Markov Decision Model for Spectrum Allocation in Elastic Optical Network Links

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ABSTRACT

Elastic Optical Networks (EONs) have been recently proposed to enhance spectrum efficiency when assigning resources to connection requests with heterogeneous bandwidth requirements. In these networks contiguous subcarriers, called slots, are allocated according to traffic demand, where each slot has a lower data rate than a wavelength in traditional Wavelength Division Multiplexing (WDM) optical networks. To establish connections, a route must be chosen and a set of slots must be assigned in every link of this route, a problem called Routing and Spectrum Allocation (RSA). In dynamic traffic scenarios, optical paths must be dynamically established on the arrival of requests, and, if no route with free slots is available, the request is blocked. In this work, we propose a Continuous-Time Markov Decision Process to model and solve the Spectrum Allocation (SA) sub-problem for a link of an EON under dynamic traffic. Given an allocation policy, the model also computes the blocking probabilities of requests.

KEYWORD. Markov Decision Process, Elastic Optical Network, Spectrum Allocation

Main Area. Tel & SI - OR in Telecommunications and Information Systems

1. Introduction

Wavelength Division Multiplexing (WDM) optical networks have been used in the past years to enable transmissions at high bit-rates between its clients, sending their data in different wavelengths in the same optical fiber. Although WDM networks improve resource utilization, they transmit optical signals at fixed line rates (10, 40 or 100 Gbps), generating a drawback due to this rigid granularity.

The continuous growth of high bit rate services combined to traffic demands with different bandwidth requirements make it difficult to efficiently use the available spectrum in a fiber. To address these issues, the concept of Elastic Optical Networks (EONs) has been recently proposed (Jinno et al., 2009). EONs can provide finer granularity capacity for connections by the allocation of contiguous subcarriers, called slots, according to the requested bandwidth, where each slot has a lower data rate than a wavelength.

Given a set of connection requests, an optical path must be established to meet each one in a WDM network. For this purpose, a route must be chosen and a wavelength must be assigned in every link of that route. Finding a good solution to this problem, called Routing and Wavelength Assignment (RWA), may increase the efficiency of the network. An analogous problem arises in EONs, named Routing and Spectrum Allocation (RSA). The latter one, however, is more challenging than the former, since one needs to allocate contiguous slots along each optical path and not just one wavelength. It is also needed to allocate slots for frequency guard-bands between adjacent channels, to prevent overlapping and crosstalk among signals.

Most research regarding RSA has been done for static traffic scenarios, where all connection requests are known in advance. To solve this problem, Integer Linear Programming (ILP) formulations have been proposed (Wang et al., 2011), (Klinkowski and Walkowiak, 2011), as well as heuristic algorithms (Christodoulopoulos et al., 2010). Due to its complexity RSA may be divided into two sub-problems, routing and spectrum allocation, which may be solved sequentially. As an example, Jinno et al. (2010) use Fixed-Alternate routing and then apply First-Fit frequency assignment algorithm.

In dynamic scenarios, on the other hand, optical paths must be dynamically established on the arrival of requests, and if no route with free slots is available the request is blocked. Therefore, dynamic RSA algorithms usually aim to minimize the blocking probability of the network. Heuristics have been proposed to solve it (Wan et al., 2011), (Wang et al., 2012).

In this paper we propose an analytical model to solve the Spectrum Allocation (SA) sub-problem for a link of an EON with dynamic traffic by finding an optimal policy that, based on ongoing connections, determine: I) if a new request will or will not be accepted; II) which slots will be used to accomodate it. Fixed a policy, our model also computes performance measures of the link.

More specifically, the link is modeled as a Continuous-Time Markov Decision Process (CTMDP) based on Carvalho et al. (2012) model, which also propose a CTMDP to find an optimal admission policy. Our model, however, takes into account the slots' positions where each ongoing connection is allocated when a new request arrives, whereas the previous one assumes that the slots can be fully reallocated at any time, and therefore their positions are irrelevant. A comparison between the two models is performed based on their complexity (e.g. state space size) and their performance measures.

