CLUSTERING SEARCH APPLIED TO THE PERIODIC VEHICLE ROUTING PROBLEM: A CASE STUDY IN WASTE COLLECTION

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ABSTRACT

The class of vehicle routing problems (VRP) involves designing the optimal set of routes for fleets of vehicles for the purpose of serving a given set of customers. A variant of the VRP is the Periodic Vehicle Routing Problem (PVRP) that consists in generating a set of routes for each day with minimal global costs and respecting all constraints of the problem. This paper presents a model for the PVRP applied to solid waste collection. To solve this problem, it is proposed an hybrid method called Clustering Search, that aims to combine metaheuristics and local search in which the search is intensified only in areas of the search space that deserve special attention. In the computational tests, a real-world case from Ponte de Lima, a Portuguese municipality is analyzed, and also some classical instances from the literature.

KEYWORDS. Periodic vehicle routing problem, solid waste collection, Clustering Search.

MAIN AREA. Metaheuristics.

1. Introduction

The rate of population growth leads to real problems in the urban environment, affecting the quality of life for millions of people around the world. One problem arising from population growth is the accelerated production of municipal solid waste, especially in recent decades.

The destiny of municipal solid waste is a problem whose solution becomes increasingly difficult. In New York, the city that produces more garbage in the world, the daily average amount of waste produced is 13 tons (Coelho and Soares, 2001). It is natural then to ask yourself how to manage as efficiently as possible, in a city like New York, 1.5 kilograms of waste per capita per day.

A bad management of this waste can compromise the environment, the health of population in the surrounding areas, in addition to significant costs. According to Cunha and Caixeta (2002) there are three crucial factors for the study of the management of this waste:

- The large amount of waste generated, as the example of New York, the Brazilian rate per capita is around 0.5 to 1 kg / capita / day;
- Environmental impacts and population in the surroundings where waste is deposited, mainly caused by manure, which is a liquid of strong odor and high contamination potential resulting from the process of putrefaction of waste that can contaminate the soil, rivers and groundwater;
- High financial costs of managing city solid waste. In Brazil, the cleaning services correspond on average by 7% to 15% of the budget of the cities, and that approximately 50% of total cost management is due to the collection and transportation.

This paper focus on part of the waste administration problem, which is collected and transported. The problem can be viewed as a periodic vehicle routing problem for waste collection.

In Vehicle Routing Problem (VRP) (Christofides *et al*., 1979), which has many practical applications and it is one of the most important problems in optimization, we have a fleet of vehicles with known capacity, we should visit customers who also have a known demand. The objective is to minimize the sum of travel time, and the total demand for each route not exceed the capacity of the vehicle which servers that route (Toth and Vigo, 2002).

The Periodic Vehicle Routing Problem (PVRP) (Christofides and Beasley, 1984) is a generalization of VRP by extending the planning period to *m* days. The objective is to minimize the sum of travel time need to supply all customers. In this paper, according to Bianchi-Aguiar *et al*. (2012), some features are added to PVRP for solving the waste collection problem: the fleet of vehicles is heterogeneous, *i.e*. the capacity of each vehicle and/or costs are distinct, there are two locations to be used as a disposal facilities; daily demand of customers can be variable, each daily route of a vehicle is limited due to its distance that does not exceed the maximum working time of the driver.

The PVRP belongs to the class of NP-hard problems (Golden *et al*. 1995). Thus, heuristic methods are the most used technical proposals for finding good solutions to the PVRP. Among the solution methods, we can include Genetic Algorithms (Holland, 1975), Greedy Randomized Adaptive Search Procedure (GRASP) (Feo and Resende, 1995) and Tabu Search (Glover, 1986).

In the literature, Carvalho *et al*. (2003) proposed a method to solve the VRP that creates an initial solution and applies local search heuristics. Gonçalves *et al*. (2005) presented a routing problem of units mobile recovery of oil modeled as PVRP, which differs from the classical problem mainly by not having a prior definition of the number on requests of each customer. The authors also used heuristics to achieve good solutions. Higino *et al*. (2012) presented a method to find good solutions to the problem of solid waste collection, applying the Simulated Annealing (SA). This method is also used in this paper as a component of the proposed method, and it is responsible for generating solutions to the clustering process.

