

MATHEMATICAL MODELING APPROACH FOR OPTIMIZING THE PLANNING OF AGRICULTURAL AND INDUSTRIAL STAGES IN THE PROCESSING TOMATO SECTOR

Cleber Damião Rocco

Federal University of Sao Carlos
Rod. Washington Luís, km 235 - SP 310 – Sao Carlos – Brazil
cdrocco@gmail.com

Bernardo Almada-Lobo

University of Porto
Rua Doutor Roberto Frias s/n, Porto - Portugal
balobo@fe.up.pt

Reinaldo Morabito

Federal University of Sao Carlos
Rod. Washington Luís, km 235 - SP 310 – Sao Carlos – Brazil
morabito@ufscar.br

ABSTRACT

This study presents a mathematical modeling approach to represent and optimize some of the main decisions in the processing tomato sector. Agricultural and industrial activities have many interconnected and costs based decisions which are organized as stages. Agricultural decisions comprise to choose when, where and how much tomato should be produced and transported to processing plants; while industrial decisions include managing the capacity allocation of plants as well as the logistic tasks, such as transportation and inventories of raw materials and final products. Field research revealed that the processing tomato companies in Brazil do not use any cost optimization tool to plan these activities. Instead they just follow existing traditional industry practice and document the results. The modeling approach is tested by using real data from one large tomato company and the outcomes are promising. The optimization approach provides information regarding potential process bottlenecks since the agricultural and industrial stages are jointly modeled by linear programming where the CPLEX/GAMS is employed to solve the model.

KEYWORDS: linear programming, mathematical programming, food industry.

Main area: OR in Industry, OR in Administration & Production Management, Mathematical Programming.

1. Introduction

The tomato processing sector is one of the most important agricultural and food processing activities in several countries. Tomatoes are the second most produced vegetable after potatoes in the world, excluding cereals and fibers (FAOSTAT, 2013). According to data from the World Processing Tomato Council (WPTC), approximately 85% of all industrial tomato is produced in the Northern Hemisphere, where the major producers are United States, China and Italy. In the Southern Hemisphere, Brazil is the leading producer, followed by Chile which has around half of the total Brazilian production. Worldwide Brazil is the 7th largest producer (WPTC, 2013).

In the academic environment some researches have been done in several directions to improve the overall understanding of this sector. Schultz et al. (1971), Miers et al. (1971), Neumann et al. (1978) and Marsh et al. (1978) were pioneers in describing the technological aspects of the industrial processing. Later the concentration of tomato juice by water evaporation in multiple effect concentrators attracted the attention of researches due to its high energy consumption and modification in tomato organoleptic properties and it seems that this attraction has remained until the present day. Dale et al. (1982), Choi and Okos (1983), Rumsey et al. (1984), Runyon et al. (1991), Hayes et al. (1998), Miranda and Simpson (2005), Simpson et al. (2008) with their studies related to energy consumption, thermal and quality properties of tomato juice concentration and optimal operation of multiple effect evaporators in tomato industry.

The economic aspect of the sector motivated Plummer (1999) to prepare an economic modeling study of the United States processing tomato industry. The challenge of how to manage the tomato industrial plant was addressed by Thompson et al. (2001) by using simulation techniques. In Brazil, Melo and Vilela (2004, 2005) described the performance of the Brazilian industrial tomato sector along the 90s and discussed the future perspective. More recently Engindeniz (2006) did an economic analysis of pesticide use on the processing tomato sector in Turkey, Arazuri et al. (2007) studied the influence of mechanical harvest on the physical properties of tomatoes and Rocco and Morabito (2012) approached the operations management of steam production in the tomato industry.

As noticed, some research effort has been done in several directions. However, to the best of our knowledge there is not any study which modeled the processing tomato sector from its agricultural fields to its processing operations as a single system composed of different stages aiming to fully optimize them. This study presents a mathematical modeling approach to represent and optimize the agricultural and industrial stages simultaneously in the processing tomato sector, considering also multi industrial plants.