The rest of the paper is organized as follows: EONs and the SA problem for dynamic traffic are described in Section 2; the proposed model is shown in Section 3; and the results are discussed in Section 4. Conclusions and future work are presented in Section 5.

2. Elastic Optical Network and Spectrum Allocation

As traditional WDM networks operate within a fixed-size frequency grid, in which the minimum granularity to accomodate traffic demands is a wavelength, even if the required bandwidth of a connection request is not sufficient to fill the capacity of a wavelength, an entire one will be allocated to it. Moreover, if multiple wavelengths are needed to allocated one connection, a guard-band frequency must be allocated between any two adjacent wavelengths to ensure signal quality. These characteristics may lead to inefficient resource utilization.

In EONs, heterogeneous traffic demands can be efficiently accommodated by dividing the optical frequency spectrum into multiple frequency slots, which have a finer granularity than a single wavelength. Each connection is assigned to one or more slots, and, if multiple slots are required, they need to be contiguous. Furthermore, in contrast to WDM networks, frequency guard-bands between two adjacent optical paths (i. e. sets of slots) are not fixed and their size may be in the order of one or multiple slots (Wang et al., 2011).

An example of a WDM fixed-grid is shown in figure 1, where each wavelength is able to accomodate up to 100 Gbps connections. As one may note, the same traffic demands may be established with fewer resources using a flexible grid.

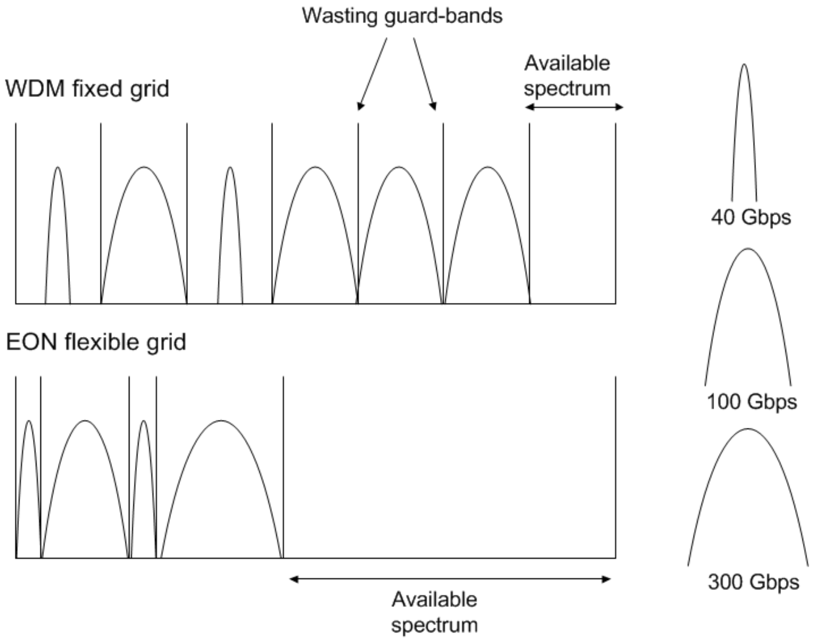

Figure 1. Heterogeneous traffic demands in WDM networks and EONs (adapted from Gerstel et al. (2012)).

The required bandwidth for a given connection in EONs can be understood as a requested number of slots between a source and a destination node (Pagès et al., 2012). In the example shown by figure 2, the spectrum is divided into 10 slots and there are three types of connections which require 1, 2 or 3 slots, respectively. Furthermore, one slot is assigned for each guard-band.

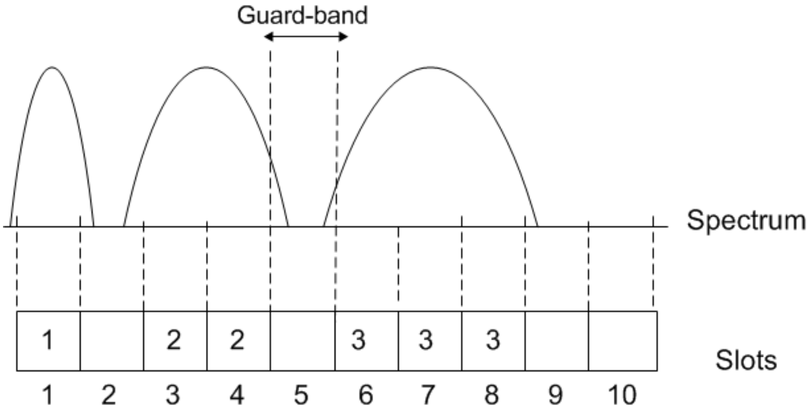

Figure 2. Division of the optical spectrum into ten slots.