In this paper, it is proposed the Clustering Search (CS) method to find good solutions to the problem of solid waste collection, which is modeled as the VPRP. The CS is a hybrid method that aims to combine metaheuristics and local search in which the search is intensified only in areas of the search space that deserve special attention. To evaluate the efficiency of the CS, it is used a real case test proposed by Bianchi-Aguiar *et al*. (2012) and classical instances for the VPRP.

This paper is organized as follows. Section 2 presents a mathematical formulation for the waste collection problem. Section 3 describes the basic ideas and conceptual components of CS. Section 4 describes in details, the proposed heuristic procedure for solving the problem. Computational results obtained by applying the proposed method are presented and discussed in section 5. At last, the findings and conclusions are summarized in section 6.

2. Mathematical formulation

This paper approaches a real case of the waste collection problem proposed by Bianchi-Aguiar *et al.* (2012), that studied the case of Ponte de Lima, a municipality located in Portugal. Ponte de Lima owns and operates five vehicles with different capacities, there are 994 waste containers distributed by the municipality and the collection is performed 6 days a week. The main objective is to find routes for each vehicle with different frequencies in the collection and distribution containers on weekdays in order to perform such task with minimal costs. The containers were distributed in 51 customers and for each customer there is a frequency of visits during a week. For each visiting frequency, a list of possible schedules was generated, trying to balance the time between consecutive visits. For example, for a frequency of two visits a week, the following schedules were considered: {Monday, Thursday}, {Tuesday, Friday}, {Wednesday, Saturday}, {Monday, Friday} and {Tuesday, Saturday} (Bianchi-Aguiar *et al.*, 2012).

Bianchi-Aguiar *et al.* (2012) presents a mathematical formulation for the waste collection problem, modeling it as a PVRP. Table 1 presents the indices, parameters and sets of the model. It is assumed that y_{ir} is the variable that performs the assignment of customer i to schedule r (getting value 1 if customer i is visited by schedule r) and x_{ijkl} is the variable that indicates if the vehicle k visits customer j immediately after visiting customer i on day l (getting value 1 if the arc ij is on the route). The complete formulation is presented by equations (2.1) to (2.10).

Table 1 – Notation

$i; j; h$	Customer
k	Vehicle
l	Day
r	Schedule
L	Planning horizon (days)
N	Number of customers
P	Number of disposal facilities
K_l	Number of vehicles available on day l
d_{ij}	Distance between customers i and j (km)
q_i	Total waste to be collected in customer i (kg)
s_i	Length of service in the customer i (minutes)
t_{ij}	Length of route i to j (minutes)
Q_k	Capacity of vehicle k (kg)
T_l	Maximum duration of a route on day l (minutes)
C_i	Number of possible schedules for customer i
a_{rl}	Constant that indicates 1 if the day l belongs to schedule r
$\mathcal{L}$	Days of the planning horizon, $\mathcal{L} = \{1; \dots; L\}$
$\mho$	Locations, $\mho = \{v_0; v_1; \dots; v_{N+P}\}$, where v_0 corresponds to garage
$\mho_c$	Customers, $\mho_c = \{v_1; v_2; \dots; v_N\}$
$\mho_p$	Discharge locations (warehouses) $\lvert\mho_p\rvert = P$, $\mho_p = \{v_{N+1}; v_{N+2}; \dots; v_{N+P}\}$
K_l	Vehicles available on day l, $\lvert\mathrm{K}_l\rvert = K_l$
C_i	Set of possible schedules for customer i, $\lvert\mathrm{C}_i\rvert = c_i$

$$minimize \sum_{i\in\mho}\sum_{j\in\mho,j\neq i}\sum_{l\in\mathcal{L}}\sum_{k\in K_l} d_{ij}x_{ijkl} \tag{2.1}$$