A two-stage production process appears in this industry. The agricultural stage should be understood as the part of the production chain in charge of producing tomato fruits and delivering them to the processing plants. The industrial stage is the part of the production chain responsible for the transformation of tomato fruits in their final products to consumers. The model was conceived to support tactical decisions (monthly decisions) along a planning horizon of over one year.

2. Problem description

The current practice of planning activities in the tomato processing sector is specific for each company and it is not well known and diffused in the literature. Two large tomato processing

companies were visited in Brazil and their current practice of agricultural and industrial planning activities were documented for this study. It can be said that these two companies have their own management information system, such as SAP and DATASUL (Totvs), and these systems are used by several departments of the company, sometimes not completely well integrated. Nevertheless the agricultural and industrial departments do not employ these systems. Commonly these systems are more useful for monitoring business costs and accomplished tasks of some departments, such as purchasing, sales, equipment maintenance, without any capability for system planning or optimizing.

Aiming to a better understanding of the tomato processing sector, we look at agricultural and industrial activities as stages. The first one is the 'agricultural stage', which consists of producing tomato fruit in agricultural fields, harvesting and transportation to industrial plants. The second stage is the 'industrial stage', which can also be split in two stages. The 'first industrial stage' consists of producing concentrated raw materials from tomato juice (crushed tomato fruit), in which water is evaporated by simple or multiple effect concentrators. The 'second industrial stage' is dedicated to the production of final products; in which concentrated raw materials produced at the 1st industrial stage are used as inputs for the 2nd industrial stage which produces to final consumers.

Usually agribusiness companies, such as the processing tomato, maize, soybean sectors, have their own agricultural and industrial teams. These two teams have very close communication and periodic meetings are held to plan and monitor the crop planting and later the harvesting. The management information systems cited before are not useful for them in planning the agricultural and industrial operations, which are planned and monitored by means of spreadsheets like Microsoft Excel. The target of the agricultural team is to supply high quality tomatoes to industrial plants at its maximum level capacities as long as possible throughout the year. The goal of the industrial team is to process all delivered tomato aiming to meet the current and also the whole year consumers' demand. Basically the agricultural stage pushes the industrial stage while the latter is pulled by final consumers.

The tomato production is seasonal and interesting in the fact that the harvest season in the Northern and Southern Hemispheres takes place almost in the same time period due to climate conditions, approximately from the end of June to the beginning of November. An industrial plant can process thousands of tons of tomatoes each day, which implies that the plant receives hundreds of trucks from different tomato producing regions with tomatoes of distinguishable levels of quality. Some agricultural parameters are especially important for the quality of final products

The mass of soluble solids measured in percentage, known also as 'brix degree', reflects the maturation level, which is also observed on the tomato color. High agricultural yield is fundamental for business profitability. Tomato brix and yield are parameters that change along the cultivation time and are directly influenced by climate. Usually high brix degree and high yields are observed in the middle of the harvesting season. Figure 1 shows an illustrative example of brix and yield parameters using figures from the Brazilian context.

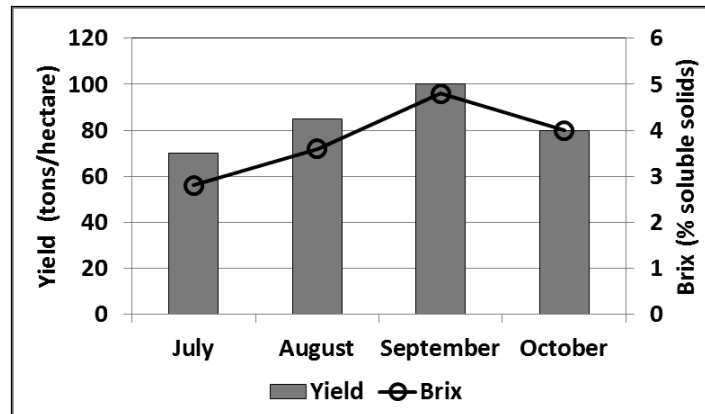


Figure 1. Example of tomato agricultural parameters.