In this context, given a route of an optical path, the problem of assigning contiguous slots in each link of that route, named Spectrum Allocation, is one of the main challenges in EONs (Gerstel et al., 2012).

3. Proposed Model

Consider an EON with $K \in \mathbb{N}^*$ different types of connection requests, where each type k, $1 \leq k \leq K$, has the following properties:

- w_k: number of required contiguous slots to accomodate it;
- λ_k: arrival rate according to a Poisson distribution;
- $1/\mu_k$: mean transmission time of a connection currently in the link according to an exponential distribution.

It is assumed that the optical spectrum is divided into $N \in \mathbb{N}^*$ slots and $w_k \leq w_{k+1}$ for $1 \leq k \leq K-1$, where $w_k \in \mathbb{N}^*$. Furthermore, $g \in \mathbb{N}$ slots are assigned for guard-bands.

When a k-type connection request arrives in a node, as shown in figure 3, one must decide if the request will be rejected or accepted; and, if accepted, which slots will be allocated to transmit it. It is worth noting that in our model it is considered that network nodes have wavelength conversion capabilities, and, as a consequence, an optical path with one or multiple slots does not need to use the same slots at each link of its route.

A Continuous-Time Markov Decision Process is proposed to model this problem, whose solution is an optimal allocation policy from which the decisions are taken. For each state s of the state space S the policy chooses an action a from the set of possible actions $A(s)$ in order to maximize the long-run average reward.

Each state $s \in S$ is composed by a charge c and an event ev. A charge describes the slot grid as n pairs, $n \in \mathbb{N}$, where each pair (c_i, k_i) indicates, respectively, the position of the first slot allocated for the *i-th* connection and its type. An example is shown in table 1, in which 0 represents an empty slot and k a slot used by a k-type call. In this example there is a 5-slot grid and two types of connections ($k = 1$ and $k = 2$) with $w_1 = 1$, $w_2 = 2$ and $g = 1$.

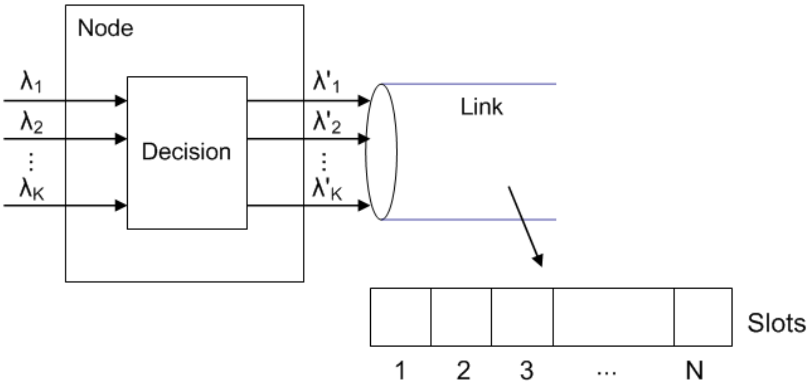

Figure 3. Model for spectrum allocation in a link.