Subject to:

$$\sum_{r\in C_i} y_{ir} = 1 \; ; \; i \in \mho_c \tag{2.2}$$

$$\sum_{j\in\mho,j\neq i}\sum_{k\in K_l} x_{ijkl} - \sum_{r\in C_i} a_{rl}y_{ir} = 0 \; ; \; i \in \mho_c ; \; l \in \mathcal{L} \tag{2.3}$$

$$\sum_{i\in\mho,i\neq h} x_{ihkl} - \sum_{j\in\mho,j\neq h} x_{hjkl} = 0 \; ; \; h \in \mho\backslash\{v_0\}; \; l \in \mathcal{L} \; ; \; k \in K_l \tag{2.4}$$

$$u_{ikl} - u_{jkl} + Q_k x_{ijkl} \leq Q_k - q_j \; ; \; i,j \in \mho\backslash\{v_0\} : j \neq i; \; q_i + q_j \leq Q_k; \; l \in \mathcal{L} \; ; \; k \in K_l \tag{2.5}$$

$$q_i \leq u_{ikl} \leq Q_k \; ; \; i \in \mho\backslash\{v_0\}; \; l \in \mathcal{L} \; ; \; k \in K_l \tag{2.6}$$

$$\sum_{j\in\mho\backslash\{v_0\}} x_{0jkl} \leq 1 \; ; \; l \in \mathcal{L} \; ; \; k \in K_l \tag{2.7}$$

$$\sum_{i\in\mho}\sum_{j\in\mho,j\neq i} (t_{ij} + s_i)\, x_{ijkl} \leq T_l \; ; \; l \in \mathcal{L} \; ; \; k \in K_l \tag{2.8}$$

$$\sum_{i\in\mho_p} x_{i0kl} - \sum_{j\in\mho\backslash\{v_0\}} x_{0jkl} = 0 \; ; \; l \in \mathcal{L} \; ; \; k \in K_l \tag{2.9}$$

$$x_{ijkl}, y_{ir} \in \{0,1\}; \; i,j \in \mho_c : j \neq i; \; l \in \mathcal{L} \; ; \; k \in K_l ; r \in C_i \tag{2.10}$$

According to Bianchi-Aguiar *et al*. (2012) the objective function (2.1) minimizes the total distance traveled by the vehicles in all days of the planning period. Constraints (2.2) ensure that a feasible schedule is attributed for each customer. Constraints (2.3) guarantee that customers are visited only on the days corresponding to the assigned schedules. Constraints (2.4) ensure connectivity by stating that when a vehicle arrives to a customer in a given day, it also leaves him. Constraints (2.5)-(2.6) are subtour elimination constraints. Constraints (2.7) state that each vehicle is used at most once a day and also that routes must start on the garage. Limits on route duration are imposed in constraints (2.8). Finally, constraints (2.9) state that vehicles must pass through a disposal facility immediately before returning to the garage.

3. Clustering Search

The Clustering Search (CS) (Chaves and Lorena, 2010) is a hybrid method that aims to combine metaheuristics and local search heuristics, in which the search is intensified only in areas of the search space that deserve special attention (promising regions). The CS introduces an intelligence and priority to the choice of solutions to apply local search, instead of choosing randomly or applying local search in all solutions. Therefore, it is expected an improvement in the convergence process associated with a decrease in computational effort by reason of more rational employment of the heuristics.

The CS attempts to locate promising search areas by framing them by clusters. A cluster is defined by a *center*, *c*, that is generally, initialized randomly and, posteriorly, it tends to progressively slip along really promising points in the search space. The number of clusters, *NC*, can be fixed *a priori*.

CS can be split off in four conceptually independent parts: the search metaheuristic (SM), the iterative clustering (IC) component, the analyzer module (AM), and the local searcher (LS). Figure 1 brings the conceptual design of CS.

