Reliability evaluation of power distribution systems through interruption time flows

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Abstract

Reliability analysis of power systems has attracted a growing attention, specially in primary distribution networks where most of the failures occur. Regulatory agencies define minimum reliability levels for distribution systems that can be achieved by a number of alternatives available to the distribution engineer. This includes allocation of sectionalizers, improvements in maintenance policy and alternative operating policies. To compare quantitatively the merits of such alternatives and their effect per monetary unit expended, an efficient reliability evaluation technique is required. This paper proposes a new reliability evaluation technique which brings the concept of interruption time flows. The approach can be applied to reliability optimization problems that are economically relevant for the utilities. A study case on the minimization of energy not supplied through the allocation of sectionalizing switches is investigated. A mixed integer linear model using interruption time flow variables is proposed and tests are performed with a benchmark set of distribution networks. Results illustrate the benefits of the approach and indicates its potential to solve reliability optimization problems.

KEYWORDS: reliability optimization, switch allocation, energy not supplied, interruption time flows

1 Introduction

An electric power distribution system contains a set of components that are subject to failure, including cables, poles, breakers, switches, transformers, capacitors and voltage regulators. It is estimated that about 70% of the overall interruption duration of power systems is associated to outages in primary distribution networks (Billinton and Allan 1996).

The probability of costumers being disconnected for any reason can be reduced by investments on distribution networks. Overinvestment can lead to excessive operating costs, while underinvestment leads to power supply service under poor continuity standards. The conflicting economic and reliability objectives demand planning and operating decisions that ought to be based on solid reliability information. Efficient evaluation techniques are required to quantify the benefits of various reinforcement schemes and thus to achieve the best improvements in the system under limited capital resources (Brown 2008).

A new reliability evaluation methodology for power distribution networks is the main contribution of this paper. The approach is supported by the introduction of *interruption time flows*, an alternative representation of the network reliability state. The main concepts are described in Sec. 2. The procedure to determine each customer interruption time is described in Sec. 3. The notion of interruption time flows is introduced in Sec. 4, formalizing the proposed reliability evaluation methodology as a network flow problem. Sec. 5 gives numerical examples to illustrate the approach, and its application on reliability optimization problems is presented in Sec. 6. Finding the minimum energy not supplied through the allocation of sectionalizing switches is investigated. This optimization problem has a straightforward formulation as a network flow problem, using the interruption time flow variables. Computational tests are performed with five benchmark networks in Sec. 7, and the proposed mixed integer linear model is solved by a commercial optimization solver. Optimal solutions are obtained for several case studies within suitable computational time.

Results show the effectiveness of the approach and points to a viable evaluation scheme that should be considered for other reliability optimization problems.

2 Reliability evaluation of power distribution systems

2.1 Assumptions

The proposed evaluation procedure adopts the following assumptions:

- The distribution network is radially operated, meaning there is an unique path linking the substation to each customer.
- All failures are non-transient short circuits that open the first upstream breaker.
- All failures are independent from each other and at most one failure happens at any time.
- The failure frequency is a stochastic variable, and its value represents the expected amount of failures that should occur in a one year period. This extends to all variables and indices that depend on the failure frequency.
- Circuit breakers and sectionalizing switches do not fail.
- The effects of load transfer through tie-lines are neglected.

2.2 Main concepts

A radially operated distribution system, using graph terminology, can me modeled as a weakly connected directed tree $T(V,A)$, rooted at the substation (node 0). Node $i \in V$ denotes either a network bifurcation point or a load point with power load l_i (kW), failure rate λ_i (failures/year) and number of customers n_i. An arc $(i,j) \in A$, $i,j \in V$, is orientated in the same direction as the power flow, which traverses from the root to the customers. Each node $j \in V \setminus \{0\}$ has a predecessor node i, or simply, $i = pred(j)$. The set of arcs in which a circuit breaker is installed is denoted by A_b. The set of arcs in which either a circuit breaker or a sectionalizer is installed is denoted by A_{sb} ($A_b \subseteq A_{sb} \subseteq A$).

There is an unique directed path connecting the root to every node in the tree. The set of nodes representing the directed path connecting two nodes i and j is represented by $path(i,j)$. If no such path exists, then $path(i,j) = \emptyset$; also, $path(i,i) = i$. For every pair of nodes i and j, if $path(i,j) \neq \emptyset$, then j is downstream of i, otherwise j is upstream of i. The set of downstream nodes of i is represented by V_i. If $V_i = \{i\}$ then node i is called a leaf. Eq. (1) shows how to determine the downstream power load $\tilde{l}_i$ of a node i.

$$\tilde{l}_i = \sum_{j \in V_i} l_j \tag{1}$$

2.3 Sectionalizers and circuit breakers

Sectionalizers and circuit breakers have an important role on the reliability of distribution network. A breaker opens immediately after a short circuit flows through it. This disconnects all downstream loadpoints but does not affect or cause disconnection of any upstream load point. The breaker can be reclosed as soon as the failure is repaired. A sectionalizer is generally not a fault-breaking switch, which means any short circuit still causes the first upstream breaker to operate. However, after a fault has been detected, a suitable sectionalizer can be opened and the breaker reclosed. This procedure allows restoration of all load points between the breaker and the point of isolation before the repair process is completed (Billinton and Allan 1996).