Rainy conditions determine the agricultural fieldwork activities since it is not possible to perform the crop sowing and harvesting under high soil moisture. During the sowing season, rains mean failures of fieldworks and consequently lack of tomatoes three months later. Otherwise if climate conditions are good, the agricultural team often performs more tomato planting as previously programmed and this afterwards means an excess of tomatoes in the industry. This is a typical conflict between agricultural and industrial stages which is monitored by using spreadsheets.

Every year the industrial stage faces difficult decisions for planning its activity related to concentrated raw material production since there are several types of concentrated raw materials which should be produced. Tomato season takes around four months while consumers' demand is all year round. Since agribusiness companies often have several industrial plants wherein each one has its own features related capacities and costs, another hard decision is to choose where the production should take place. Figure 2 shows a scheme of stages with some of main questions that are often present in the processing tomato industry. It is important to remember that agricultural and industrial stages are deeply attached and the decisions in one stage affect directly the other.

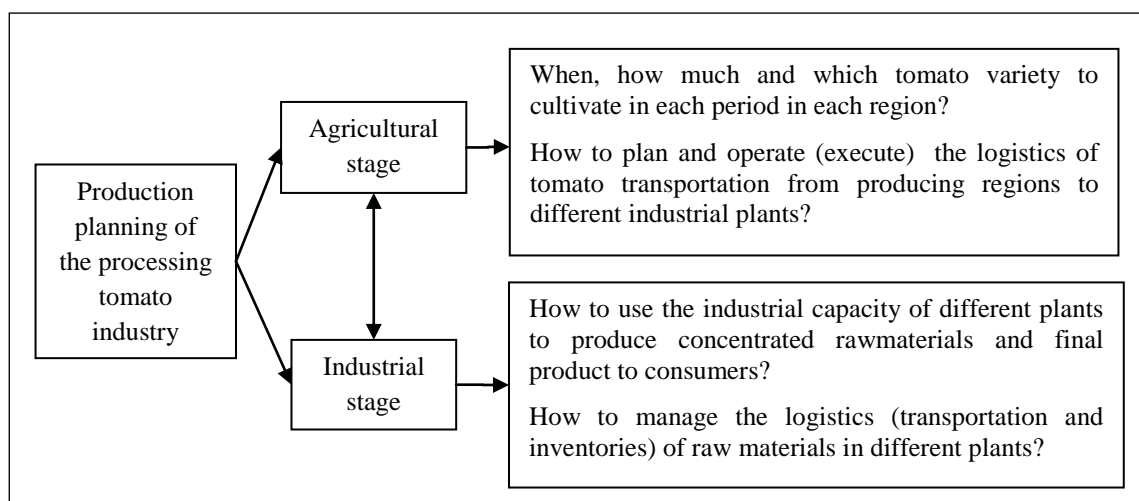


Figure 2. Scheme of stages and some questions faced by the processing tomato sector.

3. Modeling approach

Before presenting the mathematical modeling approach it is important to have a clear understanding of which decisions we want to support and how the whole system looks like. Figure 3 shows an example of a system representing the processing tomato industry in Brazil. From the left to the right side of the figure first appear several producing regions that are available to cultivate tomato of different varieties. ‘Harvest scheduling’ decision variables indicate the tomato area harvested to supply the industry over time. ‘Tomato transportation’ variables perform the logistics of transporting from tomato fields to industrial plants. All tomatoes (fruit) delivered to the industry are converted into concentrated raw materials (pulp and crushes) which are then consumed to produce final products or stored to be consumed in the intercrop period. Essentially the difference between tomato pulp and crush is that the first one is a type of paste with soluble solids concentration usually above 18% and does not contain tomato seeds, while crush has more tomato fragments containing tomato seeds and lower brix around 16%.