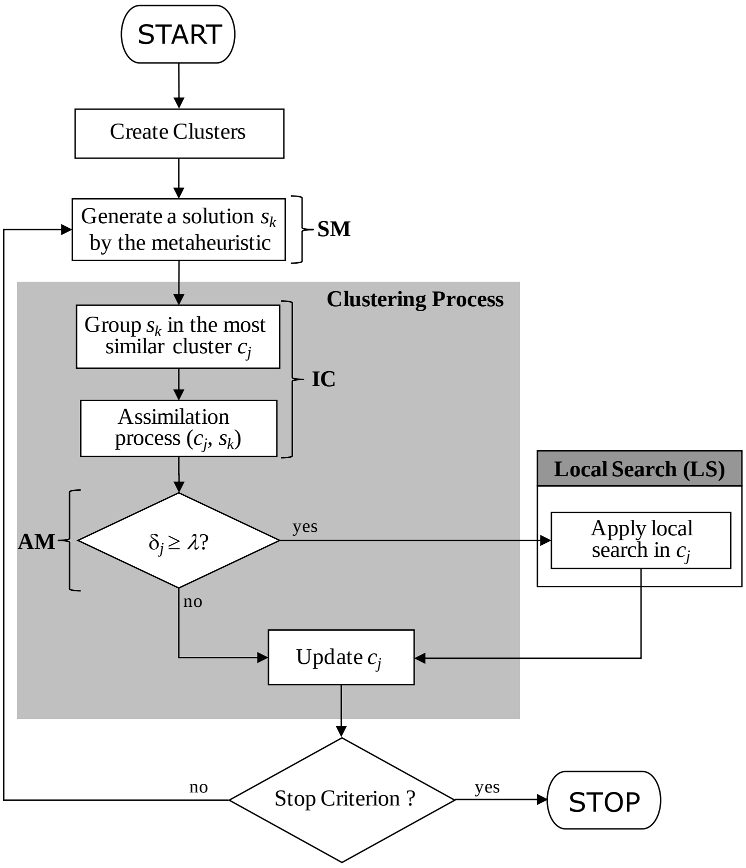

Figure 1 – CS components
Fonte: Oliveira *et al*. (2013)

The SM component can be implemented by any optimization algorithm that generates diversified solutions of the search space. It must work as a full-time solution generator, exploring the search space by manipulating a set of solutions, according to its specific search strategy.

IC component aims to gather similar solutions into groups, maintaining a representative cluster center for them. A *distance metric*, Δ, must be defined, *a priori*, allowing a similarity measure for the clustering process. For example, in combinatorial optimization, the similarity can be defined as the number of movements needed to change a solution into the cluster center (Oliveira and Lorena, 2007). The assimilation process is applied over the closest center c_i, considering the new generated solution s_k. The path-relinking (Glover *et al*., 2000) can be used to generate several points, holding the best evaluated one to be the new center.

AM component examines each cluster, in regular intervals, indicating a probable promising cluster. A *cluster density*, δ_j, is a measure that indicates the activity level inside the cluster *j*. For simplicity, δ_j can count the number of solutions generated by SM and grouped into c_j. Whenever δ_j reaches a certain *threshold*, λ, meaning that some information template becomes predominantly generated by SM, such cluster must be better investigated to accelerate the convergence process on it.

At last, the LS component is an internal searcher module that provides the exploitation of a supposed promising search area, framed by a cluster.

4. CS algorithm for PVRP

A solution of the PVRP is represented by a matrix, in which each row of the matrix represents a day of the week. The route is determined for each vehicle, that is represented by a number with a negative sign and its following visits come shortly thereafter to find another vehicle or get to the end of the line. The representation also includes a vector indicating the schedule that each customer is served. The schedules for 1-6 frequencies one day, for two days 7-11, 12-15 for three days, 16-18 for 4 days, and 19 for 6 days. Figure 2 shows an example of a solution with two vehicles, five customers and a planning period of six days. In this example, the route of the vehicle (-1) in day two is 3-5-1, and this vehicle goes from the garage to the customer 3 and after visiting the customer 1 goes to the nearest disposal facility and returns to the garage.

Day	Route					
1	-1	2	4	-2	1	3
2	-1	3	5	1	-2	
3	-1	1	4	-2		
4	-1	-2	1	4		
5	-1	2	3	-2	1	
6	-1	2	3	1	-2	4

Customers	c1	c2	c3	c4	c5
Frequency	6	3	4	4	1
Schedule	19	13	17	18	2

Figure 2 – An example of a solution representation.