The average interruption time t taken to restore power supply to all nodes affected by a fault can be decomposed into an average location time t^l and an average repair time t^r. The location time considers identifying the failure location, setting the maintenance team and opening a relevant sectionalizer. The repair time considers fixing all defective network components and reclosing the breaker. From now on, whenever a variable has the superscript l or r, it means that only the portion related to the location time or repair time, respectively, is being considered. IEEE Std 493-1997 (1998) shows how to estimate the average interruption time and interruption frequency of a distribution network.

Suppose that a fault occurs at node 2 of the network shown in Figure 1. This opens the breaker on arc $(0,1)$ interrupting power supply from nodes 1, 2 and 3. After the location time, the sectionalizer on arc $(1,2)$ is opened and the circuit breaker is reclosed, restoring nodes 1 and 3. Node 2 is restored as soon as the fault is repaired and the sectionalizer is reclosed. The interruption duration of each node depends on its relative position to the fault origin. The nodes downstream of the breaker and upstream of the sectionalizer are disconnected for a duration t^l. The nodes downstream of the sectionalizer are disconnected for a full interruption duration of $t^l + t^r$.

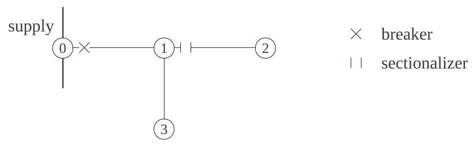


Figure 1: Roles of sectionalizers and breakers.

2.4 Reliability indices

To evaluate quantitatively how reliable a distribution system is, there are several indices that can be used. Some examples are the *system average interruption frequency index* (SAIFI), *system average interruption duration index* (SAIDI) and *energy not supplied* (ENS) (Eq. 2). These indices are not deterministic values but expectations of a probability distribution (Billinton and Allan 1996).

$$SAIFI = \frac{\sum\limits_{i \in V} n_i \lambda_i}{\sum\limits_{i \in V} n_i}, \qquad SAIDI = \frac{\sum\limits_{i \in V} n_i u_i}{\sum\limits_{i \in V} n_i}, \qquad ENS = \sum_{i \in V} l_i u_i \tag{2}$$

Parameters n_i, λ_i and l_i are, respectively, the number of customers, failure rate $(failures/year)$ and power load (kW) of node i. The interruptions times u_i are the topic of Sec. 3.

Regulatory agencies adopt some of these indices to define minimum levels of reliability that, if not regarded, can trigger costly fines to the utilities. Moreover, reliability indices can also be used to (i) identify areas of the network that require more investment; (ii) determine the reliability tendency over time; (iii) compare historical values with the current network state; (iv) compute the benefit/loss of any proposed change to the network (Brown 2008).

3 Interruption times

The interruption times u are described by Def. 1.

Definition 1 *The interruption time u_i is the expected duration of interruptions from power supply a node i will endure in a one-year period, due to all faults occurred in the network.*

The expected time each node will be interrupted due to faults occurred on its own components is the subject of Def. 2.

Definition 2 *The self-interruption time θ_i (Eq. 3) is the expected duration of interruptions from power supply a node i will endure in a one-year period, due to local faults (occurred at node i).*

$$\theta_i = \theta_i^l + \theta_i^r, \qquad \theta_i^l = \lambda_i t^l, \qquad \theta_i^r = \lambda_i t^r \qquad\qquad i \in V \tag{3}$$

Def. 3 describes how to compute downstream interruption times.

Definition 3 *The downstream interruption time $\tilde{\theta}_i$, determined by Eq.* (4)*, is the expected time node i will be interrupted due to downstream faults, in a one-year period.*

$$\tilde{\theta}_i = \tilde{\theta}^l_i + \tilde{\theta}^r_i, \qquad \tilde{\theta}^l_i = \theta^l_i + \sum_{(i,j)\in A\setminus A_b} \tilde{\theta}^l_j, \qquad \tilde{\theta}^r_i = \theta^r_i + \sum_{(i,j)\in A\setminus A_{sb}} \tilde{\theta}^r_j \qquad i \in V \tag{4}$$

The downstream interruption time follows the functionality of sectionalizers and breakers described in Sec. 2.3. Breakers avoid the accumulation of downstream interruption times and sectionalizers avoid the accumulation of the repair portion of downstream interruption times. Lemma 1 shows how to determine the interruption time for any node using downstream interruption times.

Lemma 1 *The interruption time u_i of a node i can be determined according to the following base case and recursion (Eq. 5).*

$$u_0 = \tilde{\theta}_0, \qquad u_j - u_i = \begin{cases} 0 & (i,j) \in A \setminus A_{sb} \\ \tilde{\theta}^r_j & (i,j) \in A_{sb} \setminus A_b \\ \tilde{\theta}_j & (i,j) \in A_b \end{cases} \tag{5}$$

Proof:
Base case:
The faults that cause interruption of the root are originated downstream. Therefore, the root interruption time matches the downstream interruption time (Def. 3).

Case 1: $(i,j) \in A \setminus A_{sb}$
Node i interruption causes the interruption of node j and vice-versa, thus $u_i = u_j$.