Concentrated raw materials are produced and transported among processing plants and also eventually purchased on the market. Final products demand is throughout the year and should be met by instant production or by stocks. Some products use only pulp in their composition while others, for instance sauces, uses pulp and crush. The agricultural stage and the first industrial stage may be seen as a problem of lot sizing while the second industrial stage is a typical blending problem.

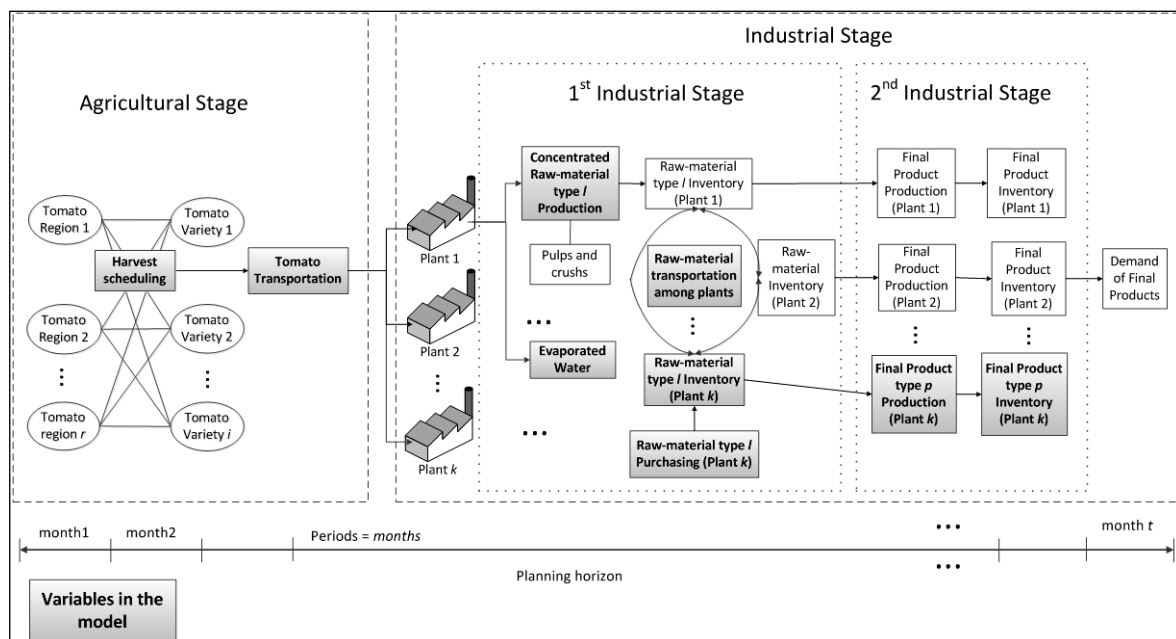


Figure 3. Modeling design of the tomato processing sector.

The mathematical formulation which is presented below was implemented in the General Algebraic Modeling System (GAMS) and solved by the CPLEX 12.4 using default settings.

Sets:

- t : Discrete time periods (months);
 i : Discrete tomato varieties (variety A, variety B etc.);
 r : Regions of tomato production (region A, region B etc.);
 k, kd : Industrial plants (industry A, industry B etc.);
 l : Types of concentrated rawmaterials (pulp18, pulp20, pulp30, crush16, crush20 etc.);
 $g(l)$: Group of rawmaterials – subset of set l (pulp, crush);
 p : Type of final products (ketchup, sauce, extract etc.);
 $j(p)$: Set of plants that can produce product p .