The objective function (4.1) is the sum of the distances of all routes, i.e., the distance that each vehicle travels each day by leaving the garage, visiting clients in a particular sequence, going to a disposal facility and returning to the garage. In the proposed algorithm there are certain penalties (δ e *β*) added to the objective function regarding solutions that do not satisfy any of constraints imposed by the problem (limits on vehicle capacity, $\delta*Ep$, or limits on route duration, $\beta*Et$). The initial distance is represented by D_i, the distance from the customer i to j, which is represented by D_{ij}, and the return distance by D_r.

$$\sum_i (D_i + D_{ij} + D_r + (\delta * Ep) + (\beta * Et)) \tag{4.1}$$

The CS method starts with a fixed number of clusters (NC = 10) that are initialized with random solutions. The metaheuristic Simulated Annealing (SA) (Kirkpatrick *et al*., 1983) was responsible for generating solutions that are grouped in CS. In this paper we used the SA algorithm implemented by Higino *et al*. (2012). We use the solution s_k of SA at each N (number of customers) neighbors to increase the diversity among the solutions sent to the clustering process.

At each iteration of CS, one solution s_k is grouped into the closest cluster C_j; that is, the cluster that minimizes the distance between the solution and the cluster center. The volume δ_j is increased in one unit and the center c_j should be updated with new attributes of s_k (assimilation process).

The assimilation process uses the path-relinking method. The procedure starts by computing the symmetric difference between the center c_j and the solution s_k, $\Delta(c_j,s_k)$; i.e., the set of moves needed to reach s_k from c_j. A path of solutions is generated, linking c_j and s_k. At each step, the procedure examines all moves $m_i \in \Delta(c_j,s_k)$ from the current solution s and selects the one that results in the best-cost solution, applying the best move to solution s. The set of available moves is updated. The procedure terminates when 50% of the solutions in the path have been analyzed. The new center c_j is the best solution along this path. In this paper, one move is to swap one schedule of c_j by one schedule of s_k, changing the route of the customer with a new schedule.

After performing the assimilation, we must conduct an analysis of the volume δ_j, verifying if this cluster can be considered promising. A cluster becomes promising when its volume reaches the threshold λ. The value of λ was define as 5.

The 2-Opt method (Croes, 1958) is implemented as a local search of CS, intensifying the search in neighborhood of a promising cluster C_j. The 2-Opt consists in 2-changes over a route, deleting two arcs and replacing them by two other arcs to form a new route. This method continues while there is improvement in the route through this movement. The center c_j is updated if the new solution is better than the previous one.

Figure 3 presents the CS pseudo-code.

algorithm CS
    create the initial clusters of CS
    { Simulated Annealing }
    generate initial solution s_k
    $IterT \leftarrow 0 \quad T \leftarrow T_0$
    while termination condition not satisfied **do**
        while ($IterT < SA_{max}$) **do**
            $IterT \leftarrow IterT + 1$
            generate a solution $s' \in N(s_k)$
            calculate $\Delta_{s',sk} = f(s') - f(s_k)$
            if ($\Delta_{s',sk} \leq 0$) **then**
                $s_k \leftarrow s'$
            else
                $s_k \leftarrow s'$ with probability $exp(-\Delta_{s',sk} / T)$
            { clustering process }
            if ($IterT \bmod N = 0$)
                find the most similiar cluster c_j to the solution s_k
                insert s_k into c_j ($\delta_j \leftarrow \delta_j + 1$)
                $c_j \leftarrow$ *Path-Relinking* (c_j, s_k)
                if $\delta_j \geq \lambda$ **then**
                    $\delta_j \leftarrow 0$
                    { local search }
                    $c_j \leftarrow$ *2-Opt* (c_j)
                end if
            end if
        end while
        $T \leftarrow \alpha \times T$
        $IterT \leftarrow 0$
    end while
end algorithm

Figure 3 – CS pseudo-code

5. Computational Results

The CS algorithm was coded in C/C++, using the CodeBlocks platform, and the computational tests carried out on a PC Intel Core i5 2.67 GHz with 2 GB of RAM.

Two problem sets were used in these tests. First, a real instance, proposed by Bianchi-Aguiar *et al*. (2012), that represents the actual situation of waste collection in the municipality of Ponte de Lima in Portugal. This instance contains: five vehicles, each with its own capacity and maximum time provided for use, one garage, from where the vehicles start and finish their routes, two disposal facilities, to be visited on end of the route and before returning to the garage, and 51 customers, each with its requirement for the amount of waste to be collected and specific frequency, which will be visited at most six days a week.