Case 2: $(i,j) \in A_{sb} \setminus A_b$
Node i interruption causes the interruption of node j, but due to the sectionalizer on arc (i,j), node i does not have to wait for downstream repair times, thus $u_j = u_i + \tilde{\theta}^r_j$.

Case 3: $(i,j) \in A_b$
Node i interruption causes the interruption of node j. However, due to the breaker on arc (i,j), node i is immune to any downstream faults, thus $u_j = u_i + \tilde{\theta}_j$. ■

4 Flow-based reliability evaluation

The new reliability evaluation procedure proposed in this paper uses the concept of interruption time flows f (Def. 4). This section shows that reliability indices, such as the ENS, can be expressed in terms of the f variables (Theo. 1). The proposed methodology represents a less natural approach than using the interruption times u. However, the interruption time flows turn the reliability evaluation into a network flow problem that is more suitable to formulate within related optimization problems (Sec. 6.2).

Definition 4 *The interruption time flow f_{ij}, from node j to node i (inverse of the arc orientation), is the expected time node i will be interrupted due to faults originated at nodes $k \in V_j$. Eq.* (6) *shows that f_{ij} is the sum of the location f^l_{ij} and repair f^r_{ij} interruption time flows.*

$$f_{ij} = \begin{cases} 0 \\ \tilde{\theta}^l_j \\ \tilde{\theta}_j \end{cases}, \qquad f^l_{ij} = \begin{cases} 0 \\ \tilde{\theta}^l_j \\ \tilde{\theta}^l_j \end{cases}, \qquad f^r_{ij} = \begin{cases} 0 & (i,j) \in A_b \\ 0 & (i,j) \in A_{sb} \setminus A_b \\ \tilde{\theta}^r_j & (i,j) \in A \setminus A_{sb} \end{cases} \tag{6}$$

The presence of breakers and sectionalizers prevents the usual input equals output flow conservation laws for each node. To capture the flow deviation of the nodes, an interruption residue F is defined next.

Definition 5 *The interruption residue F_j represents an interruption flow balance of node j, calculated according to Eq. (7).*

$$F_j = \begin{cases} \theta_0 + \sum_{(0,k)\in A} f_{0k} & j = 0 \\ \theta_j + \sum_{(j,k)\in A} f_{jk} - f_{ij} & (i,j)\in A \end{cases} \tag{7}$$

Lemma 2 and Corollary 1 show that the interruption times u can be determined by an expression of the interruption residues F.

Lemma 2 *The interruption time u_j of a node j can be determined according to the following base case and recursion (Eq. 8).*

$$u_0 = F_0, \qquad u_j - u_i = F_j \qquad (i,j)\in A \tag{8}$$

Proof:
Base case:

$$F_0 \overset{\text{Def. 5}}{=} \theta_0 + \sum_{(0,i)\in A} f_{0i} \overset{\text{Def. 4}}{=} \theta_0 + \sum_{(0,i)\in A_{sb}\setminus A_b} \tilde{\theta}^l_i + \sum_{(0,i)\in A\setminus A_{sb}} \tilde{\theta}_i \tag{9}$$

$$\overset{\text{Def. 3}}{=} \theta_0 + \sum_{(0,i)\in A\setminus A_b} \tilde{\theta}^l_i + \sum_{(0,i)\in A\setminus A_{sb}} \tilde{\theta}^r_i \overset{\text{Def. 3}}{=} \tilde{\theta}_0 \overset{\text{Lem. 1}}{=} u_0 \tag{10}$$

Case 1: $(i,j)\in A\setminus A_{sb}$

$$F_j \overset{\text{Defs. 5,4}}{=} \theta_j + \sum_{(i,j)\in A\setminus A_b} \tilde{\theta}^l_j + \sum_{(i,j)\in A\setminus A_{sb}} \tilde{\theta}^r_j - \tilde{\theta}_j \overset{\text{Def. 3}}{=} \tilde{\theta}_j - \tilde{\theta}_j = 0 \overset{\text{Lem. 1}}{=} u_j - u_i$$

Case 2: $(i,j)\in A_{sb}\setminus A_b$

$$F_j \overset{\text{Defs. 5,4,3}}{=} \tilde{\theta}_j - \tilde{\theta}^l_j \overset{\text{Def. 3}}{=} \tilde{\theta}^r_j \overset{\text{Lem. 1}}{=} u_j - u_i$$

Case 3: $(i,j)\in A_b$

$$F_j \overset{\text{Defs. 5,4,3}}{=} \tilde{\theta}_j \overset{\text{Lem. 1}}{=} u_j - u_i$$

■

Corollary 1 *The interruption time u_j is the sum of interruption residues on the path from the root to node j (Eq. 11).*

$$u_j = \sum_{k\in path(0,j)} F_k \qquad j\in V \tag{11}$$

Proof: The case in which node $j = 0$ is trivial. All other cases are considered next.

$$\begin{aligned} u_j - u_i &\overset{\text{Lem. 2}}{=} F_j & (i,j)\in A \\ u_j - u_{(pred(i))} &\overset{\text{Lem. 2}}{=} F_i + F_j & (pred(i),i)\in A \\ &\dots \\ u_j &\overset{\text{Lem. 2}}{=} F_0 + \dots + F_{pred(i)} + F_i + F_j \\ u_j &= \sum_{k\in path(0,j)} F_k \end{aligned}$$

■

Theorem 1 represents the core of the reliability evaluation methodology. It shows the ENS as an expression of the interruption time flows f, which means the ENS can be computed by solving a network flow problem.