Parameters:

- β_k : Capacity of industry k to receive tomatoes (tons/month);
 γ_r : Total area available in region r (hectares);
 $\delta_{i,r,t}$: Tomato yield of variety i in region r in period t (tons/hectare);
 $\varepsilon_{i,r,t}$: Percentage of soluble solids (brix) of tomato variety i in region r in period t (%);
 $\eta_{r,k}$: Transportation cost of tomatoes from region r to industrial plant k (\$/ton);
 θ_l : Percentage of soluble solids (brix) of concentrated raw material type l (%);
 μ_l : Price of raw material type l purchased on the market (\$/ton);
 $\kappa_{k,kd}$: Shipping cost of raw materials from industrial plant k to another kd (\$/ton);
 λ_l : Unity inventory cost for raw material type l (\$/ton);
 t_p : Unity inventory cost for final product type p (\$/ton);
 $\nu_{g,p}$: Percentage of soluble solids from raw material group g into product p (%);
 π_k : Capacity of water evaporation in industrial plant k (tons/month);
 $d_{p,t}$: Demand of final product p in period t (tons).

Decision variables:

- Z : Total logistic costs in the system along the planning horizon (\$);
 $H_{i,r,k,t}^{tom}$: Area of tomato variety i in region r destined to industrial plant k in period t (hectares);
 $T_{i,r,k,t}^{tom}$: Tomato variety i transported from region r to industrial plant k in period t (ton);
 $M_{k,l,t}^{raw}$: Raw material type l purchased on the market by industrial plant k in period t (tons);
 $T_{k,kd,l,t}^{raw}$: Raw material type l transported from plant k to another plant kd in period t (tons);

$I_{k,l,t}^{raw}$: Inventory of raw material type l in plant k in period t (tons);

$P_{k,l,t}^{raw}$: Production of concentrated raw material type l in plant k in period t (tons);

$C_{k,l,p,t}^{raw}$: Raw material type l consumed to produce product type p in plant k in period t (tons);

$U_{k,t}^{tom}$: Total of soluble solids from tomatoes in plant k in period t (tons);

$W_{k,t}^{tom}$: Total of water from tomatoes in plant k in period t (tons);

$U_{k,l,t}^{raw}$: Total of soluble solids in concentrated raw material type l in period t (tons);

$W_{k,l,t}^{raw}$: Total of water contained in concentrated raw material type l in period t (tons);

$W_{k,t}^{evap}$: Total evaporated water in plant k in period t (tons);

$I_{k,p,t}^{prod}$: Inventory of product type p in plant k in period t (tons);

$P_{k,p,t}^{prod}$: Production of product type p in plant k in period t (tons);

$PG_{k,p,g,t}^{prod}$: Fraction of production of product type p using raw materials from group g in plant k in period t (tons);

$C_{k,p,t}^{prod}$: Consumption of product type p from plant k in period t to meet the demand (tons).

Objective function:

$$Z = \sum_{i,r,k,t} \eta_{r,k} T_{i,r,k,t}^{tom} + \sum_{k,l,t} \mu_l M_{k,l,t}^{raw} + \sum_{k,kd,l,t} \kappa_{k,kd} T_{k,kd,l,t}^{raw} + \sum_{k,l,t} \lambda_l I_{k,l,t}^{raw} + \sum_{k,p,t} l_p I_{k,p,t}^{prod} \quad (1)$$

Constraints:

$$\sum_{i,k,t} H_{i,r,k,t}^{tom} \leq \gamma_r \quad \forall r. \quad (2)$$

$$T_{i,r,k,t}^{tom} = H_{i,r,k,t}^{tom} \cdot \delta_{i,r,t} \quad \forall i, r, k, t. \quad (3)$$

$$\sum_{i,r} T_{i,r,k,t}^{tom} \leq \beta_k \quad \forall k, t. \quad (4)$$

$$\sum_{i,r} T_{i,r,k,t}^{tom} \cdot \varepsilon_{i,r,t} = U_{k,t}^{tom} \quad \forall k, t. \quad (5)$$

$$\sum_{i,r} T_{i,r,k,t}^{tom} - U_{k,t}^{tom} = W_{k,t}^{tom} \quad \forall k, t. \quad (6)$$

$$\theta_l \cdot P_{k,l,t}^{raw} = U_{k,l,t}^{raw} \quad \forall k, l, t. \quad (7)$$