Table 1. Examples of charges in a 5-slot grid

Slot Grid	# of connections	Pairs
$(0\ \ 0\ \ 0\ \ 0\ \ 0)$	0	$\{\}$
$(0\ \ 2\ \ 2\ \ 0\ \ 0)$	1	$\{(2,2)\}$
$(1\ \ 0\ \ 1\ \ 0\ \ 1)$	3	$\{(1,1),(3,1),(5,1)\}$

The charge set C, containing all feasible charges, is defined as

$$\begin{aligned} C = \{\ (n,(c_1,k_1),...,(c_n,k_n)) \mid\ & n \in \mathbb{N}, \\ & 1 \le k_i \le K \text{ for } 1 \le i \le n, \\ & c_1 \ge 1,\ c_n \le N - w_{k_n}, \\ & c_{i+1} \ge c_i + w_{k_i} + g \text{ for } 1 \le i \le n\ \ \}. \end{aligned}$$

An event ev comprises a type-value pair (ev_t, ev_v), where ev_t indicates whether the last event was a request arrival (IN) or a connection termination (OUT). Given $ev_t = IN$, ev_v is the type of the request which arrived, i.e. the number of slots it requires; if $ev_t = OUT$, ev_v represents the *i-th* call in progress. Thus, the event set E is given by

$$\begin{aligned} E = \{\ (ev_t, ev_v) \mid\ & ev_t \in \{IN, OUT\}, \\ & \text{if } ev_t = IN \text{ then } ev_v \in \{1,...,K\}, \\ & \text{if } ev_t = OUT \text{ then } ev_v \in \{1,...,M\}\ \}, \end{aligned}$$

where M, the maximum number of current connections, is equal to the maximum number of $k = 1$ calls the fiber can hold added to its guard-bands. Therefore, M is the maximum value for which the following inequality holds

$$M(w_1 + g) \le N + 1.$$

Consider the function $\phi_s(p)$,

$$\phi_s(p) = \begin{cases} N - p + 1 & if\ n = 0\ \vee\ (n > 0\ \wedge\ p > c_n + w_{k_n} + g - 1) \\ c_1 - g - p & if\ n > 0\ \wedge\ p < s_1 - g \\ c_i - g - p & if\ n > 0\ \wedge\ \exists i \in \{2,...,N\} \\ & \text{such that } c_{i-1} + w_{k_{i-1}} + g \le p < c_i - g \\ 0 & \text{otherwise} \end{cases}$$

which returns the number of contiguous available slots from position p at state s. The set of all possible positions where a k-type request may be allocated at state s is

$$\psi_s(k) = \{p \in \{1, 2, ..., N\} |\ \phi_s(p) \geq w_k\}\ .$$

The state space S is defined as

$$\begin{aligned} S = \{\ (c, ev) \mid c \in C, ev \in E,& \\ &\text{if } ev_t = IN\ \wedge\ ev_v = k \text{ then } \#\psi_s(k) > 0,\ 1 \leq k \leq K \\ &\text{if } ev_t = OUT\ \wedge\ ev_v = i \text{ then } n > 0,\ 1 \leq i \leq n \qquad \}, \end{aligned}$$

in which there should be at least i calls in the system for an OUT event to occur. Moreover, a k-type call arrival is considered only when there is one or more sets of contiguous slots which can accomodate it.

The system, by hypothesis, is observed continuously in time and a decision must be made after the occurrence of an event. If the event is an arrival, $ev_t = IN$, the request may be rejected (REJ) or accepted (ACC_p) in any available position p. No action (NOA) is taken when there is a connection termination ($ev_t = OUT$). Therefore, the action set for a state $s \in S$ is

$$A(s) = \begin{cases} \{ACC_p, REJ\} & if\ ev_t = IN,\ \ \forall\, p \in \psi_s(k) \\ \{NOA\} & if\ ev_t = OUT \end{cases}$$

When an action $a \in A(s)$ is chosen, state $s = (c, ev)$ assumes a post-decision charge $c' \in C$ where it remains until the arrival of a new event. This new charge varies according to tables 2 and 3. If a k-type connection is accepted at position p, w_k contiguous slots starting from p are allocated to accomodate it; if a connection is terminated its slots become empty; and if it is rejected the charge remains unchanged.