Theorem 1 *The ENS can be expressed by interruption flow variables, according to Eq.* (12).

$$ENS = \sum_{(i,j)\in A} (\tilde{l}_i - \tilde{l}_j) f_{ij} + \sum_{i\in V} \tilde{l}_i \theta_i \tag{12}$$

Proof:

$$\sum_{i\in V} l_i u_i \overset{\text{Cor. 1}}{=} \sum_{i\in V}\left(l_i \sum_{j\in path(0,i)} F_j\right) \overset{(a)}{=} \sum_{j\in V}\left(F_j \sum_{i\in V_j} l_i\right) \overset{\text{Eq. 1}}{=} \sum_{i\in V} \tilde{l}_i F_i$$
$$\overset{\text{Def. 5}}{=} \sum_{i\in V}\left(\tilde{l}_i\left(\theta_i + \sum_{(i,j)\in A} f_{ij}\right)\right) - \sum_{(i,j)\in A} \tilde{l}_j f_{ij} = \sum_{(i,j)\in A} (\tilde{l}_i - \tilde{l}_j) f_{ij} + \sum_{i\in V} \tilde{l}_i \theta_i$$

Equality (a) is derived by noticing that $j \in path(0,i)$ if and only if $i \in V_j$. ■

It is worth mentioning that other reliability indices, such as the SAIDI, can also be expressed in terms of variables f. This can be made by replacing the SAIDI interruption times u from Eq. (2) with the interruption residues F, according to Eq. (11).

Lem. 3 shows how to determine the feasible ENS range, expressed by a lower and upperbound. The relative reliability state of a network can be established in terms of the distance from these bounds.

Lemma 3 *A lowerbound E_{lb} and upperbound E_{ub} for the ENS are described by the following inequality (13).*

$$\sum_{i\in V} \tilde{l}_i \theta_i = E_{lb} \leqslant ENS \leqslant E_{ub} = \tilde{l}_0 \sum_{i\in V} \theta_i \tag{13}$$

Proof: The lowerbound proof is trivial given that E_{lb} is a constant present in Eq. (12). The scenario in which ENS $= E_{lb}$ implies $A_b = A$ and $f_{ij} = 0$ for all $(i,j) \in A$.

By inspecting Eq. (6), the interruption time flows f are maximum if $A_{sb} = \emptyset$. Using the assumption $A_{sb} = \emptyset$, the maximum interruption time flows f^{max} are given by Eq. (14).

$$f_{ij}^{max} \overset{\text{Def. 4}}{=} \tilde{\theta}_j \overset{\text{Def. 3}}{=} \theta_j + \sum_{(i,j)\in A\setminus A_b} \tilde{\theta}_j^l + \sum_{(i,j)\in A\setminus A_{sb}} \tilde{\theta}_j^r \overset{A_{sb}=\emptyset}{=} \sum_{k\in V_j} \theta_k \qquad (i,j)\in A \tag{14}$$

The ENS upperbound E_{ub} is derived by replacing the interruption flows f_{ij} in Eq. (12) by the maximum interruption flows f_{ij}^{max} of Eq. (14).

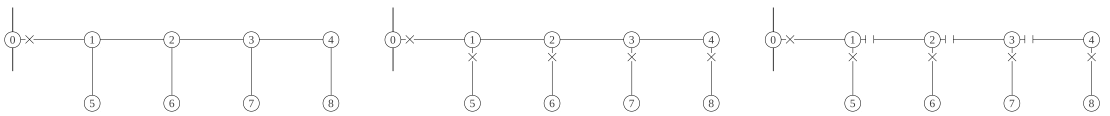
$$E_{ub} = \sum_{(i,j)\in A} (\tilde{l}_i - \tilde{l}_j) f_{ij}^{max} + \sum_{i\in V} \tilde{l}_i \theta_i = \sum_{(i,j)\in A}\left(\tilde{l}_i \sum_{k\in V_j} \theta_k\right) - \sum_{(i,j)\in A}\left(\tilde{l}_j \sum_{k\in V_j} \theta_k\right) + \sum_{i\in V} \tilde{l}_i \theta_i$$
$$= \sum_{i\in V}\left(\tilde{l}_i\left(\sum_{j\in V_i} \theta_j - \theta_i\right)\right) - \sum_{i\in V\setminus 0}\left(\tilde{l}_i \sum_{j\in V_i} \theta_j\right) + \sum_{i\in V} \tilde{l}_i \theta_i$$
$$= \tilde{l}_0\left(\sum_{i\in V} \theta_i - \theta_0\right) + \sum_{i\in V\setminus 0} \tilde{l}_i\left(\sum_{j\in V_i} \theta_j - \theta_i\right) - \sum_{i\in V\setminus 0}\left(\tilde{l}_i \sum_{j\in V_i} \theta_j\right) + \sum_{i\in V} \tilde{l}_i \theta_i$$
$$= \tilde{l}_0 \sum_{i\in V} \theta_i - \tilde{l}_0 \theta_0 - \sum_{i\in V\setminus 0} \tilde{l}_i \theta_i + \sum_{i\in V} \tilde{l}_i \theta_i = \tilde{l}_0 \sum_{i\in V} \theta_i$$

