leq a_j^T x \leq c_j, \qquad j = 1, \ldots, m_{l_1} \\ & \alpha_j^T x \leq \gamma_j, \qquad j = 1 \ldots, m_{l_2} \\ & \beta_j \leq \delta_j^T x, \qquad j = 1 \ldots, m_{l_3} \\ & S_{m_q+1} \bullet Y = 1, \\ & Y \succeq 0, \end{cases}
$$

where $S_{m_q+1} = e_0 e_0^T$ and $e_0 \in R^{1+n}$.

Let's consider now the following relaxation of SDP, which is therefore a relaxation of QCQP.

$$
(\text{SDP}_I) \begin{cases} \text{maximize} & S_0 \bullet Y \\ \text{subject to} & S_j \bullet Y \leq 0, \qquad j = 1, \ldots, m_q \\ & S_{m_q+1} \bullet Y = 1, \\ & Y \succeq 0. \end{cases}
$$

Our goal in this work is to add different sets of valid inequalities to this initial SDP relaxation and analyze which ones lead to an improvement on the bounds given by the relaxation that compensates the increase on the computational effort required to solve it. In the remainder of this section, we present the valid inequalities used in our computational experiments.

Bounding SDP$_I$

In order to guarantee the boundedness of SDP$_I$, we impose the following upper bounds to the diagonal of Y

$$Y_{ii} \leq \max\{l_i^2, u_i^2\}, \ i = 1, \ldots, n, \tag{3.1}$$

where l_i and u_i are respectively, lower and upper bounds for x_i. We also include in SDP$_I$ the nonnegative constraints on x_i, given by

$$Y_{0j} \geq 0, \ j = 1, \ldots, n, \tag{3.2}$$

Adding the linear constraints

Using the idea introduced in Sherali (1995) we multiply the linear constraints among each other and also by each variable of the problem generating valid quadratic constraints to strengthen the identity between Y_{ij} and $x_i x_j$ for $i, j = 1, \ldots, n$.

Considering the first type of linear inequality constraints in QCQP, given by

$$b \leq a^T x \leq c, \tag{3.3}$$

we derive the valid convex quadratic inequality

$$(a^T x - b)(a^T x - c) \leq 0 \Leftrightarrow x^T a a^T x - (b + c) a^T x + bc \leq 0. \tag{3.4}$$

Considering now the second type of inequality constraints:

$$\alpha^T x \leq \gamma \tag{3.5}$$

we derive the valid quadratic inequalities

$$(\alpha^T x - \gamma) x_i \leq 0 \quad \forall x_i \geq 0. \tag{3.6}$$

For the third type of linear inequality constraint

$$\delta^T x \geq \beta, \tag{3.7}$$

we derive the valid quadratic inequalities

$$(\delta^T x - \beta)x_i \geq 0 \quad \forall x_i \geq 0. \tag{3.8}$$

Finally, for the equalities constraints

$$p^T x = v \tag{3.9}$$

we derive the valid quadratic inequalities

$$(p^T x - v)x_i = 0,$$

for each variable x_i in the problem and include in the relaxation

$$p^T x = v, \quad \text{and} \quad (p^T x - v)x_i = 0, \forall i = 1, \ldots, n. \tag{3.10}$$

Adding RLT inequalities

To strengthen the SDP relaxation, we also consider the well known Reformulation-Linearization Technique (RLT), McCormick (1976), Sherali (1999) and Sherali (1995). Specifically, the following valid bilinear inequalities

$$\begin{array}{r} (x_i - u_i)(x_j - l_j) \leq 0 \\ (x_i - l_i)(x_j - u_j) \leq 0 \\ (x_i - l_i)(x_j - l_j) \geq 0 \\ (u_i - x_i)(u_j - x_j) \geq 0 \end{array}$$

generate the well known RLT inequalities, given by

$$\begin{array}{r} Y_{ij} - l_j x_i - u_i x_j + l_j u_i \leq 0 \\ Y_{ij} - l_i x_j - u_j x_i + l_i u_j \leq 0 \\ Y_{ij} - l_j x_i - l_i x_j + l_i l_j \geq 0 \\ Y_{ij} - u_j x_i - u_i x_j + u_i u_j \geq 0. \end{array} \tag{3.11}$$

Naturally, the bound constraints (3.1) are contained in the set of RLT inequalities.

We note that all valid quadratic inequalities are introduced in the SDP relaxation as a linear constraint on Y, given by $S \bullet Y \leq 0$, where $S = \begin{pmatrix} r & q^T \\ q & Q \end{pmatrix}$, with properly chosen vectors q and r and submatrix Q. For the convex quadratic inequalities (3.4), for example, we have $S = \begin{pmatrix} bc & -\frac{1}{2}(b+c)a^T \\ -\frac{1}{2}(b+c)a & aa^T \end{pmatrix}$. Furthermore, we note in this case, that since $Q = aa^T$ is positive semidefinite, the projected solution x contained in the first row and column of the semidefinite relaxation solution Y, necessarily satisfies the quadratic constraint (3.4). Indeed, considering X as the submatrix of Y obtained when we eliminate the first row and column of Y, i.e.,

$$Y = \begin{pmatrix} 1 & x^T \\ x & X \end{pmatrix}, \tag{3.12}$$

then, since $Y \succeq 0$, we have that $X - xx^T \succeq 0$. Therefore, $Q \succeq 0$ implies that $Q \bullet (X - xx^T) \geq 0$ and $x^T Q x + 2q^T x + r \leq Q \bullet X + 2q^T x + r = S \bullet Y \leq 0$.