In the second data set, we used classical instances for PVRP, introduced by Cordeau *et al*. (1997), that have similar characteristics with the instance proposed by Bianchi-Aguiar *et al*. (2012). Each vehicle contains a maximum capacity and maximum duration of a route, one garage that also serves as a disposal facilities, and each customer with a demand and specific frequencies, which will be visited at most a fixed number of days in the week.

Table 2 compares the results obtained (from CS and SA) with the current practice for solid waste collection in Ponte de Lima. The entries in the table are the current solution used in Ponte de Lima, the data of the instance (as number of customers, disposal facilities, vehicles and days), the best solution (*best*), average solution (*avge*), the percentage deviation from the best solution found (*dev*), and the average running times (*time*) that CS and SA found their solutions. The percentage deviation is calculated by *dev* = 100*(*avge* - *best*)/*best*.

Table 2 – Computational Results of *Ponte de Lima* instance.

Instance	Current Solution	Customers	Vehicles	Days	CS			SA			Time(s)
					Best	Avge	Dev(%)	Best	Avge	Dev(%)	
Ponte de Lima	2389	51	5	6	1680,80	1729,84	2,92	1748,10	1852,08	5,95	147,89

The results show the efficacy of the CS. For this instance, the CS found the best-known solution and improved the route plans (30% of reduction in operational costs). Besides, the number of routes was also improved. Currently, in Ponte de Lima there are 26 routes and CS found solutions with 25 routes.

We can observe that SA without the clustering process gave solutions of poorer quality than CS (the best solutions found by CS were about 4% better than the best solutions found by SA).

The CS algorithm was robust, producing low deviations (the average deviation, column *dev*, found by CS was 2.92% in 20 runs). And, the computational times of CS were competitive, finding good solutions in a reasonable time.

Table 3 reports a comparison with the best solutions found by Cordeau *et al*. (1997), which proposed a Tabu Search (TS) algorithm. The CS presents a best solution for the *PR01* instance. And for the others instances, the solutions obtained by CS were close to the best solutions of Tabu Search. We can observe that the average deviation from the best solution of Tabu Search was 8,4%.

Table 3 – Computational Results of Cordeau's instances.

Instance	TS (Cordeau *et al*., 1997)	Customers	Vehicles	Days	CS			SA			Time(s)
					Best	Avge	Dev(%)	Best	Avge	Dev(%)	
PR01	2234,23	48	2	4	2216,68	2295,31	3,55	2313,51	2404,44	7,62	163,51
PR02	3836,49	96	4	4	3865,33	4188,51	8,36	4058,06	4400,92	14,71	435,38
PR03	5277,62	144	6	4	5773,70	6197,19	7,33	5983,94	6409,58	21,45	966,85
PR04	6072,67	192	8	4	6855,78	7160,21	4,44	7027,29	7365,42	21,29	1897,86
PR05	6769,80	240	10	4	7329,56	7577,34	3,38	7469,34	7746,62	14,43	2904,54
PR06	8462,37	288	12	4	9323,27	9986,68	7,12	9651,80	10363,50	22,47	3485,45
PR07	5000,90	72	3	6	5171,39	5256,42	1,64	5416,63	5486,81	9,72	326,54
PR08	7183,39	144	6	6	7956,61	8117,48	2,02	8139,61	8313,64	15,73	934,43
PR09	10507,34	216	9	6	11706,61	12204,50	4,25	12092,93	12626,41	20,17	2614,09
PR10	13629,25	288	12	6	14575,61	15286,48	4,88	14852,95	15585,71	14,35	3246,73
average	6897,41				7477,45	7827,01	4,70	7700,61	8070,31	16,19	

Figure 4 illustrates runtime distributions, or time-to-target (TTT) plot (Aiex *et al*., 2007), for the Ponte de Lima instance. The experiment consists in running the SA and CS 100 times on the instance. Each run is independent of the other and stops when a solution with cost at least as good as a given target solution value is found. We observe visually that CS dominates SA. The probability of the CS finding a solution at least as good as the target value in at most 10 seconds is about 40%, in at most 20 seconds is about 70%, and in at most 30 seconds is about 90%. All solutions of the SA were found with more than 30 seconds.