■

5 Illustrative numerical examples

The networks shown in Figure 2 are used as application examples of the methodology. These networks were extracted from Billinton and Allan (1996) and they all contain nine nodes from which four are load points (nodes $5-8$). Three scenarios are tested to measure the effect of breakers and sectionalizers on the network reliability. All parameters used in the tests were replicated in Table 1 and the evaluation results are shown in Table 2. The ENS was computed to all scenarios through Eq. (12), which for these networks results in the following expression:

$$\begin{aligned} ENS =& (\tilde{l}_0-\tilde{l}_1)f_{01}+(\tilde{l}_1-\tilde{l}_2)f_{12}+(\tilde{l}_1-\tilde{l}_5)f_{15}+(\tilde{l}_2-\tilde{l}_3)f_{23}+(\tilde{l}_2-\tilde{l}_6)f_{26}+(\tilde{l}_3-\tilde{l}_4)f_{34} \\ &+(\tilde{l}_3-\tilde{l}_7)f_{37}+(\tilde{l}_4-\tilde{l}_8)f_{48}+\tilde{l}_0\theta_0+\tilde{l}_1\theta_1+\tilde{l}_2\theta_2+\tilde{l}_3\theta_3+\tilde{l}_4\theta_4+\tilde{l}_5\theta_5+\tilde{l}_6\theta_6+\tilde{l}_7\theta_7+\tilde{l}_8\theta_8 \end{aligned}$$



(a) Scenario 1: clear network
(b) Scenario 2: lateral distribution protections
(c) Scenario 3: sectionalizers and lateral distribution protections

Figure 2: Scenarios tested in the numerical examples.

Table 1: Parameters used in the numerical examples.

		Node i								
		0	1	2	3	4	5	6	7	8
l_i	kW	0	0	0	0	0	5000	4000	3000	2000
$\tilde{l}_i$	kW	14000	14000	9000	5000	2000	5000	4000	3000	2000
λ_i	f/y	0	0.2	0.1	0.3	0.2	0.2	0.6	0.4	0.2
t^l	h/f	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
t^r	h/f	3.5	3.5	3.5	3.5	3.5	1.5	1.5	1.5	1.5
θ_i^l	h/y	0	0.1	0.05	0.15	0.1	0.1	0.3	0.2	0.1
θ_i^r	h/y	0	0.7	0.35	1.05	0.7	0.3	0.9	0.6	0.3

f – failure, h – hour, kW – kilowatt, y – year

Scenario 1 – clear network: The network has no switches with the exception of the substation main breaker on arc $(0,1)$ (Figure 2(a)). Every fault will cause the main breaker to operate. Given that the failure cannot be isolated by a sectionalizer, the main breaker can only be reclosed after the failures are repaired. The network ENS in this scenario is equal to $84,000$ $kWh/year$.

Scenario 2 – lateral distribution protections: The network contains breakers on each predecessor arc of a load point (Figure 2(b)). A fault on a load point causes its corresponding breaker to operate so that no other node is affected. This explains the interruption time flows equal to zero on arcs $(1,5)$, $(2,6)$, $(3,7)$ and $(4,8)$. The network ENS in this scenario is equal to $54,800$ $kWh/year$.

Scenario 3 – sectionalizers and lateral distribution protections: The network reliability is strengthened by considering sectionalizers on arcs $(1,2)$, $(2,3)$ and $(3,4)$ (Figure 2(c)). The new possibilities of reconfiguration brought by the sectionalizers reduced significantly the interruption time flows, as shown in Table 2. The network ENS in this scenario is equal to $35,200$ $kWh/year$.

6 Application to reliability optimization

This section shows how the proposed methodology can be used to solve reliability optimization problems. The switch allocation problem is investigated in this paper and it can be stated as the problem of finding the best locations to install sectionalizers on a power distribution network to optimize a given criterion.

Table 2: Reliability evaluation for the numerical examples.