4 Numerical Results

Considering the QCQP formulation (2.3) of the strategic pricing problem in energy markets, we have applied to it the SDP relaxations discussed in Section 3. The first relaxation considered, denoted in the remainder of this section by SDP_{I_1} is the weaker one, corresponding to the initial relaxation SDP_I with only the bounding constraints (3.1, 3.2) included. Then we add each set of valid inequalities to the relaxation, one by one, in the following order. We add the original linear constraints of the problem (3.3, 3.5, 3.7, 3.9) generating the relaxation SDP_{I_2}. Then we add the RLT inequalities (3.11), generating the relaxation SDP_{I_3}. Finally, we add the quadratic constraints obtained by the product of the original linear constraints by the non-negative variables generating SDP_{I_4}. We note that when adding the RLT constraints (3.11) to the relaxation, we disconsider the diagonal bounds (3.1) already included. Furthermore, we also consider the MILP formulation of the problem, presented in Fampa (2008), to obtain the optimal solution of the instances considered, and compute the relative gap between the optimal solution of the MILP problem and the upper bounds obtained with the solution of the SDP relaxations.

Our main goal with these numerical experiments is to analyze the impact of each set on valid inequalities on the quality of the bounds and also on the computation time required to solve the SDP relaxations.

We present preliminary computational results considering some small instances of the strategic bidding problem with configurations derived from the Brazilian power system.

Our code was implemented in C and compiled with gcc (GNU COMPILE C). All runs were conducted on a 8GB Ram, 1.9GHz Intel Core processor running under Windows, Version: 8. The solution of the SDP relaxations was obtained with the solver CSDP 6.1.1 (Borchers (1999)). The solver CPLEX, v12.2 (Gay (2009)) was used to obtain the optimal solution of the instances, considering the MILP formulation mentioned above.

Our set of test problems include instances with configurations derived from the south subsystem of the Brazilian power system, which contains a total of 28 plants that account for 16% of the national installed capacity. The south subsystem has an installed capacity of 11 GW. Hydro generation accounts for 68% of the installed capacity with 19 hydro plants. Thermal generation accounts for 32% of the installed capacity with 9 plants.

The input for the test problems is related to the year of 2008 and is available at the website of the Brazilian Electric System National Operator (Operador Nacional do Sistema - ONS) (http://www.ons.org.br). We consider the data for the last week of each month of 2008 given at the weekly report of operation (Boletim Semanal de Operação - BMO) and take into account only the 10 dispatched generators, located in the state Rio Grande do Sul, that have in fact contributed to attend the demand of the south of Brazil on the given period. The generation capacity of each generator $j \in J$ is considered as its maximum sampled generation on the period. Let's denote it by G_j.

In all test problems we consider CEEE (Companhia Estatual de Energia do Rio Grande do Sul) as the bidding agent. CEEE controls 4 hydroelectric plants totaling 596 MW of installed capacity which corresponds to 20% of the capacity of the south subsystem. Table 1 shows the plant names, capacities (in MW) and operational costs in R$/MWh, where R$ stands for real, the Brazilian currency. The operational cost of each plant $j \in E$ controlled by CEEE is denoted in the following by C_j and is based on the information available at the website of the Brazilian electricity regulatory agency, ANEEL (http://www.aneel.gov.br).

To generate 11 instances of different sizes we consider either the 2 first generators in Table 1 or the 3 first or all 4 of them, generating instances with $|E| = 2$, $|E| = 3$, or $|E| = 4$ and $|J| = 8$, $|J| = 9$, or $|J| = 10$, respectively. We also consider different number of scenarios varying from 2 to 4. For all instances, the generation capacities $\bar{g}_j^*, j \in E$ and

Table 1: CEEE Generation System

Plant $j \in E$	Capacity G_j	Cost C_j
Itaúba	344	126
Dona Francisca	124	108
Passo Real	99	111
Canastra	29	122

Table 2: Relative gap between SDP bounds and optimal solutions

$\text{Inst}_{\lvert J\rvert,\lvert E\rvert,\lvert S\rvert}$	SDP_{I_1}	SDP_{I_2}	SDP_{I_3}	SDP_{I_4}
$\text{Inst1}_{08,02,02}$	1156.78%	1151.18%	450.38%	13.56%
$\text{Inst2}_{08,02,03}$	923.55%	918.58%	386.70%	0.33%
$\text{Inst3}_{08,02,03}$	847.05%	843.56%	351.72%	0.07%
$\text{Inst4}_{08,02,03}$	824.61%	821.09%	315.22%	0.00%
$\text{Inst5}_{08,02,03}$	682.16%	674.38%	275.21%	9.12%
$\text{Inst6}_{08,02,03}$	829.54%	822.46%	351.90%	21.27%
$\text{Inst7}_{08,02,04}$	1405.89%	1400.29%	588.51%	10.43%
$\text{Inst8}_{09,03,02}$	1014.47%	1010.47%	428.90%	158.31%
$\text{Inst9}_{09,03,04}$	898.64%	898.32%	346.48%	85.37%
$\text{Inst10}_{10,04,02}$	447.65%	443.33%	175.90%	105.57%
$\text{Inst11}_{10,04,04}$	1099.32%	1096.77%	476.05%	297.88%
Mean	920.88%	916.40%	377.00%	63.81%