Table 2. Post-decision charges for $ev = IN$

Event	Condition	Decision	New charge c'
(IN, k)	$\#\psi_s(k) > 0$	$ACC_p,\ p < c_1$	$(n+1, (p, k), (c_1, k_1), ..., (c_n, k_n))$
		$ACC_p,\ c_i < p < c_{i+1}$	$\begin{pmatrix} n+1, (c_1, k_1), ..., (c_i, k_i), (p, k), \\ (c_{i+1}, k_{i+1}), ..., (c_n, k_n) \end{pmatrix}$
		$ACC_p,\ p > c_n$	$(n+1, (c_1, k_1), ..., (c_n, k_n), (p, k))$
		REJ	$(n, (c_1, k_1), ..., (c_n, k_n))$

Table 3. Post-decision charges for $ev = OUT$

Event	Condition	Decision	New charge c'
(OUT, i)	$i = 1$	NOA	$(n-1, (c_2, k_2), ..., (c_n, k_n))$
	$1 < i < n$		$\begin{pmatrix} n-1, (c_1, k_1), ..., (c_{i-1}, k_{i-1}), \\ (c_{i+1}, k_{i+1}), ..., (c_n, k_n) \end{pmatrix}$
	$i = n$		$(n-1, (c_1, k_1), ..., (c_{n-1}, k_{n-1}))$

Given the post-decision charge, transition rates to the next state s' depends on the occurence of the next event (table 4). When a k-type request arrives and there are enough contiguous empty slots to accomodate it, the transition rate to the next state $s' = (c', (IN, k)$ is λ_k. If a connection is terminated, the transition rate to $s' = (c', (OUT, i))$ is μ_{k_i}. Hence, non-zero transition rates between any two states $s \in S$ and $s' \in S$ given an action $a \in A(s)$, $q_{ss'}(a)$, can be computed from tables 2, 3 and 4.

Table 4. Transition rates

Condition	New Event	Rate	New state
$\#\psi_s(k) > 0$	(IN, k)	λ_k	$(c', (IN, k))$
$1 \leq i \leq n$	(OUT, i)	μ_{k_i}	$(c', (OUT, i))$

Lastly, the total expected reward $r(s, a)$ when action $a \in A(s)$ is taken at state $s \in S$ is computed based on two measures:

- r_k: reward rate for each k-type current connection;
- f_k: fixed reward for each k-type connection already transmitted.

Therefore,

$$r(s, a) = \tau(s, a) \sum_{i=1}^{n} (r_{k_i} + f_{k_i}\mu_{k_i}) \ ,$$

where $\tau(s, a)$ is the expected time until the next decision epoch, given by

$$\tau(s, a) = \frac{1}{\sum\limits_{s' \in S} q_{ss'}(a)} \ .$$

3.1. Performance Measures

Given a CTMDP and fixed a policy, it is possible to compute the limiting probability distribution π, which may be interpreted as the fraction of time the system is in each state (Tijms, 2003). Performance measures can be obtained from these probabilities. In this work, two measures were considered: throughput of the link, i.e. its capacity to process and transmit data; and blocking probabilities for each type of connection.

Let n_{sk} be the number of k-type current connections in a state $s \in S$,

$$n_{sk} = \sum_{i=1}^{n} 1_{\{k\}}(k_i).$$

The throughput of each type of connection is then computed based on its transmission rate μ_k, and the limiting probability π_s, where

$$T_k = \sum_{s \in S} n_{sk} \, \mu_k \, \pi_s.$$

Furthermore, the blocking probability PB_k is defined as

$$PB_k = 1 - \frac{T_k}{\lambda_k}.$$

4. Numerical Results

In order to obtain numerical results the model was implemented in C++ using ModEsto library, developed at LAC/INPE. Value Iteration Algorithm was used to solve the CTMDP and the limiting probabilities were computed by Successive Over-Relaxation (SOR) (Tijms, 2003). The following experiments were performed in a AMD Turion X2 Mobile TL-60 processor with 3GB of RAM.

Initially, a comparison between the model proposed by Carvalho et al. (2012), which we call model 1, and the one proposed in this work, model 2, regarding their state space size as the number of slots increase is shown in figure 4. The following parameters were set: 1 slot for guard-bands ($g = 1$); 3 types of connection requests which required, respectively, 1, 2 and 3 slots; and arrival (λ_k) and transmission (μ_k) rates at 5 units per time for all types of requests.