Scenario 1 – clear network										
		Node i								
		0	1	2	3	4	5	6	7	8
$\tilde{\theta}_i^r$	h/y	0	4.90	3.90	2.65	1.00	0.30	0.90	0.60	0.30
$\tilde{\theta}_i$	h/y	0	6.00	4.80	3.20	1.20	0.40	1.20	0.80	0.40
F_i	h/y	0	6.00	0	0	0	0	0	0	0
u_i	h/y	0	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
		Edge (i,j)								
		$(0,1)$	$(1,2)$	$(1,5)$	$(2,3)$	$(2,6)$	$(3,4)$	$(3,7)$	$(4,8)$	
f_{ij}^r	h/y	0	3.90	0.30	2.65	0.90	1.00	0.60	0.30	
f_{ij}	h/y	0	4.80	0.40	3.20	1.20	1.20	0.80	0.40	
Scenario 2 - lateral distribution protections										
		Node i								
		0	1	2	3	4	5	6	7	8
$\tilde{\theta}_i^r$	h/y	0	2.80	2.10	1.75	0.70	0.30	0.90	0.60	0.30
$\tilde{\theta}_i$	h/y	0	3.20	2.40	2.00	0.80	0.40	1.20	0.80	0.40
F_i	h/y	0	3.20	0	0	0	0.40	1.20	0.80	0.40
u_i	h/y	0	3.20	3.20	3.20	3.20	3.60	4.40	4.00	3.60
		Edge (i,j)								
		$(0,1)$	$(1,2)$	$(1,5)$	$(2,3)$	$(2,6)$	$(3,4)$	$(3,7)$	$(4,8)$	
f_{ij}^r	h/y	0	2.10	0	1.75	0	0.70	0	0	
f_{ij}	h/y	0	2.40	0	2.00	0	0.80	0	0	
Scenario 3 – sectionalizers and lateral distributor protections										
		Node i								
		0	1	2	3	4	5	6	7	8
$\tilde{\theta}_i^r$	h/y	0	0.70	0.70	0.70	0.70	0.30	0.90	0.60	0.30
$\tilde{\theta}_i$	h/y	0	1.10	1.00	0.95	0.80	0.40	1.20	0.80	0.40
F_i	h/y	0	1.10	0.35	1.05	0.70	0.40	1.20	0.80	0.40
u_i	h/y	0	1.10	1.45	2.50	3.20	1.50	2.65	3.30	3.60
		Edge (i,j)								
		$(0,1)$	$(1,2)$	$(1,5)$	$(2,3)$	$(2,6)$	$(3,4)$	$(3,7)$	$(4,8)$	
f_{ij}^r	h/y	0	0	0	0	0	0	0	0	
f_{ij}	h/y	0	0.30	0	0.25	0	0.10	0	0	

h – hour, y – year

6.1 Previous works

Many heuristics have been proposed to solve the switch allocation problem. Levitin et al. (1995) proposed a genetic algorithm to allocate sectionalizing switches on a network with 52 nodes. Billinton and Jonnavithula (1996) used simulated annealing to determine the number and locations of switches for a network with 7 feeders and 67 nodes. Moradi and Fotuhi-Firuzabad (2008) proposed a particle swarm optimization algorithm to allocate sectionalizers and breakers for an IEEE feeder with 123 nodes.

Carvalho et al. (2005) addressed the allocation of automatic switches, which require less time to isolate faults and reconfigure the network compared to manual switches. A three step heuristic is proposed: *(i)* evaluate the benefit brought by each switch; *(ii)* partition the network into independent subsets; *(iii)* allocate the switches on each partition. This heuristic was tested on a network with 11 candidate locations to install switches. Benavides et al. (2013) also allocate automatic switches and propose an iterated sample construction with path relinking to solve networks up to 873 nodes. Optimal solutions were obtained for instances with 10 switches and 135 nodes. Comparing the efficiency of different methods is not straightforward, as Benavides et al. (2013) well observed, and the reason is the use of different objectives and reliability indices.

An exact algorithm, based on dynamic programming, was presented by Celli and Pilo (1999). However, the methodology is only applicable to small problems, such as the illustrative cases with 36 and 47 nodes, presented in the paper.

The following section proposes a mixed integer linear programming (MILP) model for the switch allocation problem, based on the interruption time flows introduced in Sec. 4.

6.2 MILP model

Different criteria can be adopted when solving the switch allocation problem. For example, minimizing the ENS, SAIDI, allocation costs or a combination of these, are equally interesting investigations. In this paper, it is considered a fixed number of N switches that must be allocated on the best possible locations in the network so as to minimize the ENS.

In the following MILP model, a switch is allocated on arc (i,j) if and only if $x_{ij} = 1$. Variables f^l_{ij} and f^r_{ij} gives the location and repair interruption time flows (Def. 4) on arc (i,j). Parameters $\tilde{l}$, θ and E_{lb} are described by Eqs. (1,3,13). A large parameter M is also defined: $M_i = \sum_{j \in V_i} \theta^r_j$.

(switch allocation problem)

$$MIN \quad \sum_{(i,j)\in A\setminus A_b} (\tilde{l}_i - \tilde{l}_j)(f^l_{ij} + f^r_{ij}) + E_{lb} \tag{15}$$

s.t.

(number of switches)

$$\sum_{(i,j)\in A\setminus A_b} x_{ij} \leqslant N \tag{16}$$

(interruption time flows)

$$f^l_{ij} - \sum_{(j,k)\in A\setminus A_b} f^l_{jk} \geqslant \theta^l_j \qquad (i,j) \in A \setminus A_b \tag{17}$$

$$f^r_{ij} - \sum_{(j,k)\in A\setminus A_b} f^r_{jk} + M_j x_{ij} \geqslant \theta^r_j \qquad (i,j) \in A \setminus A_b \tag{18}$$

(variables bounds and integrality)

$$f^l_{ij}, f^r_{ij} \geqslant 0 \qquad (i,j) \in A \setminus A_b$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A \setminus A_b$$

The objective function (15) represents the solution ENS. The number of switches is constrained by (16). The location and repair portions of the interruption time flows are given by (17,18). The model correctness relies on the accuracy of the interruption time flows. Lemmas 4 and 5 show that the flow values of an optimal solution f^*_{ij} match Def. 4.