$$(1 - \theta_l) \cdot P_{k,l,t}^{raw} = W_{k,l,t}^{raw} \quad \forall k, l, t. \quad (8)$$

$$\sum_l U_{k,l,t}^{raw} = U_{k,t}^{tom} \quad \forall k, t. \quad (9)$$

$$W_{k,t}^{evap} = W_{k,t}^{tom} - \sum_l W_{k,l,t}^{raw} \quad \forall k, t. \quad (10)$$

$$W_{k,t}^{evap} \leq \pi_k \quad \forall k, t. \quad (11)$$

$$I_{k,l,t}^{raw} = I_{k,l,(t-1)}^{raw} - \sum_p C_{k,l,p,t}^{raw} + P_{k,l,t}^{raw} + \sum_{kd} T_{kd,k,l,t}^{raw} - \sum_{kd} T_{k,kd,l,t}^{raw} \quad \forall k | k \neq kd, t. \quad (12)$$

$$\sum_{l|l \in g} \theta_l \cdot C_{k,l,p,t}^{raw} \geq v_{g,p} \cdot PG_{k,p,g,t}^{prod} \quad \forall k \in j(p), p, g, t. \quad (13)$$

$$P_{k,p,t}^{prod} = \sum_g PG_{k,p,g,t}^{prod} \quad \forall k \in j(p), p, t. \quad (14)$$

$$I_{k,p,t}^{prod} = I_{k,p,(t-1)}^{prod} - C_{k,p,t}^{prod} + P_{k,p,t}^{prod} \quad \forall k, p, t. \quad (15)$$

$$\sum_{k \in j(p)} C_{k,p,t}^{prod} \geq d_{p,t} \quad \forall p, t. \quad (16)$$

The objective function (1) minimizes many relevant logistic costs of the system over the planning horizon, i.e: transportation costs of tomatoes from producing fields to industrial plants, purchasing and inventory costs of concentrated raw materials and their transportation among plants, and also the inventory costs of final products. Area limitations in growing tomatoes are modeled by constraints (2) – note that it is possible to have only one harvest along the planning horizon. Constraints (3) calculate the amount of tomatoes produced in each area according to agricultural yields along the periods. Each industrial plant has its own capacity of tomato reception which is represented by constraints (4). In the industrial process all tomato fruit is converted into tomato juice which is essentially water and soluble solids. The amount of soluble solids is captured by constraints (5) while the water is captured by constraints (6). Concentrated raw materials (pulpes and crushes) are produced by means of constraints (7) and (8) – the first deals with soluble solids and the second with water remaining in raw materials. Constraints (9) make the mass balance of soluble solids in each plant over time. Equations (10) calculate the evaporated water in each industry by the difference between the mass of water in tomato juice and the mass of water remaining in concentrated raw materials. Evaporation capacity of each plant is restricted by constraints (11). Concentrated raw materials inventory equations are represented by expressions (12) – note that it is an accepted practise allowed in transportation of raw materials among the industrial plants. Expressions (13) produce fractions of the final products according to the groups of raw materials while expressions (14) assemble these fractions to produce the total mass of final products. Inventory equations of final products are represented by (15). Consumers' demand is met by constraints (16), in which all industrial plants in the system can supply final products to meet the demand.

4. Illustrative example

By using real data from one large processing tomato company in Brazil several model runs were performed to analyze its outcomes. In this illustrative example there are three tomato producing regions where two tomato varieties are available to cultivate, two industrial plants can produce five sorts of concentrated raw materials, in which three are pulps and two are crushes. Planning horizon is

one year divided in months. Harvesting season takes place from July to October and consumers' demand of final products is monthly aggregated throughout the year.