$\bar{g}_j^{*s}, j \in J\setminus E, s \in S$, are randomly selected in the range $[0.9G_j, G_j]$ and the operational costs $c_j, j \in E$ are randomly selected in the range $[0.9C_j, 1.1C_j]$. The price bids $\lambda_j^{*s}, j \in J\setminus E, s \in S$ are randomly selected in $[1.1C_j, 1.5C_j]$ and the demands $d^s, \forall s \in S$ are randomly selected in $[0.8\bar{G}^s, \bar{G}^s]$, where $\bar{G}^s$ stands for the sum of the generation capacities of all competitor generators in $J\setminus E$ in scenario $s \in S$. Note that this selection guarantees that the problems are always bounded, since the competitors can always satisfy the demand with no plant controlled by CEEE being dispatched. Uniform distribution was used in all raffles.

Table 2 presents the relative gap between the solution of each SDP relaxation and the optimal solution of the instances, given by $(z(\text{RELAX}) - z^*)/z^* \times 100$, where $z(\text{RELAX})$ denotes the solution of a given relaxation RELAX and z^* denotes the optimal solution of the problem. Table 3 presents the CPU time (in seconds) to solve each SDP relaxation.

The results on Table 2 show that the two first SDP relaxations are very weak, which is expected since there are no constraints strengthening the relation between the matrix X and xx^T, as defined in (3.12). These constraints are added in the two last relaxations. The results for SDP_{I_3} already show how effective the RLT constraints are in strengthening the relaxation and the results for SDP_{I_4}, show that considering quadratic constraints derived from the linear constraints of the problem can also be very effective to decrease the gaps. On average, the gap decreased 539.40% from SDP_{I_2} to SDP_{I_3} and 313.19% more from SDP_{I_3} to SDP_{I_4}. Also as excepted, the inclusion of the valid inequalities to the SDP formulation makes it much harder to solve. The computation time increases a lot from SDP_{I_1} and SDP_{I_2} to SDP_{I_3} and SDP_{I_4}, indicating that we should be cautious when choosing the constraints to add to the basic SDP relaxation. As future research work, we plan to study the solution of a separation problem to add the valid inequalities to the relaxation, in order to avoid the

Table 3: Time (in seconds) to solve the SDP relaxations

$\text{Inst}_{\lvert J\rvert,\lvert E\rvert,\lvert S\rvert}$	SDP_{I_1}	SDP_{I_2}	SDP_{I_3}	SDP_{I_4}
$\text{Inst1}_{08,02,02}$	0.69	0.83	168.91	565.11
$\text{Inst2}_{08,02,03}$	0.95	1.00	458.83	1875.48
$\text{Inst3}_{08,02,03}$	0.56	0.64	433.11	1990.59
$\text{Inst4}_{08,02,03}$	0.58	0.66	447.75	1936.59
$\text{Inst5}_{08,02,03}$	0.50	1.11	490.39	2502.92
$\text{Inst6}_{08,02,03}$	0.66	0.66	497.03	2266.25
$\text{Inst7}_{08,02,04}$	1.22	2.27	1024.75	5832.25
$\text{Inst8}_{09,03,02}$	0.84	1.13	266.34	926.13
$\text{Inst9}_{09,03,04}$	2.52	2.16	2339.53	7489.00
$\text{Inst10}_{10,04,02}$	1.11	1.38	517.16	1862.88
$\text{Inst11}_{10,04,04}$	2.97	3.02	5189.78	16678.95

big dimension of the SDP problems considered when we add all possible RLT constraints and quadratic constraints generated when we multiply the linear constraints of the problem.

5 Conclusion

In this paper we present the mathematical formulations of the strategic bidding problem under uncertainty as a bilevel program and as a non-convex quadratically constrained quadratic program. We discuss the application of semidefinite programming relaxations to compute bounds to the strategic bidding problem, considering four different relaxations with different strength levels, where we obtain stronger relaxations with the addition of valid inequalities to the weaker ones. As expected from the results presented in the literature for other applications of SDP relaxations, we conclude that we can obtain very tight bounds using strong SDP relaxations of the strategic bidding problem. However, the computational effort to solve the stronger relaxations is quite big. The study indicates that it is important to choose wisely the constraints to be added to the SDP formulation in order to get a good trade-off between bound quality and computational effort. A future research topic would be to investigate the solution of a separation problem to generate the best possible choices of valid inequalities to be added to an initial basic SDP relaxation.

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