Lemma 4 *An optimal solution for the switch allocation problem have its interruption time flows given by inequalities:*

$$f^{l*}_{ij} \geqslant \begin{cases} 0 & (i,j) \in A_b \\ \tilde{\theta}^l_j & (i,j) \in A \setminus A_b \end{cases} \qquad f^{r*}_{ij} \geqslant \begin{cases} 0 & (i,j) \in A_{sb} \\ \tilde{\theta}^r_j & (i,j) \in A \setminus A_{sb} \end{cases}$$

Proof:
Case 1: f^{l*}_{ij} and f^{r*}_{ij} for $(i,j) \in A_b$
The model only define flow variables for $(i,j) \in A \setminus A_b$. Without loss of generality, this can be assumed as $f^{l*}_{ij} = f^{r*}_{ij} = 0$.

Case 2: f_{ij}^{l*} for $(i,j) \in A \setminus A_b$
Variables f_{ij}^{l*} follow Constraints (17), which lead to the following inequality:

$$f_{ij}^{l*} \geqslant \theta_j^l + \sum_{(j,k)\in A\setminus A_b} f_{jk}^{l*}$$

If node j is a leaf, then it is clear that $f_{ij}^{l*} \geqslant \tilde{\theta}_j^l$. Otherwise, the inequality below considers as induction hypothesis that $f_{jk}^{l*} \geqslant \tilde{\theta}_k^l$ for all $(j,k) \in A \setminus A_b$:

$$f_{ij}^{l*} \geqslant \theta_j^l + \sum_{(j,k)\in A\setminus A_b} \tilde{\theta}_k^l \overset{Eq.\ (4)}{=} \tilde{\theta}_j^l$$

Case 3: f_{ij}^{r*} for $(i,j) \in A \setminus A_{sb}$
This case implies $x_{ij}^* = 0$, transforming Constraints (18) into symmetric equivalents of Constraints (17). Therefore, using the same induction of *Case 2* gives $f_{ij}^{r*} \geqslant \tilde{\theta}_j^r$.

Case 4: f_{ij}^{r*} for $(i,j) \in A_{sb} \setminus A_b$
This case implies $x_{ij}^* = 1$ and through Constraints (18):

$$f_{ij}^{r*} \geqslant \theta_j^r + \sum_{(j,k)\in A\setminus A_b} f_{jk}^r - M_j \geqslant \tilde{\theta}_j^r - \sum_{k\in V_j} \theta_k^r \geqslant \tilde{\theta}_j^r - \tilde{\theta}_j^r = 0$$

■

Lemma 5 *An optimal solution for the switch allocation problem has interruption time flows meeting Def. 4.*

Proof: Lemma 4 gives us the following expression:

$$f_{ij}^* = f_{ij}^{l*} + f_{ij}^{r*} \overset{\text{Lem. 4}}{\geqslant} \begin{cases} 0 & (i,j) \in A_b \\ \tilde{\theta}_j^l & (i,j) \in A_{sb} \setminus A_b \\ \tilde{\theta}_j & (i,j) \in A \setminus A_{sb} \end{cases} \tag{19}$$

The switch allocation problem implies the minimization of the interruption time flows. Therefore, optimal flow values must hold expression (19) in equality. ■

7 Computational studies

The switch allocation problem was solved for a set of benchmark networks (Kavasseri and Ababei 2013), whose attributes are described in Table 3. The MILP model was loaded in Gurobi 5.5 and solved on an Intel i7 3930k with 16 GB of RAM, and Ubuntu 12.04 as operating system.

Table 3: Benchmark networks attributes.

network	feeders	$\lvert V \rvert$	$\lvert E \rvert$	$\tilde{l}_0$
R3	1	34	33	3,708.27
R4	11	95	94	28,342.96
R5	8	144	143	18,315.82
R6	3	205	204	27,571.37
R7	7	881	880	124,920.01

Each network was tested five times by fixing $N = [5, 10, 15, 20, 25]$. The ENS evaluations (E_N) and execution times (t_N) are shown in Table 4. The ENS lowerbound (E_{lb}) and upperbound (E_{ub})

were determined using Eq. (13). Optimal solutions were obtained for all instances within small execution times (<1 second) for most cases. The largest network R7 was solved in under 8 minutes, which is adequate considering the instance size (881 nodes and 25 switches). Formulating the switch allocation problem as a network flow problem contributed to the solution time efficiency.

It is worth noticing that the incremental reliability benefit of switches is reduced as more switches are allocated. This behavior was observed on all networks of the study. For example, allocating 5 switches to network R7 reduced the ENS by 35.02%. When 5 more switches are allocated, the ENS drops to an additional 16.89%, which is less than half of the first reduction.

Considering now a greenfield scenario, in which a planner must decide on economic grounds the best number and locations of switches on an empty network. The annual investment to acquire and maintain a sectionalizer and the cost of interrupted energy are estimated in $C_s = \text{US}\$1,358.00/year$ and $C_e = \text{US}\$1.53/kWh$, respectively (Brown 2008). Figure 3 shows the ENS cost savings ($C_e \cdot (E_{ub} - E_N)$) and the switch investment ($C_s \cdot N$) for solutions of network R5 containing $N = [0, \ldots, 100]$ switches. Subtracting the switch investment from the ENS cost savings gives the annual returns promoted by the allocation of switches. For network R5, solutions with up to 83 switches have positive returns, which means the reliability investment pays itself in those cases. The economically optimum solution is to allocate 21 switches, giving a maximum return of $\text{US}\$66,247.41/year$.