Hereafter, some results are presented graphically from one dynamic equilibrium model, in which the planning horizon is in closed loop, i.e. inventory variables of December are connected to those of the previous January. This approach provides a broad picture of the system along the planning horizon and it is acceptable when resource, technology and price data are constant. Figure 4 shows the distribution of tomato areas to supply the industrial plants. Figure 5 and Figure 6 present information about concentrated raw materials – the first shows the production and the second the inventory. Note that the production occurs from July to October, during the tomato season, and the amount of raw materials is reduced from November to June because there is no harvesting and concentrated raw materials previously stored are consumed to produce final products in these periods. Figure 7 displays the monthly cost of industrial plants. Figure 8 presents the production planning of final products in each plant. In this example, final products to consumers were aggregated in three families (ketchup, extract and sauce) and one additional family is employed to represent the concentrated raw material to be stored for the following season.

As this study has a practical appeal, it is worth to mention some of the model analysis capabilities and advantages to use them. The first contribution that the model usage can bring is a systematic and necessarily high quality data collection; otherwise its outcomes are counterproductive. More directly it is possible to look into the capacities of each industrial process, such as tomato reception, tomato juice concentration, concentrated raw material and final product productions. By this inspection, capacity bottlenecks are easily identified and their costs properly measured. For example: it is possible to know how much the evaporation or the tomato reception costs of each industrial plant are and then to plan facilities investments. Sensitivity analysis may be performed with appropriate variables.

One important depicted result is the true value of resources which is usually very difficult to estimate. The opportunity cost of tomato production areas may be relevant information to the tomato company when they need to set contracts with farmers. By the model application the tomato company is able to analyze investments in new industrial plants or even the closure of others.

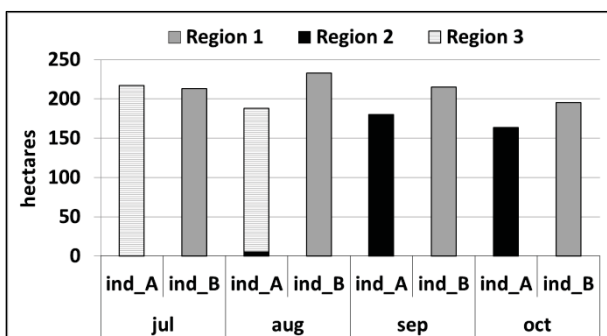


Figure 4. Harvest planning

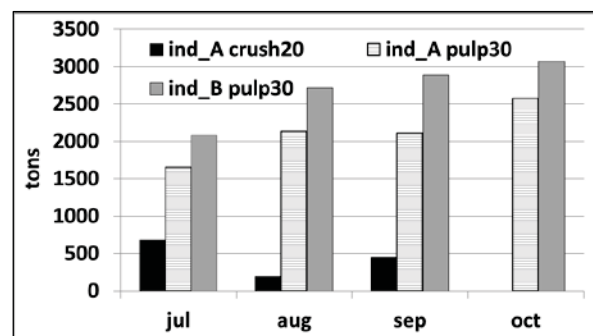


Figure 5. Production of raw materials.

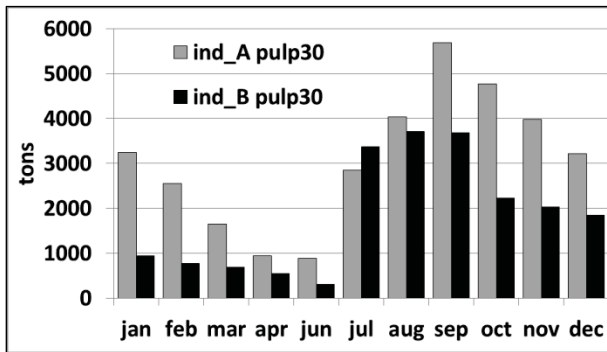


Figure 6. Raw material inventories.