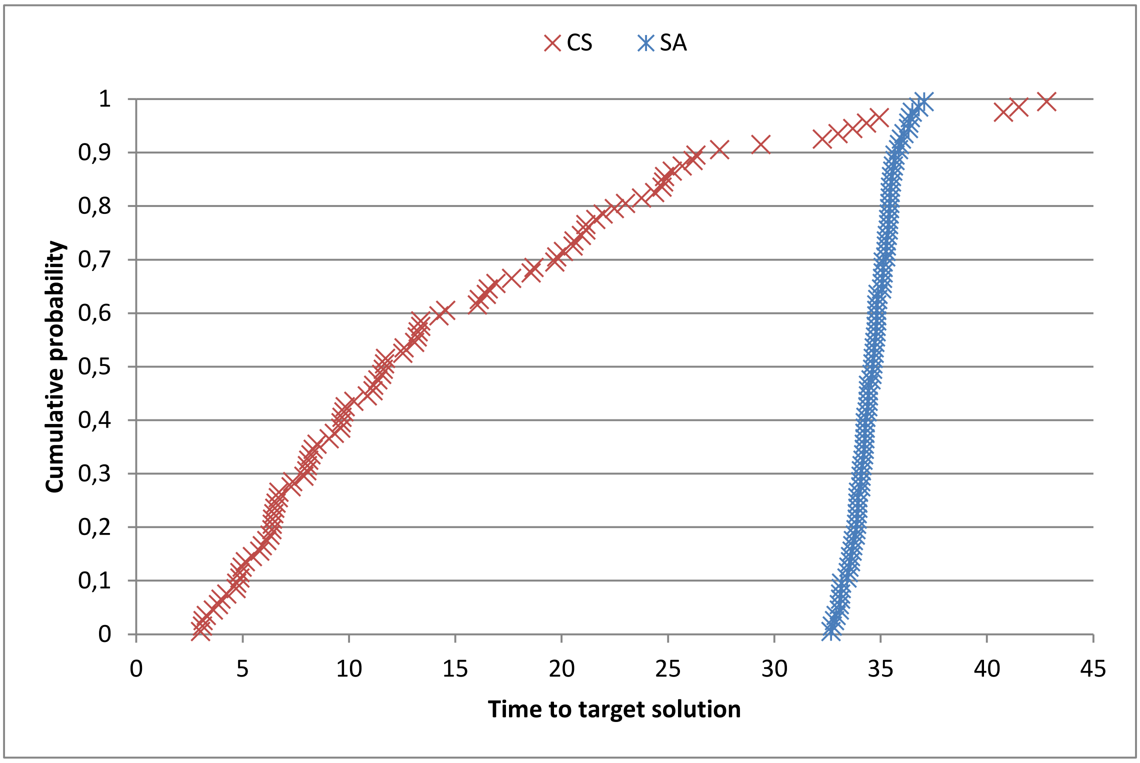

Figure 4 – Cumulative probability distribution

6. Conclusion

This paper proposes an algorithm based on the Clustering Search (CS) method to solve the waste collection problem that was modeled as periodic vehicle routing problem (PVRP). The CS has been applied with success in some combinatorial optimization problems (Oliveira *et al.*, 2013). The idea of the CS is to avoid applying a local search heuristic to all solutions generated by a metaheuristic. The CS detects the promising regions in the search space and applies local search only in these regions. Then, to detect promising regions becomes an interesting alternative, preventing the indiscriminate application of the heuristics.

This paper reports results found by CS and SA about a real instance of waste collection problem (Ponte de Lima municipality) and classical instances for the PVRP. The CS got the best solution for the real instance and good solutions for classical instances, despite not having been specifically developed to solve the classic PVRP. The results show that the CS is competitive for solving the waste collection problem.

Further studies can be done analyzing other metaheuristics to generate solutions for the clustering process of the CS, such as Ant Colony System, Tabu Search and Iterated Local Search, and also exploring other local search heuristics. We intend to use the proposed CS to solve the problem of milk collection, which has technical features similar to waste collection problem.

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