Table 4: Results summary of the computational studies.

network	E_{ub}	E_5 (ΔE_5)	t_5	E_{10} (ΔE_{10})	t_{10}	E_{15} (ΔE_{15})
R3	11,135.23	3,031.78 (-89.39)	0.01	2,372.86 (-96.66)	0.02	2,213.58 (-98.42)
R4	4,242.33	2,898.94 (-70.63)	0.02	2,691.75 (-81.52)	0.01	2,562.16 (-88.34)
R5	14,110.97	9,156.78 (-47.80)	0.01	7,468.47 (-64.09)	0.01	6,418.37 (-74.23)
R6	6,932.57	4,640.53 (-41.71)	0.03	3,388.69 (-64.49)	0.05	2,730.54 (-76.47)
R7	1,518,308.94	1,079,841.46 (-35.02)	0.34	868,343.89 (-51.91)	0.60	753,147.41 (-61.11)

network	t_{15}	E_{20} (ΔE_{20})	t_{20}	E_{25} (ΔE_{25})	t_{25}	E_{lb}
R3	0.01	2,132.2 (-99.31)	0.02	2,095.18 (-99.72)	0.03	2,069.97
R4	0.03	2,476.32 (-92.85)	0.02	2,426.22 (-95.48)	0.01	2,340.32
R5	0.03	5,618.02 (-81.95)	0.04	5,056.61 (-87.37)	0.04	3,747.42
R6	0.19	2,328.35 (-83.79)	0.16	2,038.52 (-89.06)	0.19	1,437.63
R7	5.37	680,437.23 (-66.92)	63.81	628,099.53 (-71.10)	455.34	266,293.63

E_{lb} – ENS lowerbound (kWh/year)
E_{ub} – ENS upperbound (kWh/year)
E_N – ENS evaluation of solution with N switches (kWh/year)
$\Delta E_N = 100 \cdot (E_{ub} - E_N)/(E_{ub} - E_{lb})$ – relative ENS reduction (%) of solution with N switches (kWh/year)
t_N – execution time (seconds) to obtain solution with N switches

8 Final remarks

The reliability analysis of power distribution systems typically consists on computing the expected interruption frequency, interruption duration and energy not supplied of each customer. This data can be transformed into reliability information through the use of indices that represent the reliability for a group of customers, such as SAIFI (system average interruption frequency index), SAIDI (system average interruption duration index) and ENS (energy not supplied).

A new evaluation methodology based on interruption time flows was proposed to compute reliability indices for distribution networks. The approach applicability was shown with numerical examples from literature and as background of a mixed integer linear model for the switch allocation problem. In this problem, a fixed number of switches must be installed on a network to obtain the minimum ENS. Case studies were performed using a benchmark set of distribution networks. Optimal solutions were obtained within suitable execution time for all scenarios varying the number of switches. The proposed model brings a new line of investigation for the switch allocation problem. For example, the addition of strong valid-inequalities may improve the solution time considerably.

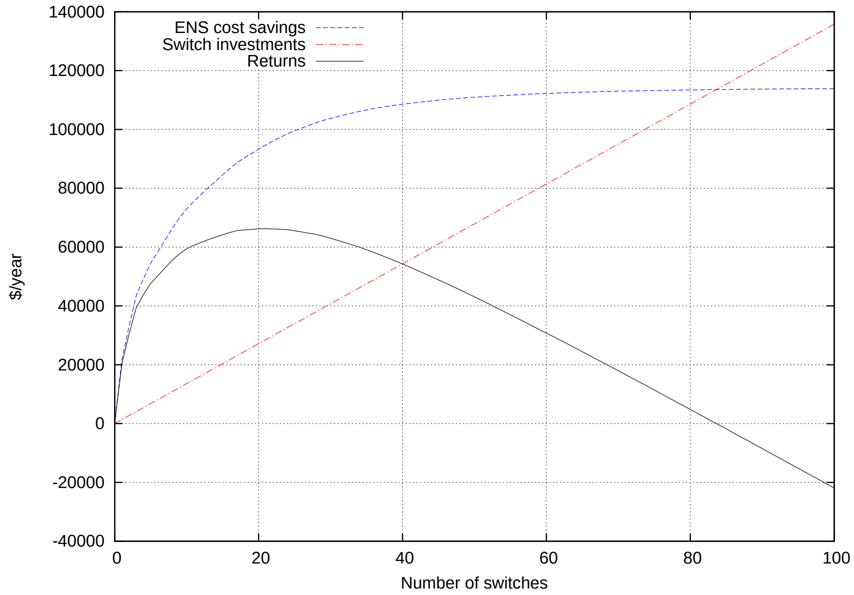


Figure 3: Cost and returns related to switch allocation on network R5.

The interruption time flows give a new perspective to assess the reliability state of a distribution system. Managerial decisions regarding investments on new devices and defining maintenance and operating policies would be better informed if the network mapping of interruption time flows is available. Moreover, the proposed evaluation methodology has been proven a viable and efficient approach to be implemented on heuristic and exact techniques for reliability optimization problems.

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