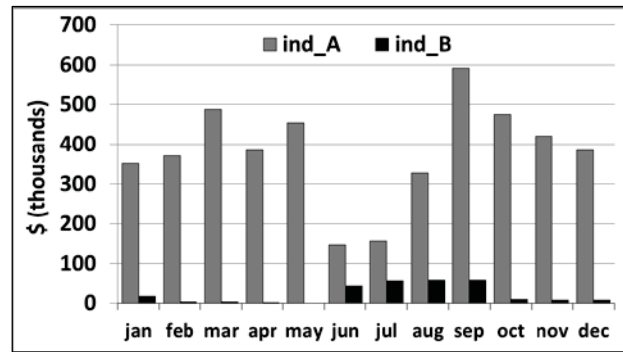


Figure 7. Monthly cost of the system.

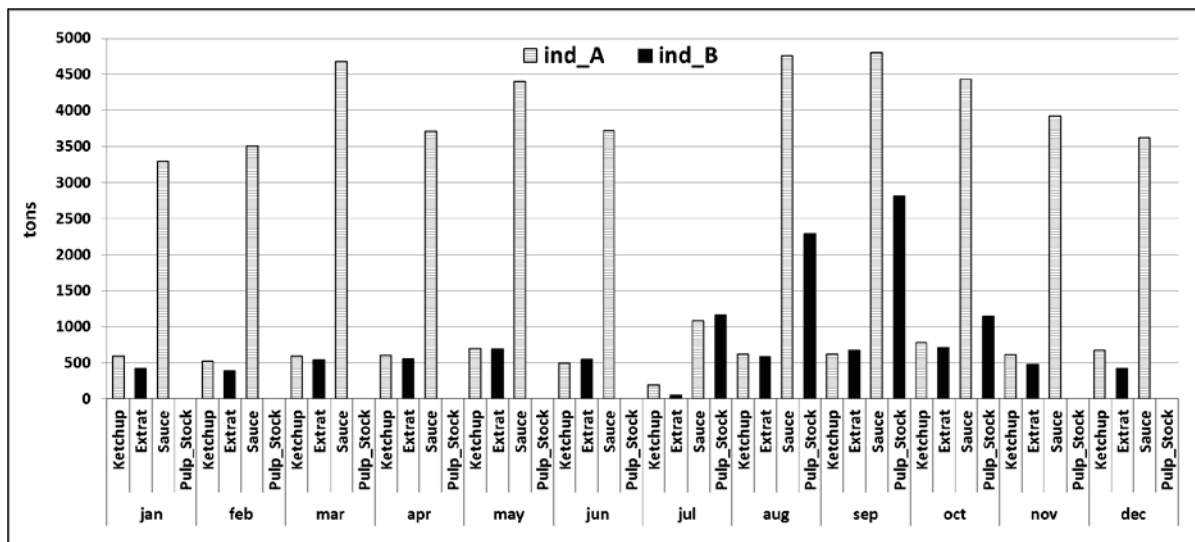


Figure 8. Production planning of final products.

5. Conclusions

The main contribution of this study is to present a mathematical formulation which represents adequately and simultaneously the agricultural and industrial stages of the processing tomato sector. The optimization approach provides information regarding potential process bottlenecks that could appear by changing the system configurations.

Some modeling simplifications and assumptions weremade at the currentresearch status and those are nowadays being worked on. We mention, for instance, the aggregated water evaporation capacity of industrial plants with multiple concentrators, which causes lack of setup and scheduling variables in the model. Another assumption is the production of concentrated raw materials for late consumption in intercrop periods as a model parameter instead of decision variables. This assumption implies, in using those data from previous years or alternatively forecasting them, that

both are not good approaches since the industrial capacity depends upon the product production choice.

Concentrated raw material types, pulps and crushes, and their concentrations of soluble solids are discretized in the model, but in reality it is measured on a continuous scale which depends on the evaporation time duration. Changing the capacity approach from the amount of evaporated water to the time of evaporation process is not straightforward once it depends on the setup time length.

Execution times to find optimal solutions are quite fast since the mathematical formulation is linear. Some slight time increase is observed when time scale is shifted from 'months' to 'weeks' and the 'family of products' is disaggregated in single products. Improvements and extensions of this research are in the authors' agenda.

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