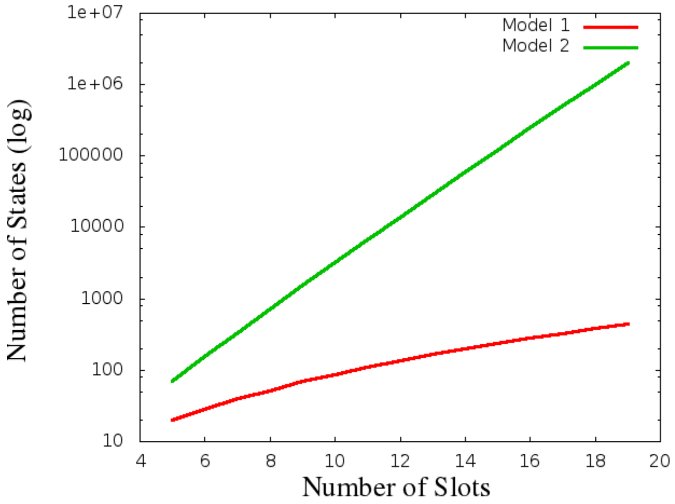

Figure 4. Number of states as the number of slots increases.

One may note that model 2 has a greater number of states, and therefore, it is harder to find an optimal policy. As a consequence, we were only able to solve it for up to 19 slots, which is a small grid for a real EON link.

Two objective functions were then tested. The first one maximizes the throughput of the link weighting equally all types of connections, no matter how many slots they use. This is done fixing f_k, $\forall k \in K$, as 1 and r_k as 0. The second objective maximizes utilization of resources, setting r_k as the number of slots used by a k-type connection and f_k as 0. Link parameters are summarized in table 5.

Table 5. Link parameters

k	w_k	λ_k	μ_k
1	1	from 10 to 50	5
2	2	from 10 to 50	5
3	3	from 10 to 50	5

For the first objective function, the throughput and the blocking probability of the link, which are, respectively, the sum of all k-type connection throughputs and probabili-

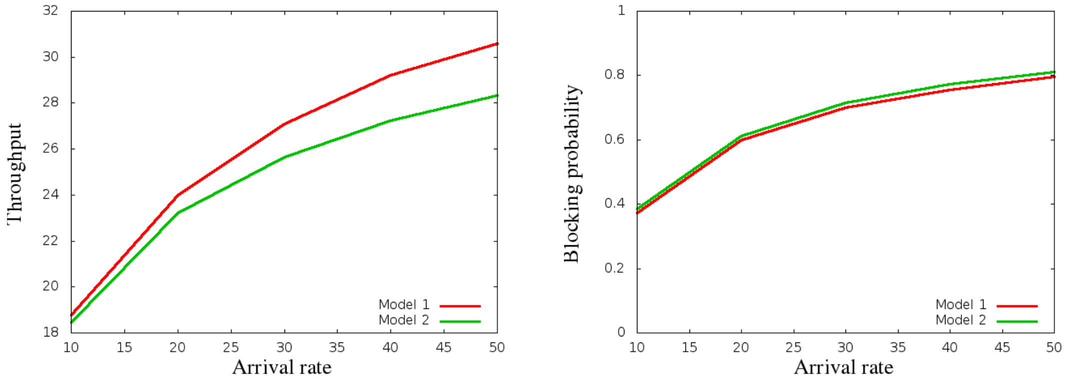

Figure 5. Objective function 1: throughput and blocking probability as arrival rates increase

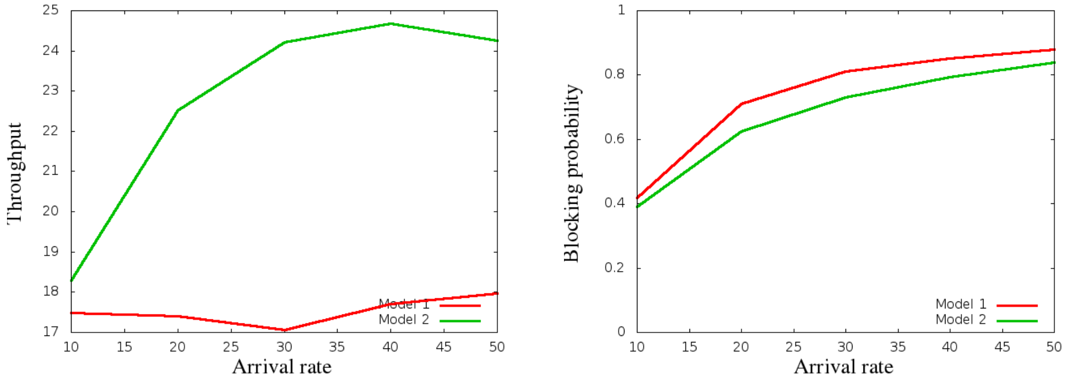

Figure 6. Objective function 2: throughput and blocking probability as arrival rates increase

ties, are shown in figure 5. As model 2 takes into account slots positions, its throughput is lower and its blocking probability is higher than model 1. This is due to grid‘s fragmentation, that occurs as connections are allocated or removed from it in a dynamic scenario. Some solutions have been proposed to measure and solve the fragmentation problem (Rosa et al., 2012), (Pagès et al., 2012).

A different result is obtained when the second objective function is used (figure 6). Since the slots are reallocated when necessary in model 1, more 3-type connections can be allocated in the grid over types 1 and 2, so the throughput is lower than model 2. Another consequence is that the blocking probability is higher for model 1, due to the rejection of $k = 1$ and $k = 2$ connection requests.

One may observe that when the throughput is maximized, connections which use less slots are prefered since each request is counted as one call. Therefore, connections demanding more slots have higher blocking probabilities. When the objective is to maximize slot utilization, however, connections which demands more slots are prefered over the others. This behavior may be seen in the graphs of figure 7, which shows the blocking probabilities for model 2 regarding objectives 1 and 2, respectively.

5. Conclusions

The proposed model aims to solve the spectrum allocation sub-problem in a link of an EON. It also provides an upper bound, which allows the comparison of different

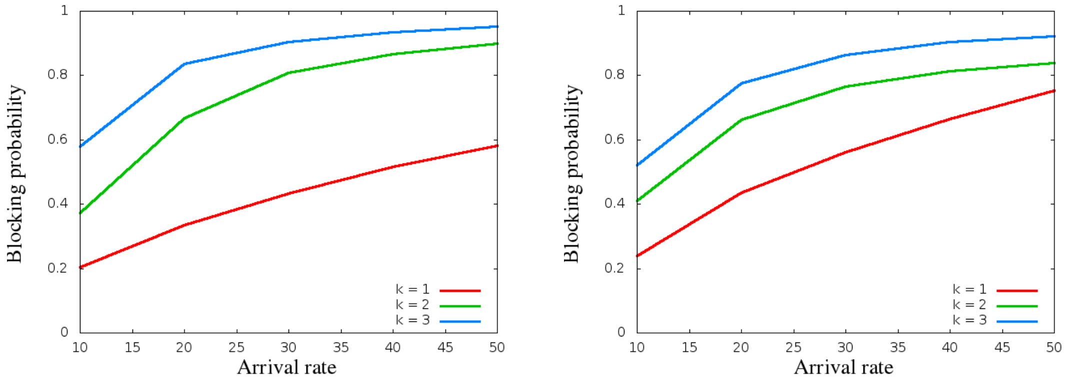

Figure 7. Blocking probabilities as arrival rates increase in model 2

allocation policies with the optimal one. As future work, other objective functions can be implemented, as well as different performance measures. Fragmentation is an important issue when dealing with dynamic traffic scenario, thus it is also an important characteristic that can be added to the model.

As shown in the numerical results, the size of the model grows rapidly as the number of slot increases. Real network topologies, however, operates with a larger grid and may have dozens of links. Consequently, an efficient way to find near-optimal policies for the CTMDP is crucial to its application to real scenarios, and, to do so, approximate methods to solve CTMDPs can be studied and implemented.

Beyond solving a link model, formulations to integrate the policy of each link need to be developed, so the routing sub-problem is also solved for the entire network.

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